SPACE AND TIME COMPARISONS OF PURCHASING POWER PARITIES AND REAL VALUES

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The calculation of purchasing power parities and quantity comparisons for a given year provides interesting information about the relative importance of countries. However, it is necessary to make these estimates annually in order to enable users to apply these parities for international comparison of annual data expressed in national currency. The paper deals with the problems related to merging spatial comparisons and temporal volume and price movements for the countries of the European Community. For these countries full information was collected in 1975 and in 1980, whereas in the intermediate years some price data were collected and price indices at a detailed level have also been collected. First the theoretical problems of consistency between the spatial results and temporal indices are discussed. Because no immediate consistency can be obtained, several methods are proposed to achieve consistency, by estimating one unique set of spatial and temporal indices. The available information for the period 1975–80 has been used in order to test the numerical differences between two sets of parities and price indices over time. Besides theoretical reasons for inconsistency, it is also necessary to take into account errors in the price observations or in the price indices. The results presented in the paper should be considered as provisional and further work will be undertaken to obtain better insights into the inconsistency between these sets of data.

INTRODUCTION

To introduce time as an additional dimension in international comparisons of real product it is necessary to enlarge the concept of transitivity by taking into account "time." In order to justify the conclusions, the objectives behind the calculation of purchasing power parities for comparisons of real products are briefly discussed. At the same time problems will be described which arise from the calculation of expenditure breakdowns for groups of countries when time is introduced. Solutions are given in 1.4.

In section 2 new methodological tools are presented which allow the calculation of spatial and time transitive purchasing power parities. Depending on the data available and the level of aggregation different methods can be applied to derive transitive purchasing power parities.

In section 3 the data available at EUROSTAT are described and some preliminary results of EUROSTAT 1980 purchasing power parity calculations are discussed compared with the 1975 results.

1. ON THE ROLE OF PURCHASING POWER PARITIES IN INTERNATIONAL COMPARISONS OF GROSS DOMESTIC PRODUCT

The calculation of Purchasing Power Parities (PPP) has two closely related objectives:

(1) Values expressed in national currencies can be converted by PPP's into a common currency unit. Ratios of these converted values between countries can

then be interpreted as quantity ratios reflecting differences in quantities (and qualities) of the countries compared ("Quantity Comparisons").

(2) In order to obtain data for groups of countries (such as the European Community) converted values can be aggregated over countries.

1.1. *Quantity Comparisons*

Quantity comparisons of values expressed in different national currencies do not raise serious problems if expenditures for a homogeneous item are to be compared. Ratios of average prices can be used as the appropriate PPP's.

$$\frac{V_{i1}}{V_{i2}} \Big/ \frac{p_{i1}}{p_{i2}} = \frac{q_{i1}}{q_{i2}} = \frac{q_{i1}p_{i1}}{q_{i2}p_{i1}} = \frac{q_{i1}p_{i2}}{q_{i2}p_{i2}} = \frac{q_{i1}p_{iE}}{q_{i2}p_{iE}}$$

V_{ij} = expenditure on item i in country j

p_{ij} = average price of item i (per unit) in j

q_{ij} = quantity (in units) of item i in j

E = any third country or group of countries.

Index number problems arise if analogous quantity comparisons are carried out for basic headings or aggregates of basic headings.

As in the case of a homogeneous item the aim is to isolate the quantity component included in the expenditures in different countries.

(1.1) $$\sum_i V_{ij} \Big/ \sum_i V_{ih} = {}_hP_j \, {}_hQ_j$$

${}_hP_j$ = PPP between base country h and country j

${}_hQ_j$ = Volume ratio between base country h and country j.

Since different formulas can be employed to calculate PPP's, different quantity ratios ${}_hQ_j$ are found. In all quantity ratios, national quantities are evaluated at a common set of prices which are either implicitly or explicitly defined.

If for example the quantity component in the expenditures on food in France and Germany is to be compared, appropriate PPP's could be of the Laspeyres-, Paasche-, Fisher-, Geary-, Van Yzeren-, etc. type. Each formula leads to a different quantity ratio; each formula has its own economic interpretation. On the one hand balanced quantity ratios can be determined (via Fisher-type PPP's), which are based on average prices and average quantities (implicit in PPP's), where averaging of prices and quantities follows the same principles.

On the other hand unbalanced quantity ratios can be determined, which are based on average prices and average quantities, where averaging of prices and quantities follows opposite principles. Problems concerning these issues are discussed elsewhere in detail (Hill (1982), Kravis, Heston, Summers (1983), Faerber (1980), Gerardi (1982)). In this report they are only discussed in so far as they are related to the extrapolation of PPP's.

1.2. *Aggregation of Data for Groups of Countries*

Once national values for a single item or a group of commodities are converted into a common currency they can be aggregated for groups of countries. For this exercise specific PPP's are employed at all levels of aggregation: basic headings, the three, two and one digit level and for final domestic uses and GDP as a whole. However an expenditure breakdown for a group of countries determined in this way lacks, in most cases, any meaningful economic interpretation. As a matter of fact it should be noted that a breakdown of expenditure for a given country is only meaningful in prices and quantities of that same country. If quantities are evaluated at prices of another country, or at some average prices, the derived expenditure structure has no economic meaning for this country as such, since the applied price structure does not fit the country's quantities.

This case corresponds to a similar situation in temporal comparisons when the quantities for a given year are evaluated at base year prices (values at constant prices). The derived expenditure breakdown at constant prices does not correspond to a real expenditure structure (Neubauer (1978)). Moreover, it varies with the base year chosen.

These arguments must be kept in mind when one wants to determine a meaningful expenditure breakdown for a group of countries. While the quantities for a group of countries are given by the sum of quantities over the countries, a corresponding price structure has to be defined. On the one hand the average prices should reflect some sort of average preferences over the countries; on the other hand the resulting expenditure breakdown of the group of countries should not depend on the currency unit into which the national expenditures are converted before they are aggregated over the countries.

Both arguments exclude the application of specific PPP's for the calculation of the expenditure breakdown for a group of countries. This can be illustrated with an example, in which expenditures of two countries, and for two items, are aggregated.

AGGREGATION BY SPECIFIC PPP'S

	Country 1		Country 2	
Item	Expenditures	% of total	Expenditures	% of total
1	2,000	66.7	5,000	50
2	1,000	33.3	5,000	50
Total	3,000	100	10,000	100

The specific PPP's are assumed to be

$$p_{11}/p_{12} = 0.5 \qquad p_{21}/p_{22} = 0.2.$$

If the expenditure of country 2 is converted by specific PPP's into expenditures expressed in the currency of country 1 and vice versa the expenditure breakdowns obtained for the sum of the two countries are as follows:

EXPENDITURE BREAKDOWN IN THE CURRENCY OF COUNTRY 1

Item	Expenditures	in %	Formula
1	4,500	69	$p_{11} \sum_{\alpha} q_{1\alpha}$
2	2,000	31	$p_{21} \sum_{\alpha} q_{2\alpha}$

EXPENDITURE BREAKDOWN IN THE CURRENCY OF COUNTRY 2

Item	Expenditures	in %	Formula
1	9,000	47	$p_{12} \sum_{\alpha} q_{1\alpha}$
2	10,000	53	$p_{22} \sum_{\alpha} q_{2\alpha}$

As can be seen from this example the total expenditure structure of 1 and 2 is not only different but even out of the range of possible expenditure breakdowns obtained from averaging of the original structures. From the formula used for the calculation of total expenditures it can be seen that the application of specific PPP's not only changes the currency unit but also replaces the prices of a country by the prices of that country whose currency was chosen as a unit. These prices correspond neither with the quantities of the country whose currency unit was not chosen, nor with the sum of quantities for both countries. The total expenditure breakdown derived in this way is distorted, and depends on the currency unit chosen.

Expenditure breakdowns which are independent of the currency unit chosen can only be achieved when national expenditures are converted by one set of overall PPP's. The overall PPP's applied should be the same as those used for the quantity comparisons of the totals (i.e. PPP's for GDP) so that at that level results consistent with the original quantity comparisons are obtained.

National expenditure breakdowns calculated in this way remain unchanged. The implicit "international" prices for the total expenditure structure are of Geary's (1958) type, although obviously "international" prices and overall parities can be determined separately.

However if Geary's system is applied, the total expenditure breakdown derived from specific PPP's and from overall PPP's are identical as long as the specific PPP's are expressed as relations between national currency units and the "international" unit. For sub-groups of countries or specific parities expressed as relations of national currencies to another national currency this property of the Geary system does not apply.

So far it has been assumed that national expenditure data and specific PPP's are available at the level of individual products. However specific PPP's and expenditure data are usually available only at the level of basic headings (or detailed categories). The conclusions drawn above concerning the calculation of total expenditure breakdowns need no fundamental revision in this case. The application of specific PPP's at the level of basic headings replaces the original national prices by the prices of the base country (B).

$$_{h}\mathrm{P}_{B}^{m} \sum_{i \in m} p_{ih} q_{ih} = \sum_{i \in m} p_{iB} q_{iB\,B} Q_{h}^{m} \quad \forall h, \forall m.$$

$_{B}Q_{h}^{m}$ = Volume ratio between B and H for the basic heading m. National quantities, however, are now replaced by estimated quantities

$$q_{iB\,B}Q_{h}^{m} \quad \forall h.$$

From these considerations it is possible to draw some important conclusions on the use of PPP's.

1. Specific parities are needed in order to compare quantities (or "real" values) between countries for basic headings or aggregates of basic headings.

2. For the determination of total expenditure breakdowns for a group of countries only one set of overall PPP's should be employed. Otherwise the expenditure breakdowns of the countries are distorted and the total expenditure breakdown of the group depends on the country taken as a base for the specific PPP's.

3. The results can be summarized in two basic matrices (rows: basic headings or aggregates of basic headings, columns: countries)
   (a) Volume ratios (quantity comparisons) between row elements. Ratios are derived by using specific PPP's. The data contained in the columns do not provide any meaningful information for a single country.
   (b) Comparable expenditure breakdowns and a total expenditure breakdown for a group of countries derived by applying one set of overall parities for each country. Elements of rows do not indicate quantity ratios, except for the total (i.e. GDP).

An additional comment should be made on the use of overall parities for conversion of figures in national currency. For commodity flows the choice between specific parities and overall parities is open but for figures not corresponding to commodity flows the use of specific PPPs is not possible. This applies to a great number of aggregates from national accounts and other data outside national accounts. In these cases the use of an overall parity is the most appropriate because it will give comparable values between countries which can be summed up by taking into account the general price level as a conversion factor.

