NOTES AND COMMENTS

CLARIFICATION OF MILLER ON CAPITAL AGGREGATION IN THE PRESENCE OF OBSOLESCENCE-INDUCING TECHNICAL CHANGE

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I. INTRODUCTION

In a recent article appearing in this journal, Professor Edward Miller addressed the issue of how improvements in the quality of capital purchased should be treated in a vintage aggregation model. He was concerned with how improvements in the quality of capital purchased affect the weighting of older vintages within a perpetual inventory model of vintage aggregation. Specifically, he argued that "gross" measures of capital "do not allow for the higher marginal product of more modern capital" and that older vintages of capital should receive less weight when there has been capital embodied technological progress (p. 283). It is argued below that Professor Miller's description of the current measures of capital stock is in error. The gross and net measures of capital stock as calculated by the Bureau of Economic Analysis (BEA) and the "productive capital stock" measure calculated by the Bureau of Labor Statistics (BLS) already incorporate implicitly measures of technological progress and reduced utilization of older vintages of capital caused by technological progress.

Furthermore, it is argued that determining the exact weighting of older vintages is an empirical matter, not a theoretical one. Thus, while the BEA or BLS may be measuring obsolescence imperfectly, it is not clear *a priori* whether this would raise or lower the weights on older vintage capital. To fully comment on these issues, we begin in section II with some definitions and considerations critical to any discussion of capital measurement. In section III, a formal model of vintage aggregation is presented. In section IV the perpetual inventory method is explored and some final comments relating to Professor Miller's article are made.

II. DEFINITIONS

The central focus of Professor Miller's paper is the correct weighting of different vintages of capital within the same time period and over different time periods. Vintage aggregation constitutes a major element of developing a time series for capital inputs. For example, in deriving an index of capital inputs for a broad variety of capital goods over time using a Tornqvist index, the growth

*The usual disclaimer applies.

rate of the capital aggregate, KA, between time $t-1$ and t is defined as:

$$(1) \qquad \ln\left[\frac{KA(t)}{KA(t-1)}\right]=\sum_{i=1}^{z}\left\{\left[\frac{B_i(t)+B_i(t-1)}{2}\right]\ln\left[\frac{K_i(t)}{K_i(t-1)}\right]\right\}$$

where there are z types of capital goods, K_i is the ith type of capital, r_i is the rental payment on K_i, and B_i is defined as

$$(2) \qquad B_i(t)=\frac{r_iK_i}{\sum r_iK_i},$$

which is the ith share of rental payments in total rental payments for capital services. In a model like (1), each $K(t)$ is derived using direct aggregation or a perpetual inventory model. The model developed in the remainder of this comment is to measure K_i in (1). It is a vintage aggregation model employing a direct aggregation scheme. We will assume we are aggregating across similar types of capital goods but not necessarily identical capital goods. When a capital good requires one or more variable inputs to operate, this means that similar goods will have different variable input/output ratios. To the extent that newer vintages are of higher quality, we will assume this means newer vintages have at least one variable input/output ratio which is lower than older vintages.[1]

III. A General Model of Vintage Aggregation

In this section we construct a general vintage aggregation model and describe the measurement issues any such model must address. In section IV it will be shown that the perpetual inventory model is this general model whenever flows are a constant proportion of stocks.

The starting point for constructing a single year's measure of capital input is the observation of capital purchases for T consecutive periods, where T is the maximum number of years a particular type of capital good will remain in service. Thus, we have

$$(3) \qquad \widehat{Pk_tK_t}=[Pk_tK_t, Pk_{t-1}K_{t-1},\ldots, Pk_{t-T}K_{t-T}]$$

where Pk_iK_i is the cost of capital purchased in year i.

Given the desire to measure capital in time period t, K_t, we can define K_t as follows:

$$(4) \qquad K_t=\widehat{Pk_tK_t}\Omega_t$$

where Pk_tK_t is defined in equation (3) and $\Omega_t=(w_{0t}, w_{1t}, w_{2t},\ldots, w_{Tt})$ is a column vector of weights needed to convert observations of the market value of capital purchases to a single quantity measure. Each weight is a measure to account for

[1]The problem of "proportionality" is also assumed away here. When two capital goods have intersecting service profiles, no price index for capital goods will be able to capture these differences. This is what Jorgensen often refers to as quality change.

all of the following:

the conversion of stock measures to flow measures[2]
changes in relative prices
differences in the quality of new capital
changes in the utilization of capital
the physical deterioration of capital goods as they age.

Given the above list of measurement issues that are incorporated in w_{it}, we specify it as follows:

$$w_{it} = \frac{\phi_i^t}{Px_{t-i}} \tag{5}$$

where the superscript t denotes the year in which K is being measured and the subscript $t-i$ denotes the vintage of the capital good. The "i" in the subscript is the age of the capital good. Ignoring the super and subscripts for the moment, ϕ is the relative efficiency function; and Px is the price index of capital goods.

The price index for capital, Px in (5), should account for quality change between different vintages of capital. How quality change should be measured and whether it is currently measured correctly has been discussed by Gordon (1983) and Triplett (1983). Triplett has advocated that price indexes for inputs employ an output based or user value criterion. Since the BEA currently uses a production-cost criterion to measure differences in quality for capital goods over time, the price indexes for capital goods need to be treated with some skepticism.

There are other reasons, however, that price indexes for capital goods need to be treated with great caution. Currently, these indices are constructed for broad categories of capital. The weight for a specific component within each index is its proportion to the total amount of the composite *produced* at the *national* level. Bias is thus imparted to the capital stock measures when the price indexes are applied to *purchases* of capital goods at a sectoral level.

