A BALANCED SYSTEM OF NATIONAL ACCOUNTS FOR THE UNITED KINGDOM

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This paper describes the construction of a disaggregated system of 262 national accounts for the U.K. economy in 1975. The objective is to remove the discrepancies between income, expenditure, production and financial estimates which occur in practice. This is done with the aid of a generalised least squares algorithm for adjusting national accounts with subjective estimates of reliability of the various account items. The balanced system of accounts provides the cross-section data base needed for the estimation of a consistent multisectoral dynamic model of the U.K. economy and yields the classification converters and input–output tables necessary for such a model.

1. INTRODUCTION

The statistics collected for national accounts are subject to different sources of error: sampling bias, problems with timing, omissions, under-reporting and simple mistakes. The reduction of these errors and their treatment in the accounts are major problems for national income accountants. The usual procedure seems to be to reduce the errors as far as possible using the accounting identities of different parts of the system, then present one or two residuals between the main aggregates (e.g. U.K., CSO *National Income and Expenditure*).

This paper proposes a different approach in which the accounts are initially left unbalanced in a disaggregated System of National Accounts (SNA) (UN, 1968). Each account is then given a subjective estimate of its reliability and the whole system is balanced subject to the accounting identities and restrictions of the system. These methods have been proposed and developed by Stone *et al.* (1942), Stone (1982), Byron (1978), van der Ploeg (1982a, b, c) and Weale (1982), and applied by Stone (1982) and van der Ploeg (1982a) to small consolidated systems of U.K. accounts, and by Weale (1984) to the development of an accounting matrix showing the transactions of the world economy. This paper applies them to a full scale SNA for the U.K. for 1975 with 262 accounts, showing that their practical application in the context of detailed national accounts is now feasible and relatively cheap.

2. THE UN SYSTEM OF NATIONAL ACCOUNTS

Although national accounts, as a means of presenting economic data in a coherent framework, have a history going back to the 1930s and 1940s (Stone, 1981), it is only more recently that the accounts themselves have been presented

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in an ordered way. The guidelines laid down by UN (1968) present a coherent framework for incorporating detailed production, income–outlay, accumulation and financial accounts.

The SNA is presented as an accounting matrix in which payments are shown in columns and receipts are shown in rows. If the accounts balance each row total equals the corresponding column total. Although many countries claim to present their national accounts in SNA format there is a reluctance to adopt matrix presentation.

The Norwegian system of national accounts (Norway, Central Bureau of Statistics, 1980; Flottum, 1981) is one of the most detailed to embody the principles of the UN SNA. It includes opening and closing balance sheets as well as hundreds of production sectors. The system is updated each year and used for model building and planning (Bjerkholt and Longva, 1980; Barker, 1981). France (Institut National de la Statistique et des Études Économiques) adopts a framework fairly close to that prescribed in the Tableaux Économique and the Tableaux Entrées-Sorties in that it is possible to see not only commodity sales and purchases by industry of the type presented in the input–output tables, but also the receipts and payments of each institutional sector in a framework where payments of each type of transfer are balanced against receipts. This approach provides a useful means of identifying discrepancies which can otherwise pass unnoticed. The OECD National Accounts, for example, often contain misprints or other errors which might be identified in a matrix presentation, and it is not always clear how, in their existing format, the various payments by each sector interrelate.

The United Kingdom has traditionally focused on analysing the income–outlay side of the economy but even here it is only recently that net transfers have been presented in a matrix framework (U.K. CSO, 1982). Input–output tables are an occasional rather than, as in the Norwegian and French cases, an annual product. Little emphasis is placed on combined production and income–outlay accounts in a coherent framework, although the input–output tables are produced to be consistent with one edition of the National Accounts.

The first step in the work described here has therefore been to cast the national accounts and the input–output table for the U.K. for 1975 into the framework of the System of National Accounts. The SNA is then used to perform a complete balancing exercise in which adjustments to production imply changes in the income–outlay accounts and vice versa.