1.3. *Quantity Comparisons in Space and Time*

If specific PPP's are available for each year quantity comparisons can be carried out as described in 1.1. Since time is now added as another dimension, each row of the quantity comparison matrix becomes a matrix itself.

In the following example quantity comparisons are carried out for a given aggregate m. Expenditures on m, expressed in national currencies, are converted by a set of specific PPP's into expenditures expressed in the currency of country 1.

TABLE 1

QUANTITY COMPARISONS IN SPACE AND TIME

Time \ Country	1	2	...	k
1	$\sum_{i\in m} p_{i11}q_{i11}$	$\sum_{i\in m} p_{i11}q_{i11}\,{}^1_1Q^m_2$	...	$\sum_{i\in m} p_{i11}q_{i11}\,{}^1_1Q^m_k$
2	$\sum_{i\in m} p_{i12}q_{i12}$	$\sum_{i\in m} p_{i12}q_{i12}\,{}^2_1Q^m_2$	...	$\sum_{i\in m} p_{i12}q_{i12}\,{}^2_1Q^m_k$
t	$\sum_{i\in m} p_{i1t}Q_{i1t}$	$\sum_{I\in m} p_{i1t}q_{i1t}\,{}^t_1Q^m_2$	...	$\sum_{i\in m} p_{i1t}q_{i1t}\,{}^t_1Q^m_k$

${}^t_hQ^m_j$ = quantity ratio between base country h and country j for period t and aggregate m.
p_{ijt} = average price of item i in j in period t.
q_{ijt} = average of item i in j in period t.

Each element of Table 1 is derived from equation (1.1)

$$ {}^t_k\mathrm{P}^m_1 \sum_{i\in m} p_{ikt}q_{ikt} = {}^t_1Q^m_k \sum_{i\in m} p_{i1t}q_{i1t} \quad \forall k, \forall t, \forall m $$

${}^t_k\mathrm{P}^m_1$ = PPP between base country k and country 1 for aggregate m and period t.

Quantity changes over time can be derived from Table 1; in the case of the base country 1 the original national price and quantity indices can be used.

(1.2) $$ \frac{\sum_{i\in m} p_{i_1t_2}q_{i_1t_2}}{\sum_{i\in m} p_{i_1t_1}q_{i_1t_1}} = {}^1_{t_1}I^m_{t_2} \cdot {}^1_{t_1}M^m_{t_2} \quad \forall m, \forall (t_1, t_2) $$

${}^h_{t_1}I^m_{t_2}$ = original national price index between base period t_1 and period t_2 for country h and aggregate m

${}^h_{t_1}M^m_{t_2}$ = analogous quantity index.

For the other countries quantity indices can be derived through the base country 1,

(1.3) $$ {}^h_{t_1}M^m_{t_2} = {}^{t_1}_hQ^m_1 \cdot {}^1_{t_1}M^m_{t_2} \cdot {}^{t_2}_1Q^m_h \quad \forall h, \forall (t_1, t_2), \forall m $$

since quantity ratios are available in space for each period and between periods for country 1.

However quantity indices or quantity ratios derived in this way will vary if the base country varies, i.e. they are neither base country invariant nor transitive.

Example: Non-transitivity

$$ {}^{t_2}_1Q^m_h(1) = ({}^h_{t_1}M^m_{t_2} / {}^1_{t_1}M^m_{t_2})\,{}^{t_1}_1Q^m_h $$

$$ {}^{t_2}_1Q^m_h(2) = ({}^h_{t_1}M^m_{t_2} / {}_{t_1}M^m_{t_2}) \cdot {}^{t_2}_1Q^m_2 \cdot {}^{t_1}_2Q^m_h $$

$$ {}^{t_2}_1Q^m_h(1) \neq {}^{t_2}_1Q^m_h(2). $$

Base country invariance and transitivity are necessary properties in international comparisons. Obviously, there is no reason to give up transitivity when the time dimension is introduced. The concept of "transitivity" is therefore enlarged into time and is defined by the following equations (parities and index numbers having these properties will be denominated by an additional bar).[1]

(a) Transitivity in space and time

$$ {}^{t_2}_{h}\bar{Q}^{m}_{j} / {}^{t_1}_{h}\bar{Q}^{m}_{j} = {}^{j}_{t_1}\bar{M}^{m}_{t_2} / {}^{h}_{t_1}\bar{M}^{m}_{t_2} \quad \forall (h, j), \forall t. \tag{1.4} $$

(b) Transitivity in space

$$ {}^{t}_{h}\bar{Q}^{m}_{j} / {}^{t}_{h}\bar{Q}^{m}_{z} = {}^{t}_{z}\bar{Q}^{m}_{j} \quad \forall t, \forall (h, j, z). \tag{1.5} $$

(c) Transitivity in time

$$ {}^{j}_{r}\bar{M}^{m}_{s} = {}^{j}_{t_1}\bar{M}^{m}_{s} / {}^{j}_{t_1}\bar{M}^{m}_{r} \quad \forall (r, s), \forall j. \tag{1.6} $$

(d) General transitivity in space and time

$$ {}^{r}_{h}\bar{Q}^{m}_{j} / {}^{s}_{h}\bar{Q}^{m}_{j} = {}^{j}_{s}\bar{M}^{m}_{r} / {}^{h}_{s}\bar{M}^{m}_{r} \quad \forall (r, s), \forall (h, j). \tag{1.7} $$

Since for each period expenditures expressed in the currency of a base country can be aggregated over countries, one has to confirm that a quantity index for the group of countries does not depend on the base country chosen. The quantity index of the group of countries E is defined as the residual factor after dividing the ratio of the total expenditures by the time and space transitive price index ratio of the base country.

$$ \frac{\sum_{i \in m} p_{iBt_2} \sum_{\alpha=1}^{k} q_{iBt_2} \, {}^{t_2}_{B}\bar{Q}^{m}_{\alpha}}{\sum_{i \in m} p_{iBt_1} \sum_{\alpha=1}^{k} q_{iBt_1} \, {}^{t_1}_{B}\bar{Q}^{m}_{\alpha}} \Bigg/ {}^{B}_{t_1}\bar{I}^{m}_{t_2} = {}^{E}_{t_1}\bar{M}^{m}_{t_2} \tag{1.8} $$

k = number of countries.

By dividing (1.8) by ${}^{j}_{t_1}\bar{I}^{m}_{t_2}$ an analogous equation is obtained, in which B is replaced by j.

Quantity comparisons in time and space can be presented in three different types of tables. The first two types are given in Tables 2 and 3.

TABLE 2

SPATIAL COMPARISONS FOR AGGREGATE *m*

Time	Country 1	Country 2	...	Country k	All countries
1	${}^{1}_{E}\bar{Q}^{m}_{1}$	${}^{1}_{E}\bar{Q}^{m}_{2}$	...	${}^{1}_{E}\bar{Q}^{m}_{k}$	100
2	${}^{2}_{E}\bar{Q}^{m}_{1}$	${}^{2}_{E}\bar{Q}^{m}_{2}$	...	${}^{2}_{E}\bar{Q}^{m}_{k}$	100
⋮	⋮	⋮		⋮	⋮
t	${}^{t}_{E}\bar{Q}^{m}_{1}$	${}^{t}_{E}\bar{Q}^{m}_{1}$	...	${}^{t}_{E}\bar{Q}^{m}_{k}$	100

The third type shows spatial and time comparisons (all countries at $t = 1$ equals 100) and can be derived from Tables 2 and 3 by multiplying the elements of Table 2 by the appropriate elements of Table 3.

[1]Equations for PPP's and price indices are analogous and follow from expenditure ratios.

TABLE 3

TIME COMPARISON OF QUANTITY CHANGE IN COUNTRIES (AGGREGATE m)

Time	Country 1	Country 2	...	Country k	All countries
1	100	100	...	100	100
2	${}_1^1\bar{M}_2^m$	${}_1^2\bar{M}_2^m$	...	${}_1^k\bar{M}_2^m$	${}_1^E\bar{M}_2^m$
⋮	⋮	⋮		⋮	⋮
t	${}_1^1\bar{M}_t^m$	${}_1^2\bar{M}_t^m$	...	${}_1^k\bar{M}_t^m$	${}_1^E\bar{M}_t^m$

1.4. *Expenditure Breakdowns for a Group of Countries in Time and the Problem of the Numeraire*

With the method described in 1.2 it is possible to calculate for each period t expenditure breakdowns for a group of countries. For a time comparison of these expenditure breakdowns an overall price index should be applied representing the general price level. The use of specific price indices will change the current expenditure structure. If the expenditures of a group of countries are expressed in the currency of a base country h, the space and time transitive price index ${}_{t_1}^{h}\bar{I}_{t_2}$ is an appropriate measure for the elimination of general price developments.

Difficulties will arise if GDP or total uses are being expressed in a basket currency or an artificial currency unit.

For a given year it is suitable to express the total uses of GDP in an existing "basket" currency as in the case of the European Community the ECU (EUA). The ECU is defined as the sum of certain amounts of each community currency. These amounts were fixed in June 1974 (1 ECU = 1 SDR = 1.20635 US $). The weights of each country were determined by the country's share in the European gross national product and the country's foreign trade. The ECU is fixed at each moment by taking into account the basic basket and the exchange rate of each currency compared to the US $.

Example: ECU AND EUROPEAN CURRENCIES (1/12/78)

	Basic "Basket" (a)	Exchange rate against $ US (b)	Equivalent in $ (c) = (a):(b)	Equivalent in national currency (d) = total $ × (b)
DM	0.828	1.9358	0.4277301	2.51689
FF	1.15	4.4495	0.2584560	5.78516
LIT	109	853	0.1277842	1109.06
Flor.	0.286	2.1035	0.1359638	2.73494
BF	3.66	30.6675	0.1193445	39.8734
Lfr	0.140	30.6675	0.0045650	39.8734
£ UK	0.0885	1.5164	0.1713714	0.671443
£ Irl.	0.00759	1.5164	0.0146972	0.671443
Dkr	0.217	5.3885	0.0402709	7.00604
			1.3001831	

This basket unit can easily be introduced into international comparisons of GDP, by putting GDP of the community expressed in ECU derived through the official exchange rates equal to the total GDP expressed in Purchasing Power Standards. For a given year this is an appropriate numeraire.

However for time to time comparisons difficulties arise since the change in the price level for a group of countries is influenced also by changes in the exchange rates. Although the volume change for a group of countries is defined by (1.8) as a weighted average of the quantity indices of the countries, no similar appropriate price index for the groups of countries can be calculated.