Professor Miller argues that we are not capable of constructing price indexes which account for embodied technological progress (p. 285). He then argues that we should account for higher quality capital by lowering the weight ϕ_i rather than raising Px_{t-i} in (5). Given that the same measurement of quality change is necessary, I do not understand his position. If we can make the measurement it belongs in Px (see above). If we can't, then we can't.

The relative efficiency function has received much attention in recent years (e.g. see BLS (1979, 1983), Hulten and Wykoff (1980)). It describes a mixture of phenomena related to capital goods including retirement, deterioration, and changes in utilization. To account for retirements and deterioration, ϕ_i^t is usually specified as

$$\phi_i^t = f(i, \bar{T}, \sigma_T, \beta), \qquad 0 \leq \phi_i^t \leq 1 \tag{6}$$

where T is the mean service life of the capital good, σ_T is a proxy for the distribution of retirements about the mean service life, and β is a deterioration

[2]This issue is irrelevant when making an index of stock quantities, if all components of the index have the same coefficient or if the problem of proportionality is ignored.

parameter. All of the partial derivatives in (6) are non-negative except for $\partial f/\partial i$ which is non-positive. The two main problems with the specification in (6) are that it ignores changes in utilization and the effects of changes in relative prices on parameter values. Changes in utilization are not usually included in ϕ, but easily could be, given accurate definition and measurement of capital utilization.[3] Suppose we had such a measure, say Θ_t. We could then specify $\phi_i^t = \Theta_t f(\cdot)$.

All of the research on the specification of ϕ has been undertaken with very little data and has been oriented toward making ϕ as consistent as possible with these data. Most published research has ignored changes in the parameter values in (6) over time for a broad category of capital goods and assumes for a given category that $\phi_i^t = \phi_i^{t+j}$ for all j. This assumption is quite strong. As shown in Arnold (1982) the growth rates of capital aggregates are much more sensitive to this assumption than they are to some specific choice of parameter values.

The marginal physical product of capital for existing vintages of capital falls as capital deteriorates. A measure of deterioration is incorporated in ϕ as specified in (6). Thus, the measure of the capital aggregate is biased whenever deterioration is incorrectly specified. A biased measure also occurs when ϕ is incorrectly specified because $\bar{T}$ or some other parameter is different from its actual value. Professor Miller argues that embodied technological progress lowers the marginal product of capital for existing vintages and, therefore, ϕ should be reduced to account for this change. A careful response to this argument is in order.

To the extent that capital-embodied technological progress raises the wages of labor or lowers the price of output, the service lives of existing vintages of capital will be reduced below what they otherwise might have been. (This is by definition obsolescence.) But this has no implication for the marginal physical product of existing capital, unless producers allow deterioration to occur more rapidly as the time of scrappage nears. Most importantly, it has no specific implication for the specification of ϕ, since ϕ already incorporates measures of deterioration and retirement.

In other words, capital-embodied technological progress only implies ϕ is misspecified when it has caused the actual values of ϕ to be different from those used in constructing a vintage aggregate. This is an empirical question, not a theoretical one, and Professor Miller has produced no evidence to tell us whether ϕ is misspecified.

IV. The Perpetual Inventory Method

The purpose of this section is to show that the perpetual inventory model (PIM) is almost identical to the model described by (4), (5), and (6). It is also to clarify some of the discussion in Professor Miller's paper concerning the perpetual inventory method and when it properly and improperly accounts for capital embodied technological progress. Since this comment has dealt entirely with vintage aggregation issues, in the remainder of the discussion we will assume we

[3]Recently, Berndt and Fuss (1982) argued that changes in capital utilization should be accounted for in the weights (B_i).

are aggregating across different vintages of a single type of capital. It should be noted, however, that once this restriction is loosened the BEA and BLS use different methods for aggregating across broad capital types.

The PIM begins with the simple relation between capital at the beginning of time period, K_t, and real gross investment in time period, t, I_t. We have

$$K_t = I_t + (1-\delta)K_{t-1} \tag{7}$$

where δ is the proportion of K_t which is retired or deteriorated.

Note that describing δ and adjusting nominal gross investment flows for changes in price and quality are the two issues of concern here. Since K_t is the stock measure referred to in equation (7) and real investment is Pk_tK_t/Px_t, we have

$$K_t = \frac{Pk_tK_t}{Px_t} + \sum_{i=1}^{T} \phi_i \frac{Pk_{t-i}K_{t-i}}{Px_{t-i}}. \tag{8}$$

Thus δ is a summary measure for the weighted lagged measures of ϕ_i/Px_{t-i}. Also note that (8) is identical to (4), when (5) holds.

Note that the superscript has been removed from ϕ. This is because the BEA and BLS incorporate in ϕ_i^t the assumption that retirement and deterioration are independent of t.

V. Final Comments

One of the main components of constructing a quantity index of capital inputs is vintage aggregation. This, in turn, usually requires that capital goods of different quality be summed.

The current accounting procedures to derive measures of capital stocks for a specified category of capital goods employ the perpetual inventory method (PIM) to construct a vintage aggregate. Professor Miller asserts that this method fails to account properly for captial-embodied technological change. He argues that older vintages of capital should receive relatively less weight than they currently receive when there has been technological progress.

In response, I have argued that the PIM already accounts for capital-embodied technological change through the adjustment of price indexes for quality change and the construction of a relative efficiency function based on estimates of service lives and deterioration patterns for capital goods which implicitly incorporate an estimate of obsolescence.

The issue here is not whether the capital stocks are measured correctly. It is whether we can say *a priori* that a specific adjustment is in order. Professor Miller argued that there is an improper procedure in the accounting framework. I have argued that the issue is an empirical one.

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