The framework of the UN accounts and the way they relate to U.K. data can best be represented in the aggregates shown in the SNA in Table 1. This table shows three estimates for each SNA entry, but here we consider only the initial (prior, p) estimates, leaving the balanced ones (neutral, n, or subjective, s) to be discussed in section 7 below. The entries are aggregated from a much more detailed set of 262 accounts for the purposes of presentation: the classifications adopted at this detailed level are shown in Table 2 along with the subjective reliabilities of each item which are discussed in section 6 below.

The accounts in Table 1 follow the original UN SNA numbering. Accounts one and two are balance sheet items which have been omitted from the present analysis. The third row shows sales of commodities valued at prices net of output taxes (although the EEC suggest valuation gross of VAT) to various categories of demand such as industries, consumption and investment. Account four presents commodity taxes allocated to the category of demand purchasing the good on which the commodity taxes fall; most of these taxes therefore fall on consumption. Account five represents industries, whose output is entirely commodities sold to account three and the commodity taxes on the output of these industries allocated to the industry producing the taxed commodity. Its row totals therefore represent the value of industrial output gross of output taxes. Accounts six and seven show similar detail for non-industrial private and public "activities" such as defence or ownership of dwellings. Accounts eight, nine and ten show consumption expenditure, valued at market prices as far as possible. Thus each column in account eight represents a particular type of consumers expenditure and shows the commodity inflow and taxes paid making up this expenditure at market prices. In the example here total consumers expenditure in 1975 was £64,424m. This included £50,649m of goods valued at prices net of commodity taxes and £6,357m of commodity taxes on the goods. Consumption of "other activities" (row seven) reflects the imputed rent of owner-occupied dwellings. In addition the entry in row eleven represents the sum of subsidies (on transport etc.) and a factor tax (motor vehicle licences) paid by consumers; consumption supplied by the rest of the world shown in row twenty-four covers expenditure by British tourists abroad.

Value added in the economy is shown in account eleven. This includes not only operating surplus and employee remuneration but also indirect taxes. Those falling on factors of production, e.g. property taxes (rates), motor vehicle duties and licence fees, are shown directly together with subsidies paid to the production process. Commodity taxes are shown in aggregate allocated to the industry on whose output they fall. This ensures that the row totals of account five equal their column totals. Depreciation is deducted in column 23 and so the sum of this account represents net domestic product at market prices. Income from employment and net surplus are allocated to their sector of origin (sector owning the capital on which they are produced) in account twelve. Payments from this account go into account thirteen which classifies income by type of income and thus further breaks down value added.

Account thirteen itself represents types of income, both factor and transfer, while account fourteen provides a sectoral breakdown of those paying and receiving the transfer. Industrial stockbuilding classified by commodity added to stock rather than stocking industry is presented in account fifteen, while account sixteen shows stock changes for producers of government services. Accounts seventeen, eighteen and nineteen present investment by industries, in social capital and private non-industrial capital respectively. The finance of capital accumulation, including investment in stocks, is shown in account twenty while capital transfers and taxes appear in twenty-one and acquisition of financial assets in twenty-two. The full sectoral capital account appears as twenty-three. Its credit is saving from the sectoral current accounts in account fourteen and depreciation, appearing in row eleven as a negative payment. Investment in the capital accounts is therefore presented gross of depreciation, although saving is net. Account twenty-four completes the flow picture by representing the extent to which the economy interacts with the rest of the world. It has two rows representing current

TABLE 1

THE AGGREGATED SYSTEM OF NATIONAL ACCOUNTS FOR THE U.K. 1975 (£ million)