It is possible to solve this problem by introducing an artificial currency unit, consistent with a price index of the group defined in a particular way. In order to keep the volume index ${}^{E}_{t_1}\bar{M}^{m}_{t_2}$ (i.e. $m = \text{GDP}$) constant, the price index for a group of countries must be transitive in time and space.

$$(1.9) \qquad {}^{E}_{t_1}\bar{I}^{m}_{t_2} / {}^{h}_{t_1}\bar{I}^{m}_{t_2} = {}^{t_2}_{h}\bar{\mathrm{P}}^{m}_{E} / {}^{t_1}_{h}\bar{\mathrm{P}}^{m}_{j} \quad \forall h, \forall(t_1, t_2).$$

If the PPP's for one point in time are defined in relation to a Purchasing Power Standard, it is sufficient to define ${}^{E}_{t_1}\bar{I}^{m}_{t_2}$ to derive PPP's at other points in time.

Examples for definitions of overall price indices are:

$$(1.10) \qquad {}^{E}_{t_1}\bar{I}^{m}_{t} = \sum_{\alpha=1}^{k} {}^{\alpha}_{t_1}\bar{I}_{t} \frac{{}^{t_1}_{\alpha}\bar{\mathrm{P}}^{M}_{E}\, {}^{t_1}V^{m}_{\alpha}}{\sum_{\alpha=1}^{k} {}^{t_1}_{\alpha}\bar{\mathrm{P}}_{E}\, {}^{t_1}V^{m}_{\alpha}} \quad \forall(t_1, t)$$

${}^{t}V^{m}_{\alpha}$ = Expenditure on m in country α in t (weighted arithmetic mean of national price indices)

$$(1.11) \qquad {}^{E}_{t_1}I^{m}_{t} = \prod_{\beta=t_1}^{\beta=t-1} \left(\sum_{\alpha=1}^{k} {}^{\alpha}_{\beta}\bar{I}^{m}_{\beta+1} \frac{{}^{\beta}_{\alpha}\bar{\mathrm{P}}^{m}_{E}\, {}^{\beta}V^{m}_{\alpha}}{\sum_{\alpha=1}^{k} {}^{\beta}_{\alpha}\bar{\mathrm{P}}_{E}\, {}^{\beta}V^{m}_{\alpha}} \right) \quad \forall(t_1, t)$$

(chained formulation of (1.10)).

2. Space and Time Consistent Parities and Temporal Indices

The use of purchasing power parities for comparison and aggregation of final domestic uses or GDP for a group of countries necessitates establishing a framework in which space and time are simultaneously involved. In this framework temporal indices and parities should be transitive in time and space. In order to achieve transitivity in time and space new methodological tools for extrapolation and interpolation are necessary. It is assumed that at least one set of PPP's, calculated on the basis of special price surveys, and national price or volume indices are available. According to the quality standard all available information is used for the derivation of PPP's. Before presenting the methodological tools some of the difficulties are described which arise when price indices are related with PPP's.

2.1. *Relationship Between* PPP *Formula and Price Index Formulas*

There is a relationship between the formula used for the calculation of the PPP's and the type of temporal index used for linkage of parities at different points in time. This can easily be shown by means of a simple example taking the case of two countries A and B and two products 1 and 2 at two periods of time t_0 and t_1.

	Country *A*				Country *B*			
	Product 1		Product 2		Product 1		Product 2	
Period	*p*	*q*	*p*	*q*	*p*	*q*	*p*	*q*
t_0	5	3	6	2	35	3	30	4
t_1	6	4	9	1	25	2	15	5

The Laspeyres parity between A and B with A being the base country at t_0 is given by

$$ {}^{t_0}_{A}P_B = {}^{t_0}_{A}LAS_B = \frac{\sum_i p_{iBt_0} q_{iAt_0}}{\sum_i p_{iAt_0} q_{iAt_0}} $$

$$ {}^{t_0}_{A}P_B = 6.11. $$

For t_1 the corresponding Laspeyres-parity is equal to

$$ {}^{t_1}_{A}P_B = {}^{t_1}_{A}LAS_B = 3.49. $$

Comparing these two parities the denominators correspond to the nominal expenditure in country A, while the numerators are given by the current quantities of A evaluated by the current prices of B. A consistent "extrapolation factor" for Laspeyres parities is then given by

$$ \frac{\sum p_{iBt_1} q_{iAt_1}}{\sum p_{iBt_0} q_{iAt_0}} \Big/ {}^{A}_{t_0}V_{t_1}. $$

For the example given above this extrapolation factor equals 0.57. Full information on prices and quantities is necessary for the calculation of such a factor. Another "extrapolation factor" can be derived from Paasche type PPP's.

$$ {}^{t_0}_{A}P_B = {}^{t_0}_{A}PAS_B = \sum p_{iBt_0} q_{iBt_0} / \sum p_{iAt_0} q_{iBt_0} $$

$$ {}^{t_0}_{A}P_B = 5.77 $$

$$ {}^{t_1}_{A}P_B = {}^{t_1}_{A}PAS_B = 2.19. $$

The "extrapolation factor" is given by

$$ {}^{B}_{t_0}V_{t_1} / (\sum p_{iAt_1} q_{iBt_1} / \sum p_{iAt_0} q_{iBt_0}) $$

and equals for the example 0.38. If PPP's are of the Fisher type, parities and extrapolation factors are different again.

$$ {}^{t_0}_{A}P_B = {}^{t_0}_{A}FIS_B = 5.94 $$

$$ {}^{t_1}_{A}P_B = {}^{t_1}_{A}FIS_B = 2.76. $$

The extrapolation factor is 0.46. This simple example shows that different extrapolation factors should be applied when different formulas are used for the calculation of PPP's.

However, full information on quantities and prices is only available in examples. Usually one will only have price indices for the extrapolation and interpolation of parities. From these price indices different extrapolation factors can be derived which are almost identical if the price indices used are of the same type.

$$ {}^{B}_{t_0}LAS_{t_1} / {}^{A}_{t_0}LAS_{t_1} = 0.45 $$

$$ {}^{B}_{t_0}PAS_{t_1} / {}^{A}_{t_0}PAS_{t_1} = 0.448 $$

$$ {}^{B}_{t_0}FIS_{t_1} / {}^{A}_{t_0}FIS_{t_1} = 0.449. $$

In this case both countries obviously have the same influence on the extrapolation of the basic PPP and therefore extrapolation factors of this type fit better in the extrapolation of Fisher-type parities.[2]

It is possible to confirm this assumption with the help of a simulation model in which extrapolations are carried out with Laspeyres-, Paasche-, and Fisher-indices. Here we confine ourselves to the main conclusions.

(1) Extrapolations of Laspeyres- and Paasche-type parities give clearly greater average deviations from newly calculated Laspeyres- and Paasche-parities than Fisher type parities.

(2) Extrapolations of Fisher parities with indices of Laspeyres- or Paasche- or Fisher-type lead to results which are approximately identical since relative indices of the same type are almost identical.

(3) One can expect that the largest absolute deviations are smaller in the case of the extrapolation of Fisher type parities than they are in the case of Laspeyres- and Paasche-parities.

2.2. *Global or Detailed Extrapolation*

It is possible to carry out extrapolations of PPP's on different levels of aggregation; however, the outcome will not be the same as can be shown by means of a simple example.

Suppose that in t_0 the parity between countries h and j is of the Fisher type

$$ {}^{t_0}_{h}P_j = {}^{t_0}_{h}FIS_j $$

and that this parity is extrapolated on the basis of price indices for detailed categories for the period t given by

$$ {}^{j}_{t_0}I^{i}_{t} = \text{price index of detailed category } i \text{ for country } j \text{ from } t_0 \text{ to } t. $$

In this way a new Fisher type parity is obtained in period t

$$ {}^{t}_{h}P_j = \left(\left(\sum_i \frac{p_{ijt_0}}{p_{iht_0}} \frac{{}^{j}_{t_0}I^{i}_{t}}{{}^{h}_{t_0}I^{i}_{t}} \frac{p_{iht}q_{iht}}{\sum_i p_{iht}q_{iht}} \right) \left(\sum_i \frac{p_{iht_0}}{p_{ijt_0}} \frac{{}^{h}_{t_0}I^{i}_{t}}{{}^{j}_{t_0}I^{i}_{t}} \frac{p_{ijt}q_{ijt}}{\sum_i p_{ijt}q_{ijt}} \right)^{-1} \right)^{1/2}. $$

[2] In the case of Fisher parities both countries have the same weight in the determination of average prices or quantities.

However, extrapolation of the overall parity between country h and j using an overall price index will give

$$ {}^{t}_{h}P_j = {}^{t_0}_{h}FIS_j({}^{j}_{t_0}I_t / {}^{h}_{t_0}I_t) $$

where the base year parity is of a Fisher type.

Obviously the results are different since the overall index is not defined in the same way as the implicit index which follows from the detailed extrapolation. So the two formulas will give different results because weights are not the same in the two cases. *The detailed extrapolation is based on an implicit overall price index which differs from the national price index because weights used are bi- or multilateral weights and not pure national weights.*

It would be desirable if PPP's were extrapolated by indices on a very detailed level. In this way inconsistencies between the fomulas used for price indices and PPP's can be reduced to the level of detailed categories. Problems arising from different methods of aggregation of PPP's and different weighting conceptions for indices are avoided beyond the level of the detailed categories (or even below).

Moreover detailed extrapolation allows introduction of new information on weights or newly calculated detailed PPP's into the calculation procedure. It becomes possible to examine structural changes at the level of detailed categories and the quality of the price indices used, their coverage etc. Questions of this type cannot be solved by global extrapolation.

The same arguments are valid if global extrapolation is carried out by overall Paasche price indices:

$$ {}^{t}_{h}P_j = {}^{t_0}_{h}P_j \frac{{}^{j}_{t_0}PAS_t}{{}^{h}_{t_0}PAS_t} \quad \forall(h,j), \forall t. $$

In this case national values at constant prices of a base year are converted into real values by the PPP's of the base year.

$$ \begin{aligned} {}^{t}_{h}Q_j &= {}^{t}_{h}V_j / {}^{t}_{h}P_j \\ &= \frac{\sum_i p_{ijt_0} q_{ijt}}{\sum_i p_{iht_0} q_{iht}} \Big/ {}^{t_0}_{h}P_j \quad \forall(h,j), \forall t, \end{aligned} $$

or

$$ {}^{t}_{h}Q_j = {}^{t_0}_{h}Q_j \cdot \frac{{}^{j}_{t_0}LAS^Q_t}{{}^{h}_{t_0}LAS^Q_t} \quad \forall(h,j), \forall t $$

${}^{j}_{t_0}LAS^Q_t$ = Laspeyres quantity index for country j between t_1 and t_0.

2.3. *Consistency Problems between National Price Indices and Parities*

National price indices as well as parities are calculated on the basis of prices. However, the transformation of these prices into price indices or parities will create a consistency problem between them: parities calculated at different periods should also reflect the price change over time.

Even in the case of two countries and supposing that all indices and parities are of the same type (Fisher) it will not be possible to obtain consistency between extrapolated parities and newly calculated parities.

Consistent results can only be obtained if price indices and parities are determined simultaneously. In the next section such models will be discussed in detail.

Besides these consistency problems related to the formulas used, there are other reasons for inconsistency between price indices and parities which are due to the fact that the purpose of price indices or the parities are not the same.

Parities measure price differences in space whereas price indices measure the price change in a given geographical unit (a country). Product selection, weights, and index formulas are determined by these difference in purposes.

Moreover, even for the measurement of price change the countries of the EEC do apply different concepts in the field of formulas, the introduction of quality changes, treatment of seasonal price variations, selection of products, collection of price quotations, etc.

But there is no doubt that differences between countries are much greater than temporal differences for a single country if the time period is not too long. The product selection and definition for space comparison should take into account very great structural differences, and one fixed basket for the calculation of a "pure" Laspeyres index will not be possible. For this reason the baskets used for space comparisons and baskets used for temporal price indices are not at all the same.

Conclusions

From the foregoing considerations it can be concluded that there is a lack of consistency

—between national indices and parities,

—between detailed extrapolated parities and globally extrapolated parities,

—between a product selection for price indices and for parity calculations.

However, these inconsistencies should not be considered as negative because it is possible to find a link between them. The differences are justified by the fact that the objectives are not the same for price indices and parities.

Both calculations do provide information about price change and price levels in relation with other countries. Each calculation gives specific information, but leaves out other valuable information. The space comparison does not consider any quality changes or changes in the product selection over time. Price indices do not allow any conclusions on level comparisons. While in price indices it is difficult to introduce weighting schemes for all points in time, this is not a problem for PPP's. Therefore price changes measured via PPP's have a different interpretation from national price indices.

The direction of the described systematic differences cannot be forecast. Moreover they are overruled by random errors, since different price observations are employed for the calculation of parities and price indices.

In practice it will not be possible to calculate each year or each month PPP's for all categories of GDP on the basis of new price observations. But for the use of PPP's in practical work it is necessary to provide parities at least on a yearly basis. In order to do so, national price indices have to be used and the consistency problems described above have to be solved by new methodological tools. Depending on the quality standard of parities and indices consistent indices and parities are found

(a) by the derivation of indices from parities, or

(b) by the extrapolation and interpolation of parities with indices, or

(c) by "smoothing" both parities and indices.

Although "smoothing" in a statistical sense cannot be applied here (since there are systematic differences), it is helpful to calculate consistent parities and indices in a way which minimizes an overall distance between the original parities and indices and the consistent ones. In order to do so, in the following section, the EKS distance measure is minimized (for details see Faerber (1980)).

2.4. *Methods of Calculating Space and Time Transitive Parities and Indices*

If time is added as another dimension in international comparisons of real product it becomes necessary to enlarge transitivity and base country invariance into time (transitivity in space and time). However, as has been explained in the previous section, national index numbers and parities are not by themselves transitive in this sense. Furthermore, temporal index numbers cannot be neglected since results of price surveys are only available at long intervals while PPP's should be available at least on an annual basis. Price index numbers are indeed the only information concerning the price change between two price surveys. Since parities and indices should be transitive in space and time methodological tools are chosen for this purpose which fulfill the desired properties.

There are essentially three methods of calculating space and time transitive parities and indices:

(1) The "observed" parities remain unchanged; in this case time indices are fixed in such a way that the parity change is "explained".

(2) The time indices remain unchanged; parities of a base year are extrapolated with these indices.

(3) Parities and indices are simultaneously adapted.

It is difficult to give general rules for application of one of the three methods because it will depend on the actual situation. The choice depends to a great extent on the estimated quality of indices and parities. It may even be useful to combine different methods to derive suitable parities.

Let us examine the three methods in some detail and discuss chained applications.

(1) Calculation of Time Indices on the Basis of Parities

The change of parities over time will be due to the relative change in price level in the countries concerned. Let us assume that the transitive parities "observed" in period t_0 and t_1 are kept unchanged. It will be clear that the change in the parity from t_0 to t_1 will not correspond to the price change over time of the countries. In order to determine space and time transitive parities and indices it will be necessary to reestimate the indices.

The equation

$$\frac{{}^{t_1}_{h}P_j}{{}^{t_0}_{h}P_j} = \frac{{}^{j}_{t_0}\bar{I}_{t_1}}{{}^{h}_{t_0}\bar{I}_{t_1}} \quad \forall(h, j), \forall(t_0, t_1)$$

does not allow determination of transitive price indices and an additional condition will be necessary. A possible solution is to determine the new transitive price indices in such a way that the deviations from the original national indices are minimized. As the indices are relative indices, the EKS distance measure is very appropriate for this purpose.

Equation:

$$D = \sum_{\alpha} \left(\log \frac{{}^{\alpha}_{t_0}\bar{I}^*_t}{{}^{\alpha}_{t_0}I_t} \right)^2 \rightarrow \text{minimum} \tag{2.1}$$

${}^{\alpha}_{t_0}I_t$ are the original price indices of a country, ${}^{\alpha}_{t_0}\bar{I}^*_t$ are the new price indices of the country.

By introducing the consistency property between parities and indices it is possible to write for a given country B the minimization of

$$D_B = \sum_{\alpha} \left(\log {}^{B}_{t_0}\bar{I}^*_t \Big/ \frac{{}^{t}_{\alpha}P_B}{{}^{t_0}_{\alpha}P_B} {}^{\alpha}_{t_0}I_t \right)^2 \rightarrow \text{minimum} \tag{2.2}$$

and so the new transitive price index for country B becomes:

$${}^{B}_{t_0}\bar{I}^*_t = \left(\prod_{\alpha=1}^{k} ({}^{t}_{\alpha}P_B / {}^{t_0}_{\alpha}P_B) {}^{\alpha}_{t_0}I_t \right)^{1/k} \quad \forall(t_0, t) \tag{2.3}$$

for other countries j:

$${}^{j}_{t_0}\bar{I}^*_t = \left(\prod_{\alpha=1}^{k} ({}^{t}_{\alpha}P_j / {}^{t_0}_{\alpha}P_j) {}^{\alpha}_{t_0}I_t \right)^{1/k} \quad \forall j, \forall(t_0, t). \tag{2.4}$$

It is not necessary to minimize the distance functions over the other countries because in the distances D_B for different countries B the requested variable ${}^{B}_{t_0}\bar{I}^*_t$ does not coincide with other transitive indices; in addition the direct derivation of ${}^{j}_{t_0}\bar{I}^*_t$ $\forall_j$ will give the same result as the indirect calculation (if one index is determined ${}^{B}_{t_0}\bar{I}^*_t$ the transitive condition will then determine all other indices).

It can be easily shown that the original parities and the derived indices are transitive in space and time.

There is no need to minimize the distance measure D over all available points in time. It is also possible to confine the minimization procedure to two points in time and to chain the resulting index numbers. Transitivity in space and time is guaranteed by the chaining procedure

$${}^{j}_{s}\bar{I}^*_{s+1} = \left(\prod_{\alpha=1}^{k} ({}^{s+1}_{\alpha}P_j / {}^{s}_{\alpha}P_j) {}^{\alpha}_{s}I_{s+1} \right)^{1/k} \quad \forall s, \forall j \tag{2.5}$$

$${}^{j}_{s}\bar{I}^*_r = \prod_{\beta=t_0}^{r-1} {}^{j}_{\beta}\bar{I}_{\beta+1} \Big/ \prod_{\beta=t_0}^{s-1} {}^{j}_{\beta}\bar{I}_{\beta+1} \quad \forall(s, r), \forall j. \tag{2.6}$$

The application of this method in which price indices are adapted gives a high priority to the parities calculated from survey data. It should be used if given parities are fixed and consistent index numbers are to be determined.

(2) Indices Remain Unchanged, Parities are Estimated

The second set of models covers different possible situations which will be described successively. Three different cases are distinguished:

- Parities of a benchmark year are extrapolated with the help of indices.
- Interpolation of parities with indices.
- Interpolation under assumptions concerning relative indices or other parameters.

(a) *Extrapolation of parities.* In this case a set of parities is extrapolated by indices.

(2.7) $$ {}^{t}_{h}P^{*}_{j} = {}^{t_0}_{h}P^{*}_{j} \frac{{}^{j}_{t_0}I_t}{{}^{h}_{t_0}I_t} \quad \forall (h,j), \forall t. $$

This is the simplest method of extrapolation. Transitivity in space and time is automatically guaranteed without further assumptions.

(b) *Interpolation between two benchmark years.* Parities are observed in different years and for the intermediate periods parities are estimated. The interpolation procedure is carried out on the basis of the available national price indices.

An appropriate interpolation formula is given by the following nonlinear equation

(2.8) $$ {}^{t}_{h}P_{j} = \left({}^{t_0}_{h}P_{j} \frac{{}^{j}_{t_0}I_t}{{}^{h}_{t_0}I_t} \right)^{(t_1-t)/(t_1-t_0)} \left({}^{t_1}_{h}P_{j} \frac{{}^{j}_{t_1}I_t}{{}^{h}_{t_1}I_t} \right)^{(t-t_0)/(t_1-t_0)} \quad \forall (h,j), \forall t, t_0 \leq t \leq t_1. $$

In this model if $t = t_0$ and $t = t_1$ the observed parities ${}^{t_0}_{h}P_j$ and ${}^{t_1}_{h}P_j$ are obtained. The influence of these parities depends on the distance from the considered period t.

If the parities ${}^{t_0}_{h}P_j$, ${}^{t}_{h}P_j$ and ${}^{t_1}_{h}P_j$ are transitive and the indices ${}^{j}_{t_0}I_t$, ${}^{h}_{t_0}I_t$ and ${}^{j}_{t_1}I_t$, ${}^{h}_{t_1}I_t$ are also transitive in time and space, the parities ${}^{t}_{h}P_j$ do not change when this interpolation formula is used. Relative indices can be replaced by parities in this case.