		SNA Account		3	4	5	6	7	8	9	11	12	13	14
Production	Commodities	3 Commodities	p	0	0	92,488	8,462	0	50,649	0	0	0	0	0
			n			92,403	8,458		50,663					
			s			92,211	8,243		49,760					
		4 Commodity taxes net	p	0	0	1,175	248	0	6,357	0	0	0	0	0
			n			1,189	250		6,372					
			s			1,210	242		6,485					
	Activities	5 Industries	p	168,405	7,271	0	0	0	0	0	0	0	0	0
			n	168,852	7,070									
			s	167,001	7,208									
		6 Government services	p	0	0	0	0	0	0	23,074	0	0	0	0
			n							23,286				
			s							23,716				
		7 Other activities	p	0	0	662	−1,207	0	7,157	0	0	0	0	0
			n			662	−1,215		7,064					
			s			663	−1,248		6,976					
Consumption	Expenditure	8 Household g&s	p	0	0	0	0	0	0	0	0	0	0	64,424
			n											64,352
			s											63,493
		9 Government purposes	p	0	0	0	0	0	0	0	0	0	0	23,074
			n											23,285
			s											23,716
	Income and outlay	11 Value added	p	540	934	81,270	14,930	6,104	−6	0	0	0	0	0
			n	541	925	81,669	15,322	6,019	19					
			s	540	933	80,125	16,000	5,925	37					
		12 Institution:origin	p	0	0	0	0	0	0	0	82,573	0	0	0
			n								82,804			
			s								82,042			
		13 Form of income	p	0	0	0	0	0	0	0	0	82,576	0	66,471
			n									82,804		66,686
			s									82,042		66,643
		14 Institution: receipt	p	0	0	0	0	0	0	0	10,478	0	149,340	0
			n								10,547		149,880	
			s								10,744		148,948	

Accumulation	Increase in stocks	15 Industries	p	0	0	0	0	0	0	0	0	0	0	0
			n											
			s											
		16 Government services	p	0	0	0	0	0	0	0	0	0	0	0
			n											
			s											
	Fixed capital formation	17 Industries	p	0	0	0	0	0	0	0	0	0	0	0
			n											
			s											
		18 Government services	p	0	0	0	0	0	0	0	0	0	0	0
			n											
			s											
		19 Other fixed capital	p	0	0	0	0	0	0	0	0	0	0	0
			n											
			s											
	Capital finance	20 Industrial capital	p	0	0	0	0	0	0	0	0	0	0	0
			n											
			s											
		21 Capital transfers	p	0	0	0	0	0	0	0	0	0	0	0
			n											
			s											
		22 Financial assets	p	0	0	0	0	0	0	0	0	0	0	0
			n											
			s											
		23 Institutional sectors	p	0	0	0	0	0	0	0	0	0	0	5,831
			n											6,104
			s											5,840
		24 Rest of the world	p	28,025	0	0	476	0	233	0	0	0	3,328	0
			n	27,613			471		234				3,293	
			s	27,864			479		234				3,301	
		Residuals and errors	p	−306	−243	80	165	2	33	0	−395	−3	5	18
			n	0	0	0	0	0	0	0	0	0	0	0
			s	0	0	0	0	0	0	0	0	0	0	0
		Total	prior	196,664	7,962	175,676	23,074	6,106	64,424	23,074	92,656	82,573	152,673	159,818
			neutral	197,006	7,995	175,923	23,286	6,019	64,352	23,286	93,351	82,804	153,173	160,427
			subjective	195,405	8,141	174,209	23,716	5,925	63,493	23,716	92,786	82,042	152,250	159,692

TABLE 1 *(cont.)*

		SNA Account		15	16	17	18	19	20	21	22	23	24	Total
Production	Commodities	3 Commodities	p	−1,422	−9	13,842	5,043	2,182	0	0	0	0	25,428	196,664
			n	−1,469	−9	14,097	5,092	2,015					25,755	197,006
			s	−1,430	−9	13,892	5,097	2,168					25,474	195,405
		4 Commodity taxes net	p	0	0	186	0	0	0	0	0	0	−5	7,962
			n			189							−5	7,995
			s			208							−5	8,141
	Activities	5 Industries	p	0	0	0	0	0	0	0	0	0	0	175,676
			n											175,923
			s											174,209
		6 Government services	p	0	0	0	0	0	0	0	0	0	0	23,074
			n											23,285
			s											23,716
		7 Other activities	p	0	0	−956	−53	0	0	0	0	0	502	6,106
			n			−943	−53						505	6,019
			s			−928	−53						515	5,925
Consumption	Expenditure	8 Household g & s	p	0	0	0	0	0	0	0	0	0	0	64,424
			n											64,352
			s											63,493
		9 Government purposes	p	0	0	0	0	0	0	0	0	0	0	23,074
			n											23,285
			s											23,716
	Income and outlay	11 Value added	p	0	0	0	0	0	0	0	0	−11,116	0	92,656
			n									−11,143		93,351
			s									−10,775		92,786
		12 Institution: origin	p	0	0	0	0	0	0	0	0	0	0	82,573
			n											82,804
			s											82,042
		13 Form of income	p	0	0	0	0	0	0	0	0	0	3,626	152,673
			n										3,683	153,173
			s										3,565	152,250
		14 Institution: receipt	p	0	0	0	0	0	0	0	0	0	0	159,818
			n											160,427
			s											159,692