Example: Interpolation of parities

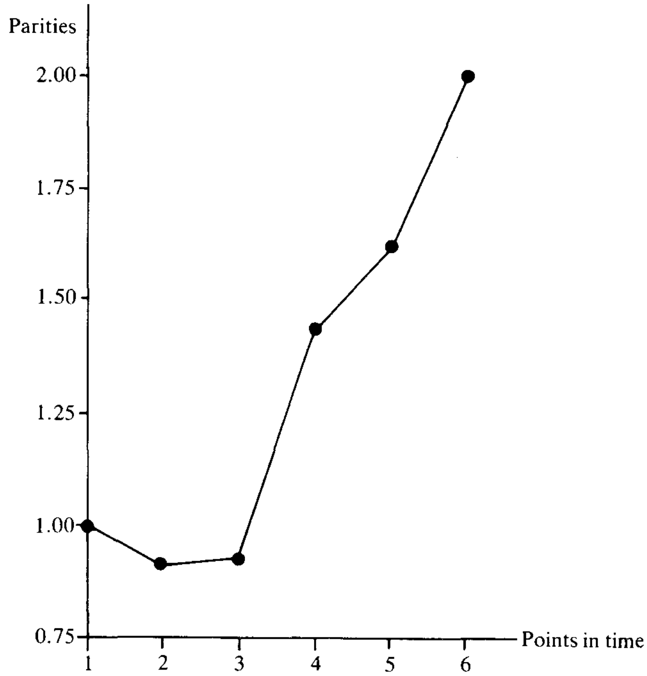
PPP at t_0 ${}^{t_0}_{h}P_j = 1$

PPP at t_5 ${}^{t_5}_{h}P_j = 2$

National price indices

	t_0	t_1	t_2	t_3	t_4	t_5
h	100	110	120	130	140	150
j	100	100	110	180	220	280

The estimated development of the parities can be taken from the following diagram.



(c) *Interpolations under conditions.* If price index numbers are not available or are not reliable, then assumptions concerning relative indices can be introduced in the interpolation formula given above. As an example it is possible to assume that price changes in one country are x times quicker than in a second country. Introducing this assumption into the interpolation formula given above leads to

$$ {}^{t}_{h}P_j = \left({}^{t_0}_{h}P_j x^{t-t_0}\right)^{(t_1-t)/(t_1-t_0)} \left({}^{t_1}_{h}P_j \left(\frac{1}{x}\right)^{t_1-t} \right)^{(t-t_0)/(t_1-t_0)} $$

$$ {}^{t}_{h}P_j = {}^{t_0}_{h}P_j^{(t_1-t)/(t_1-t_0)} \; {}^{t_1}_{h}P_j^{(t-t_0)/(t_1-t_0)} $$

$$ {}^{t}_{h}P_j = {}^{t_0}_{h}P_j^{(t_1-t_0+t_0-t)/(t_1-t_0)} \; {}^{t_1}_{h}P_j^{(t-t_0)/(t_1-t_0)} $$

(2.9) $$ {}^{t}_{h}P_j = {}^{t_0}_{h}P_j \left(\frac{{}^{t_1}_{h}P_j}{{}^{t_0}_{h}P_j} \right)^{(t-t_0)/(t_1-t_0)} \qquad \forall(h, j), \forall(t_1, t_0), \forall t. $$

Other assumptions concerning the relative price change can be substituted in this model.

(3) Adaption of Parities and Indices

In many practical situations observed parities and national price indices will be given the same priority for the calculation of the change in the price levels between countries.

It will be useful under these circumstances to use both types of information, the national price indices as well as the parities.

In the next paragraph we will develop three smoothing systems:

(a) Forward smoothing system

(b) Backward smoothing system

(c) General smoothing system.

In the three systems parities are assumed to be transitive in space, and indices are assumed to be transitive in time.

(a) *Forward smoothing system.*[3] With this system the parity between countries h and j is fixed for the base year. For all other years t the parities are defined as the geometric average of the space transitive parities observed in year t and the space transitive parities at year t extrapolated by means of price indices and parities of year t_0.

$$(2.10)\qquad \begin{aligned} {}^{t_0}_{h}\bar{P}_j &= {}^{t_0}_{h}P_j \quad \forall(h,j) \\ {}^{t_1}_{h}\bar{P}_j &= \left({}^{t}_{h}P_j\,{}^{t_0}_{h}P_j\,\frac{{}^{j}_{t_0}I_t}{{}^{h}_{t_0}I_t}\right)^{1/2} \quad \forall(h,j), \forall t \end{aligned}$$

${}^{t_0}_{h}P_j$, ${}^{t}_{h}P_j$ = transitive parity between countries j and h for t_0 and t

${}^{j}_{t_0}I_t$ = transitive price index between t_0 and t for country j

${}^{t_0}_{h}\bar{P}_j$, ${}^{t}_{h}\bar{P}_j$ = are space and time transitive parities between h and j for year t_0 and t.

The corresponding space and time transitive indices ${}^{j}_{t_0}\bar{I}_t$ are obtained as indices between the two parities ${}^{t_0}_{h}\bar{P}_j$ and ${}^{t_1}_{h}\bar{P}_j$.

$$(2.11)\qquad {}^{j}_{t_0}\bar{I}_t = \left(\prod_{\alpha=1}^{k}\frac{{}^{t}_{\alpha}P_j}{{}^{t_0}_{\alpha}P_j}\,{}^{j}_{t_0}I_t\,{}^{\alpha}_{t_0}I_t\right)^{1/2k} \quad \forall j, \forall t.$$

(b) *Backward smoothing system.* This system is just the opposite of the first one. Space and time transitive parities of year t are kept unchanged, and all the others are calculated as the geometric average of the space transitive parities observed in year t and the space and time transitive parities at year t extrapolated by means of price indices and parities of t_1.

$$(2.12)\qquad \begin{aligned} {}^{t}_{h}\bar{P}_j &= \left[{}^{t}_{h}P_j\cdot{}^{t_1}_{h}P_j\cdot\frac{{}^{j}_{t_1}I_t}{{}^{h}_{t_1}I_t}\right]^{1/2} \quad \forall(h,j), \forall t \\ {}^{t_1}_{h}\bar{P}_j &= {}^{t_1}_{h}P_j \quad \forall(h,j), \forall t_1. \end{aligned}$$

The price indices corresponding with the estimated parities are given by:

$$(2.13)\qquad {}^{j}_{t_1}\bar{I}_t = \left[\prod_{\alpha=1}^{k}\frac{{}^{t}_{\alpha}P_j}{{}^{t_1}_{\alpha}P_j}\cdot{}^{j}_{t_1}I_t\,{}^{\alpha}_{t_1}I_t\right]^{1/2k} \quad \forall(t_1, t), \forall j.$$

Space and time transitive parities for t are recalculated each time a new "observed" parity becomes available. The most recently observed parities of year t_1 are

[3]See for this system Gerardi (1978).

assumed to be the best estimates. The corresponding price indices are derived from the estimated parities.

(c) *General smoothing system.* The third system allows for a general revision of parities and price indices instead of fixing parities of a given year. If the first system is of a Laspeyres type and the second of a Paasche type, this more general model could be called a Fisher type system. This provides the basic idea of the third method: parities for each year t result from the geometric average of transitive parities obtained in system I and II. The space and time transitive indices do not change. This model can be written as

$$(2.14) \qquad {}^{t}_{h}\bar{P}_j = \left[\left({}^{t}_{h}P_j \cdot {}^{t_0}_{h}P_j \cdot \frac{{}^{j}_{t_0}I_t}{{}^{h}_{t_0}I_t} \right)^{1/2} \left({}^{t}_{h}P_j \cdot {}^{t_1}_{h}P_j \cdot \frac{{}^{j}_{t_1}I_t}{{}^{h}_{t_1}I_t} \right)^{1/2} \right]^{1/2} \quad \forall t, \forall (h,j)$$

For the parity t and for the indices for country j

$$(2.15) \qquad {}^{j}_{t_0}\bar{I}_t = \left[\prod_{\alpha=1}^{k} \frac{{}^{t}_{\alpha}P_j}{{}^{t_0}_{\alpha}P_j} \cdot {}^{j}_{t_0}I_t \, {}^{\alpha}_{t_0}I_t \right]^{1/2k} \quad \forall j, \forall t.$$

(d) *Comparison of the three systems.* In table 4 space and time transitive parities resulting from the three systems can be compared.

(4) Chained Smoothing Systems for Parities and Indices

The systems explained in the previous paragraph take the base year at t_0 or t_1 or the average of the two as a basis of the annual estimation of the parities and time indices.

It is possible that between t_0 and t_1 the distance is very great, i.e. the benchmark surveys are only carried out in very long time intervals. This means that estimated parities are influenced by parities observed in years which are very far away, and this seems to be undesirable. It is possible to improve this situation by introducing a system of chain indices in which the transitive parities of a given year are mainly influenced by parities of very close years by a regular shift of the base year t_0 or t_1 or both.

It is possible to introduce in the three smoothing systems the idea of chain indices not only for time indices but also for parities. Transitivity in space and time is not distorted by a chained application of the three systems (see Faerber (1980)).

(a) *Chained forward smoothing system.* Let us examine how this system can be written as a chained system of the Laspeyres type. Let

${}^{t}_{h}\tilde{P}_j$ = transitive parity between h and j for year t obtained from a chained system

${}^{j}_{s}\tilde{I}_r$ = transitive index for country j between years s and r calculated as a chain index

t_0 = base year

β = points in time.

TABLE 4

RUNS OF THE THREE SMOOTHING SYSTEMS

A \ B	t_0	t_1	t_2
		System I	
t_0	${}^{t_0}_{h}P_j$	—	—
t_1	${}^{t_0}_{h}P_j$	$\left[{}^{1}_{h}P_j \cdot {}^{t_0}_{h}P_j \cdot \frac{{}^{j}_{t_0}I_1}{{}^{h}_{t_0}I_1}\right]^{1/2}$	—
t_2	${}^{t_0}_{h}P_j$	$\left[{}^{1}_{h}P_j \cdot {}^{t_0}_{h}P_j \cdot \frac{{}^{j}_{t_0}I_1}{{}^{h}_{t_0}I_1}\right]^{1/2}$	$\left[{}^{2}_{h}P_j \cdot {}^{t_0}_{h}P_j \cdot \frac{{}^{j}_{t_0}I_2}{{}^{h}_{t_0}I_2}\right]^{1/2}$
		System II	
t_0	${}^{t_0}_{h}P_j$	—	—
t_1	$\left[{}^{t_0}_{h}P_j \cdot {}^{1}_{h}P_j \cdot \frac{{}^{j}_{1}I_{t_0}}{{}^{h}_{1}I_{t_0}}\right]^{1/2}$	${}^{1}_{h}P_j$	—
t_2	$\left[{}^{t_0}_{h}P_j \cdot {}^{2}_{h}P_j \cdot \frac{{}^{j}_{2}I_{t_0}}{{}^{h}_{2}I_{t_0}}\right]^{1/2}$	$\left[{}^{1}_{h}P_j \cdot {}^{2}_{h}P_j \cdot \frac{{}^{j}_{2}I_1}{{}^{h}_{2}I_1}\right]^{1/2}$	${}^{2}_{h}P_j$
		System III	
t_0	${}^{t_0}_{h}P_j$	—	—
t_1	$\left[{}^{t_0}_{h}P_j \cdot \left({}^{t_0}_{h}P_j \cdot {}^{1}_{h}P_j \cdot \frac{{}^{j}_{1}I_{t_0}}{{}^{h}_{1}I_{t_0}}\right)^{1/2}\right]^{1/2}$	$\left[{}^{1}_{h}P_j \cdot \left({}^{t_0}_{h}P_j \cdot {}^{1}_{h}P_j \cdot \frac{{}^{j}_{t_0}I_1}{{}^{h}_{t_0}I_1}\right)^{1/2}\right]^{1/2}$	—
t_2	$\left[{}^{t_0}_{h}P_j \cdot \left({}^{t_0}_{h}P_i \cdot {}^{2}_{h}P_j \cdot \frac{{}^{j}_{2}I_{t_0}}{{}^{h}_{2}I_{t_0}}\right)^{1/2}\right]^{1/2}$	$\left[\left({}^{1}_{h}P_i\, {}^{t_0}_{h}P_j \frac{{}^{j}_{t_0}I_1}{{}^{h}_{t_0}I_1}\right)^{1/2} \cdot \left({}^{1}_{h}P_j\, {}^{2}_{h}P_j \frac{{}^{j}_{2}I_1}{{}^{h}_{2}I_1}\right)^{1/2}\right]^{1/2}$	$\left[{}^{2}_{h}P_j \cdot \left({}^{2}_{h}P_j \cdot {}^{t_0}_{h}P_j \cdot \frac{{}^{j}_{t_0}I_2}{{}^{h}_{t_0}I_2}\right)^{1/2}\right]^{1/2}$

A: space and time transitive parities of t_0, t_1, t_2.
B: PPP's from price survey at t_0, t_1, t_2.

For the base year t_0 the original parities are not changed

$${}^{t_0}_{h}\tilde{P}_j = {}^{t_0}_{h}\bar{P}_j = {}^{t_0}_{h}P_j \quad \forall(h,j).$$

For all other years a recursive definition of parities is necessary.

$$(2.16) \qquad {}^{t}_{h}\tilde{P}_j = \left[{}^{t-1}_{h}\tilde{P}_j \cdot \frac{{}^{j}_{t-1}I_t}{{}^{h}_{t-1}I_t} \cdot {}^{t}_{h}P_j\right]^{1/2} \quad \forall(h,j), \forall t.$$

Indices which are consistent with these parities are given by:

$$(2.17)\qquad {}^{j}_{s}\tilde{I}_{r} = \prod_{\beta=t_0}^{r-1} {}^{j}_{\beta}\tilde{I}_{\beta+1} \Big/ \prod_{\beta=t_0}^{s-1} {}^{j}_{\beta}\tilde{I}_{\beta+1} \quad \forall(s, r), \forall j$$

$$(2.18)\qquad {}^{j}_{s}\tilde{I}_{s+1} = \left[\prod_{\alpha=1}^{k} \frac{{}^{s+1}_{\alpha}P_j}{{}^{s}_{\alpha}\tilde{P}_j} \cdot {}^{\alpha}_{s}I_{s+1} \cdot {}^{j}_{s}I_{s+1} \right]^{1/2} \quad \forall j, \forall s.$$

In Table 5 the run of this chained system is shown for three points in time. The entries are directly comparable with the entries in Table 4.

TABLE 5

A \ B	t_0	t_1	t_2
t_0	${}^{t_0}_{h}P_j$	—	—
t_1	${}^{t_0}_{h}P_j$	$\left[{}^{t_0}_{h}P_j \cdot \frac{{}^{j}_{t_0}I_1}{{}^{h}_{t_0}I_1} \cdot {}^{1}_{h}P_j \right]^{1/2}$	—
t_2	${}^{t_0}_{h}P_j$	$\left[{}^{t_0}_{h}P_j \cdot \frac{{}^{j}_{t_0}I_1}{{}^{h}_{t_0}I_1} \cdot {}^{1}_{h}P_j \right]^{1/2}$	$\left[\left({}^{t_0}_{h}P_j \cdot \frac{{}^{j}_{t_0}I_1}{{}^{h}_{t_0}I_1} \cdot {}^{1}_{h}P_j \right)^{1/2} \cdot \frac{{}^{j}_{1}I_2}{{}^{h}_{1}I_2} \cdot {}^{2}_{h}P_j \right]^{1/2}$

It is obvious that the impact of the base year parities decreases geometrically if parities of other years are entered into the system.

In this chained system, as was the case in the original forward smoothing system, the published results do not need any revision because of new information. All the results respect the conditions of space and time transitivity. This can easily be seen if we start from a year with transitive parities, then the index extrapolation provides an estimate of parities for t_1, which are then used as new estimates in order to extrapolate to year t_2, etc.

(b) *Chained backward smoothing system.* In the same way chaining was introduced into the forward smoothing system the backward smoothing system can be reformulated. The parities of the last benchmark year will be kept constant (if they are reliable) and chaining will work backwards.

$${}^{t_1}_{h}\tilde{P}_j = {}^{t_1}_{h}\bar{P}_j = {}^{t_1}_{h}P_j \quad \forall(h, j), \forall t_1$$

$$(2.19)\qquad {}^{t}_{h}\tilde{P}_j = \left[{}^{t}_{h}P_j \cdot {}^{t+1}_{h}\tilde{P}_j \cdot \frac{{}^{j}_{t+1}I_t}{{}^{h}_{t+1}I_t} \right]^{1/2} \quad \forall(h, j), \forall t$$

$$(2.20)\qquad {}^{j}_{s}\tilde{I}_{r} = \prod_{\beta=r}^{t_1-1} {}^{j}_{\beta+1}\tilde{I}_{\beta} \Big/ \prod_{\beta=s}^{t_1-1} {}^{j}_{\beta+1}\tilde{I}_{\beta} \quad \forall j, \forall s$$

$$(2.21)\qquad {}^{j}_{s+1}\tilde{I}_{s} = \left[\prod_{\alpha=1}^{k} \frac{{}^{s}_{\alpha}P_j}{{}^{s+1}_{\alpha}\tilde{P}_j} \cdot {}^{\alpha}_{s+1}I_s \cdot {}^{j}_{s+1}I_s \right]^{1/2k} \quad \forall j, \forall s.$$

For this case it is possible to establish a similar table presenting the development of parities over time and the corresponding indices. Three years are distinguished t_0, t_1 and t_2. In this system the newest parities are introduced as such, all other observed parities are chained and it is clear that the impact of old observed parities on the more recent results will decrease.

TABLE 6

A \ B	t_0	1	2
t_0	${}^{t_0}_{h}P_j$	—	—
1	$\left[{}^{1}_{h}P_j \cdot \frac{{}^{j}_{1}I_{t_0}}{{}^{h}_{1}I_{t_0}} \cdot {}^{t_0}_{h}P_j\right]^{1/2}$	${}^{1}_{h}P_j$	—
2	$\left[{}^{t_0}_{h}P_j \cdot \frac{{}^{j}_{1}I_{t_0}}{{}^{h}_{1}I_{t_0}} \cdot \left({}^{2}_{h}P_j \cdot \frac{{}^{j}_{2}I_1}{{}^{h}_{2}I_1} \cdot {}^{1}_{h}P_j\right)^{1/2}\right]^{1/2}$	$\left[{}^{2}_{h}P_j \cdot \frac{{}^{j}_{2}I_1}{{}^{h}_{2}I_1} \cdot {}^{1}_{h}P_j\right]^{1/2}$	${}^{2}_{h}P_j$

(c) *Chained application of the general smoothing system.* This third system (*III*) is simply the geometric average of the forward smoothing (*I*) and backward smoothing (*II*) system. The model can be written as follows (parities and indices of the three systems are denoted by *I*, *II*, or *III*):

$$(2.22)\quad {}^{t_0}_{h}\tilde{P}^{III}_j = \left[{}^{t_0}_{h}P_j \cdot \left({}^{t_0}_{h}P_j \cdot {}^{1}_{h}\tilde{P}^{II}_j \cdot \frac{{}^{j}_{1}I_{t_0}}{{}^{h}_{1}I_{t_0}}\right)^{1/2}\right]^{1/2} \quad \forall(h,j)$$

$$(2.23)\quad {}^{t_1}_{h}\tilde{P}^{III}_j = \left[{}^{t_1}_{h}P_j \cdot \left({}^{t_1}_{h}P_j \cdot {}^{t_1-1}_{h}\tilde{P}^{I}_j \cdot \frac{{}^{j}_{t_1-1}I_{t_1}}{{}^{h}_{t_1-1}I_{t_1}}\right)^{1/2}\right]^{1/2} \quad \forall(h,j), \forall t_1$$

$$(2.24)\quad {}^{t}_{h}\tilde{P}^{III}_j = ({}^{t}_{h}\tilde{P}^{I}_j \cdot {}^{t}_{h}\tilde{P}^{II}_j)^{1/2} \quad \forall(h,j), \forall t$$

$$(2.25)\quad {}^{t}_{h}\tilde{P}^{III}_j = \left[\left({}^{t-1}_{h}\tilde{P}^{I}_j \cdot \frac{{}^{j}_{t-1}I_t}{{}^{h}_{t-1}I_t} \cdot {}^{t}_{h}P_j\right)^{1/2} \cdot \left({}^{t+1}_{h}\tilde{P}^{II}_j \cdot \frac{{}^{j}_{t+1}I_t}{{}^{h}_{t+1}I_t} \cdot {}^{t}_{h}P_j\right)^{1/2}\right]^{1/2}$$

$$(2.26)\quad {}^{j}_{s}\tilde{I}^{III}_r = ({}^{j}_{s}\tilde{I}^{I}_r \cdot {}^{j}_{s}\tilde{I}^{II}_r)^{1/2} \quad \forall j, \forall(s,r)$$

$$(2.27)\quad {}^{j}_{s}\tilde{I}^{III}_r = \left[\left(\prod_{\beta=t_0}^{r-1} {}^{j}_{\beta}\tilde{I}^{I}_{\beta+1}\right)\left(\prod_{\beta=r}^{t_1-1} {}^{j}_{\beta+1}\tilde{I}^{II}_{\beta}\right) \Big/ \left(\prod_{\beta=t_0}^{s-1} {}^{j}_{\beta}\tilde{I}^{I}_{\beta+1}\right)\left(\prod_{\beta=s}^{t_1-1} {}^{j}_{\beta+1}\tilde{I}^{II}_{\beta}\right)\right]^{1/2}$$

$$(2.28)\quad {}^{j}_{s+1}\tilde{I}^{II}_s = \left[\prod_{\alpha=1}^{k} \frac{{}^{s}_{\alpha}P_j}{{}^{s+1}_{\alpha}\tilde{P}^{II}_j} \cdot {}^{\alpha}_{s+1}I_s \cdot {}^{j}_{s+1}I_s\right]^{1/2k} \quad \forall j, \forall s$$

$$(2.29)\quad {}^{j}_{s}\tilde{I}^{I}_{s+1} = \left[\prod_{\alpha=1}^{k} \frac{{}^{s+1}_{\alpha}P_j}{{}^{s}_{\alpha}\tilde{P}^{I}_j} \cdot {}^{\alpha}_{s}I_{s+1} \cdot {}^{j}_{s}I_{s+1}\right]^{1/2k} \quad \forall j, \forall s$$

The development over time of this model is presented in Table 7.

TABLE 7

A \ B	t_0	t_1	t_2
t_0	${}^{t_0}_{h}P_j$	—	—
t_1	$\left[{}^{t_0}_{h}P_j \cdot \left({}^{1}_{h}P_j \cdot \frac{{}^{j}_{1}I_{t_0}}{{}^{h}_{1}I_{t_0}} \cdot {}^{t_0}_{h}P_j\right)^{1/2}\right]^{1/2}$	$\left[{}^{1}_{h}P_j \cdot \left({}^{1}_{h}P_j \cdot {}^{t_0}_{h}P_j \cdot \frac{{}^{j}_{t_0}I_1}{{}^{h}_{t_0}I_1}\right)^{1/2}\right]^{1/2}$	—
t_2	$\left[{}^{t_0}_{h}P_j \cdot \left[{}^{t_0}_{h}P_j \cdot \frac{{}^{j}_{1}I_{t_0}}{{}^{h}_{1}I_{t_0}} \cdot \left({}^{2}_{h}P \cdot \frac{{}^{j}_{2}I_1}{{}^{h}_{2}I_1} \cdot {}^{1}_{h}P_j\right)^{1/2}\right]^{1/2}\right]^{1/2}$	$\left[\left({}^{t_0}_{h}P_j \cdot \frac{{}^{j}_{t_0}I_1}{{}^{h}_{t_0}I_1} \cdot {}^{1}_{h}P_j\right)^{1/2} \cdot \left({}^{2}_{h}P_j \cdot \frac{{}^{j}_{2}I_1}{{}^{h}_{2}I_1} \cdot {}^{1}_{h}P_j\right)^{1/2}\right]^{1/2}$	$\left[{}^{2}_{h}P_j \cdot \left[\left({}^{t_0}_{h}P_j \cdot \frac{{}^{j}_{t_0}I_1}{{}^{h}_{t_0}I_1} \cdot {}^{1}_{h}P_j\right)^{1/2} \cdot \frac{{}^{j}_{1}I_2}{{}^{h}_{n}I_2} \cdot {}^{2}_{h}P_j\right]^{1/2}\right]^{1/2}$

In order to obtain transitivity in space and time all parities and indices have to be recalculated when new observed parities are available.

(5) Summary and Conclusions

In the preceding paragraph some models were developed in order to bring parities and time indices into an overall system. The different systems are summarized in the following.

It is not possible to give a clear preference for one or another model. The choice depends on the relative quality of parities or indices. If parities are considered to have a high degree of precision compared with indices or if indices are lacking, then the methods used should aim at an adjustment of indices to parities. In case parities are calculated for two benchmark years, the parities for the intermediate years can be calculated with the help of indices. It may also be possible to calculate more than two benchmark years; in this case adapted indices can be calculated between each interval and then it is possible to carry out chaining between intervals.

In many cases parities are obtained through aggregation of parities on the detailed category level; it will not be useful to change these detailed parities. All basic information which is partly used for the establishment of national price indices of aggregates is already included in these parities. In this case methods of adjustment of price indices seem most appropriate.

If reliable parities are only available in one benchmark year and the reliability of the national price indices is clearly higher than that of the other observed parities, the best method is extrapolation with the help of price indices.

However, in practice, parities are available in benchmark years and national price indices for each intermediate year and it is not possible to indicate the degree of reliability of the two indicators. The smoothing systems will then be the most appropriate to apply. The two sources—parities and indices—will be treated on an equal basis for the determination of annual parities and corresponding indices.

With these systems different options are open:

(a) Base year parities remain unchanged which means that parities already published do not change.
(b) Parities of the most recent benchmark year remain unchanged, which means that all parities already published change.
(c) Mixed system in which old and new parities and all intermediate parities change.

If more than two benchmark years have to be taken into account a system of chaining can be applied in order to link different periods.

3. Some Numerical Results on Space and Time Comparisons

In the preceding section methods were developed to calculate transitive parities and indices between countries for a given period of time, say five years. In this section some results will be presented for the countries of the European Community for the period 1975–1980.

Before presenting the main results it will be necessary to describe the data which were available for this exercise.

3.1. *Description of Available Data*

Three sets of data are used: a set of observed parities, i.e. parities derived from price surveys, a set of price indices and a set of expenditure data or weights.

All the data refer to the nine member countries of the EEC.

Federal Republic of Germany	Luxembourg
France	United Kingdom
Italy	Ireland
Netherlands	Denmark
Belgium	

(a) Parities Derived from Price Surveys

Complete sets of purchasing power parities have been calculated for 1975 and 1980 on the basis of the benchmark price surveys, whereas for the intermediate years prices were collected for special areas:

(1) *Household consumption.* Very extensive price surveys have been carried out in 1975 and 1980 for all areas of household consumption. In 1975 about 700 products were covered, and in 1980 more than 1,000 products were included in the price survey. Because of the institutional differences between countries, health expenditure is only partly included in household consumption in some countries whereas it is almost completely covered in other countries. For this reason these expenditures are compared in a separate way.

For the years 1977 and 1978, price surveys were carried out for special areas of household consumption: in 1977 for durables like electrical appliances, acoustic apparatus and photographic products; in 1978 for clothing and footwear.

(2) *Gross fixed capital formation*

(a) Equipment goods. For this group of products prices were collected each intermediate year for a part of the 1975 sample. The annual sub-sample was not confined to a special area but covered all different groups of products. For each product included not only was the price collected, but model changes were also studied and the technical specifications were adapted if necessary. It turned out that in about one sixth of the cases each year the models selected previously had disappeared. In 1980 prices were collected for about 200 products, as in 1975 covering the whole range of products.

(b) Construction and civil engineering. The method of estimation of construction and civil engineering is based on detailed bills of quantities for specified buildings or works, which are given for each of the participating countries. By applying unit prices total cost is the sum of quantities × prices. In 1975 and 1980, about 20 buildings and works were priced in this way, whereas in the intermediate years only five or six buildings were priced. The unit prices derived for these five or six were applied also to the bills of quantities for the remaining specification. In each year a different set of buildings or works was priced.

Thus for each intermediate year the whole 1975 sample was priced, for a part directly and for the rest indirectly by applying unit prices; the directly estimated buildings and works varied each year. The sub-sample examined each year was also adapted to take into account changes in technical specifications of the buildings and civil engineering works.

(3) *Collective consumption.* Because of the method of estimation of this category of uses, i.e. by input prices, the emphasis is laid on wages and salaries. In 1975 and 1980 the salary cost was used for 16 different jobs in the government sector. For the intermediate years the same information was available in principle.

(b) Price Indices

There are annual price and volume indices available in the framework of national accounts. These indices have two drawbacks: their degree of disaggregation is relatively low and they are available only with a considerable delay of about 2 years.

In addition to these price or volume indices, price indices are available in the framework of the consumer price index for all countries, whereas for investment and collective consumption no price indices are available for *all* countries.

In the field of *household consumption* a great number of detailed price indices are available in the framework of the consumer price index. These indices are published by all countries with a varying degree of detail but not according to a common classification. For this reason EUROSTAT in collaboration with the member countries set up a system of price indices corresponding to the same classification as used for the calculation of the 1975 parities.

In 1975 for the calculation of the parities on the most detailed level of disaggregation 104 basic headings were adopted for household consumption. In accordance with those basic headings the price indices were constructed by using the most detailed individual price series available in each country. It turned out that for most of these basic headings these price indices are appropriate as a basis for extrapolation of parities. However, for some groups the indices are not appropriate because of a lack of coverage. In general this is the case for health expenditure and in some countries for rents whereas for some basic headings in some countries no prices were included in the price indices. In these cases a similar group was taken for the extrapolation of parities.

The system of price indices is available for the period 1971–80 (base 1975). Most national price indices are of the Laspeyres type whereas for France and the U.K. they are Laspeyres type chain indices.

For *collective consumption* no current price indices are available; the only price index available is in the national accounts framework.

In the field of *fixed capital formation* national price indices or construction cost indices are available, however, not for all the member countries and for the moment it is not possible to use these for our purpose. For equipment goods a programme of work is in progress by EUROSTAT for producer price indices but results for all countries are not yet available for the period 1975–80. This is also the situation for construction cost, but in this area price indices are available as a by-product of the parities. Furthermore price indices for equipment goods, broken down in some categories, and for construction (dwellings, non residential) and civil engineering are annually included in the detailed tables of national accounts with two years delay.

(c) Expenditure Data

The parity calculations for the benchmark years are based on very detailed expenditure data; for all domestic final uses about 150 detailed categories were used in 1975. This very detailed breakdwon is estimated because results of special surveys are available i.e. family budget surveys or detailed input/output tables. In all countries these expenditure breakdowns for the benchmark surveys are not included in the regular annual work but are specially made for this exercise. In the intermediate years the figures available are on a much more aggregated level and the level of disaggregation increases with the delay of the year of reference.

In the national accounts the data available for household consumption correspond to a breakdown of about 50 headings with a delay of two years, whereas for collective consumption and gross fixed capital formation the number of headings is very limited: two for collective consumption, only five for gross fixed capital formation.

This source of information is the same as the one for which price indices are available as described before. Expenditure data from other sources are not available for all member countries.

In this exercise, expenditure data for 1971–79 have been taken from the national accounts for the breakdown available. The 1975 expenditure structures within these items were applied in order to derive an annual updated expenditure structure for the period 1971–79. A new expenditure structure will be provided by countries for 1980 in the framework of the 1980 benchmark exercise. However, this new structure was not yet available at the time the calculations included in this paper were finished.

3.2. *Results for 1980*

For the moment only preliminary and incomplete results are available for 1980. Only for household consumption were parities calculated on the basis of price surveys, but these prices are still not final. So the comparison should be revised at a later stage.

For rents and health expenditure no results can be shown. For rents complete data are not yet available for 1980 where as for health expenditure the extrapolation of parities with price indices is not very appropriate because the price indices are very weak and not comparable between countries.

The extrapolation was carried out for about 100 basic headings belonging to household consumption in the 1975 exercise which are then aggregated to the three, two and one digit level as well as for total household consumption.

Table 8 summarizes the results for the one digit level (except group 5), between the extrapolated parities (E) and the observed parities (O).

From these results on the aggregated level it can be seen that the two parities for 1980 are not identical. Out of 63 parities 11 show a difference of more than 10 perce t; for 12 the percentage is between 5 and 10 percent; and for 40 they are less than 5 percent as can be seen from Table 9.

The difference is more important for the U.K. and Ireland but it is not yet clear what are the causes for these relatively high differences, the 1975 parities, the price indices or the 1980 parities. This certainly needs further investigation.

TABLE 8

COMPARISON BETWEEN EXTRAPOLATED PARITIES (E) AND OBSERVED PARITIES (O), 1980 HOUSEHOLD CONSUMPTION

	Germany		France		Italy		Netherlands		Belgium		Luxembourg		U.K.		Ireland		Denmark	
	E	O	E	O	E	O	E	O	E	O	E	O	E	O	E	O	E	O
1. Food, beverages, tobacco	0.320	0.318	0.703	0.694	119.5	115.7	0.300	0.313	4.40	4.66	4.78	4.60	0.075	0.070	0.072	0.070	1.18	1.19
2. Clothing and footwear	0.295	0.288	0.801	0.797	114.8	155.7	0.319	0.319	5.09	5.06	5.39	5.58	0.066	0.059	0.068	0.064	0.93	1.09
3. Lighting and heating (rents excluded)	0.361	0.366	0.777	0.767	107.5	105.7	0.343	0.340	6.17	6.08	5.13	5.23	0.064	0.074	0.057	0.055	0.84	0.81
4. Furniture, household appliances	0.311	0.303	0.693	0.725	122.7	128.2	0.318	0.302	4.91	4.55	4.77	4.71	0.073	0.697	0.076	0.083	0.92	0.94
6. Transport	0.312	0.291	0.739	0.741	110.3	102.2	0.329	0.327	4.72	4.63	4.04	4.15	0.070	0.080	0.083	0.085	1.08	1.06
7. Education, entertainment	0.306	0.300	0.840	0.754	130.7	131.1	0.304	0.310	5.29	4.71	4.73	4.46	0.063	0.065	0.057	0.071	1.11	1.13
8. Other goods and services	0.339	0.317	0.777	0.749	104.9	108.5	0.393	0.351	5.08	4.42	4.12	4.35	0.066	0.072	0.057	0.075	1.16	1.06
Total household consumption	0.328	0.319	0.734	0.724	113.8	111.9	0.329	0.325	5.02	4.88	4.68	4.76	0.702	0.727	0.065	0.069	1.03	1.02

TABLE 9

FREQUENCY DISTRIBUTION OF PERCENTAGE DIFFERENCES IN PARITIES

	Germany	France	Italy	Nether-lands	Belgium	Luxem-bourg	U.K.	Ireland	Denmark
>10%	0	1	0	1	2	0	3	3	1
5–10%	2	0	1	1	2	2	2	1	1
<5%	5	6	6	5	3	5	2	3	5
Total consumption difference in %	2.5	1.4	1.6	1.2	2.8	−1.8	−3.5	−6.4	0.0

3.3. *Gross Fixed Capital Formation*

Unfortunately the comparison for 1980 is not yet possible because price indices for gross fixed capital formation are not yet available for 1980.

In Table 10 the extrapolated parities and the observed parities are compared for 1976 and 1977, years for which partial price surveys were carried out and price indices are derived from the national accounts.

In fact the comparison is carried out for total gross fixed capital formation and separately for equipment goods and construction. The deviations are all low but the period of comparison is only one or two years. The largest differences are found here for Ireland and the U.K. as follows from Table 11.

TABLE 11

FREQUENCY DISTRIBUTION OF DIFFERENCES BETWEEN EXTRAPOLATED AND OBSERVED PARITIES, 1976 AND 1978, PERCENTAGES

	5%	5–3%	3–1%	1%
Germany	0	0	1	5
France	1	1	3	1
Italy	0	1	4	1
Netherlands	0	2	3	1
Belgium	0	1	1	4
U.K.	1	0	1	4
Ireland	0	2	4	0
Denmark	0	0	3	2

3.4. *Results of Partial Surveys 1977 and 1978*

As has been pointed out previously in 1977 and in 1978 special price surveys were carried out for some areas and so observed parities could be derived which then can be compared with the extrapolated parities.

In 1977 the areas covered were:

electrical household appliances	group 431
acoustic apparatus	group 711.1
photographic products	group 712.1.

TABLE 10

COMPARISON BETWEEN EXTRAPOLATED PARITIES (E) AND OBSERVED PARITIES (O), GROSS FIXED CAPITAL FORMATION, 1976 AND 1977

	Total Gross fixed capital formation				Equipment goods				Construction			
	1976		1977		1976		1977		1976		1977	
	E	O	E	O	E	O	E	O	E	O	E	O
Germany	0.360	0.358	0.340	0.340	0.349	0.346	0.328	0.330	0.369	0.368	0.350	0.350
France	0.661	0.685	0.656	0.670	0.657	0.660	0.643	0.652	0.666	0.704	0.668	0.686
Italy	83.2	85.5	90.3	89.5	100.2	101.8	107.4	109.9	74.6	77.2	81.8	79.8
Netherlands	0.388	0.379	0.378	0.367	0.394	0.393	0.378	0.372	0.384	0.370	0.380	0.364
Belgium	5.59	5.60	5.42	5.55	5.19	5.71	4.96	4.96	5.82	5.84	5.69	5.90
Luxembourg	5.59	5.61	5.38	5.51	5.19	5.23	4.96	4.96	5.78	5.80	5.64	5.85
U.K.	0.071	0.069	0.073	0.073	0.070	0.070	0.075	0.075	0.072	0.069	0.071	0.071
Ireland	0.065	0.063	0.068	0.066	0.071	0.071	0.076	0.075	0.061	0.059	0.063	0.060
Denmark	0.904	0.915	0.909	0.904	0.824	0.822	0.836	0.820	0.965	0.986	0.967	0.972

TABLE 12

COMPARISON OF DIFFERENCES BETWEEN EXTRAPOLATED PARITIES AND OBSERVED PARITIES, IN PERCENTAGES

	Germany	France	Italy	Netherlands	Belgium	Luxembourg	U.K.	Ireland	Denmark
1977									
Acoustic apparatus	−6.6	4.4	4.3	−8.1	0.1	−12.8	37	−1.4	19.8
Photographic products	−7.1	7.8	−19.9	−1.0	−8.3	−15.6	23.5	24.7	5.6
Refrigerators, washing machines, etc.	0.1	6.7	7.5	−3.5	1.0	1.0	−11.0	6.8	−3.6
Cookers, electric heaters	−4.2	10.1	2.6	−18.1	−5.9	−2.5	13.3	8.9	−0.4
Vacuum cleaners, sewing machines, etc.	−6.7	−4.1	8.7	−7.9	2.6	0.2	−5.9	1.8	13.2
TOTAL HOUSEHOLD EQUIPMENT	−1.8	5.5	1.1	−4.0	−5.1	−0.3	−3.4	+6.6	1.4
1978									
Men's and children's outer garments	13.4	−3.1	−10.7	−1.4	−3.7	−2.3	0.0	9.9	·
Woman's outer garments	20.5	−0.4	0	−16.5	1.1	4.3	7.0	−11.9	·
Underwear and knitwear, men and children	9.1	2.1	−5.1	−6.8	−3.6	15.2	−11.5	3.4	·
Underwear and knitwear, women	−3.0	−2.3	5.6	−13.1	10.3	−1.6	−11.6	19.9	·
Haberdashery	29.6	−22.2	17.7	17.2	−21.4	−20.7	7.9	6.9	·
Sub-total, clothing	11.3	−1.4	−0.7	−7.7	−0.8	2.8	−0.5	5.1	
Men's and children's footwear	8.5	−9.4	−11.5	−4.1	−19.0	−7.1	34.5	18.7	
Women's footwear	4.5	−13.2	15.6	−2.5	−17.1	18.7	−4.8	4.3	
Sub-total, footwear	7.2	−11.0	3.4	−3.3	−18.2	3.4	−10.4	12.7	
CLOTHING AND FOOTWEAR	10.8	−3.5	0	−7.1	−3.7	2.7	−2.2	6.4	

In 1978 the area covered was clothing and footwear (without repair). A comparison between calculated parities and observed parities is presented in Table 12. The differences between parities are given in this table for the detailed groups and for the more aggregated items household equipment, clothing, footwear.

Again for the detailed groups differences are rather big, but for the aggregated items they are much less important which means that many of these differences cancel out.

3.5. *Conclusions*

The results presented in this paper are still provisional because the results from the 1980 benchmark survey are not yet completely available. In the coming months more final conclusions can be drawn. As a first remark it can be stated that on the detailed level there are important gaps between observed parities and extrapolated parities but these tend to cancel out on a more aggregated level. For some areas price indices are not very appropriate (rents, health, investment, government consumption) and it will be necessary to make use of direct parity estimations each year.

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