Accumulation	Increase in stocks	15 Industries	p	0	0	0	0	0	−1,527	0	0	0	0	−1,527
			n						−1,469					−1,468
			s						−1,430					−1,430
		16 Government services	p	0	0	0	0	0	0	0	0	−9	0	−9
			n									−9		−9
			s									−9		−9
	Fixed capital formation	17 Industries	p	0	0	0	0	0	12,776	0	0	0	0	12,776
			n						12,938					12,938
			s						12,766					12,766
		18 Government services	p	0	0	0	0	0	0	0	0	5,466	0	5,466
			n									5,443		5,443
			s									5,446		5,446
		19 Other fixed capital	p	0	0	0	0	0	0	0	0	2,182	0	2,182
			n									2,015		2,015
			s									2,168		2,168
	Capital finance	20 Industrial capital	p	0	0	−406	406	0	0	0	0	11,250	0	11,250
			n			−406	404					11,471		11,469
			s			−406	402					11,340		11,336
		21 Capital transfers	p	0	0	0	0	0	0	0	0	2,320	0	2,320
			n									2,315		2,315
			s									2,320		2,320
		22 Financial assets	p	0	0	0	0	0	0	0	0	−1,586	1,586	0
			n									−1,673	1,673	0
			s									−2,330	2,330	0
		23 Institutional sectors	p	0	0	0	0	0	0	2,320	0	0	0	8,151
			n							2,315				8,419
			s							2,320				8,160
		24 Rest of the world	p	0	0	0	0	0	0	0	0	0	1,674	33,726
			n										1,673	33,284
			s										2,330	34,208
		Residuals and errors	p	−105	0	109	70	0	1	0	0	−356	925	0
			n	0	0	0	0	0	0	0	0	0	0	0
			s	0	0	0	0	0	0	0	0	0	0	0
		Total	prior	−1,527	−9	12,776	5,466	2,182	11,250	2,320	0	8,151	33,736	1,059,044
			neutral	−1,469	−9	12,938	5,443	2,015	11,469	2,315	0	8,419	33,284	1,062,026
			subjective	−1,430	−9	12,766	5,446	2,168	11,336	2,320	0	8,160	34,208	1,056,340

Note: The account numbering follows that of the System of National Accounts. No estimates are presented for balance-sheet entries (accounts 1, 2, 25, 26, 27 and 28).

TABLE 2

SUBJECTIVE RELIABILITIES FOR THE U.K. SYSTEM OF NATIONAL ACCOUNTS (ρ_i)

SNA Accounts 3 & 5 (Q&Y) Commodities & Industries

1	Agriculture etc.	5
2	Coal mining	5
3	Mining n.e.s.	10
4	Petroleum & nat. gas	5
5	Food manufacturing	5
6	Drink	5
7	Tobacco	5
8	Coal products	5
9	Petroleum products	5
10	Chemicals	5
11	Iron & steel	5
12	Non-ferrous metals	5
13	Mech. engineering	5
14	Instr. engineering	5
15	Elect. engineering	5
16	Shipbuilding	5
17	Motor vehicles	5
18	Aerospace equipment	5
19	Other vehicles	5
20	Metal goods n.e.s.	5
21	Textiles	5
22	Leather, clothing etc.	5
23	Bricks	5
24	Timber & furniture	5
25	Paper & board	5
26	Printing & publishing	5
27	Other manufacturing	5
28	Construction	15
29	Gas	5
30	Electricity	5
31	Water	15
32	Rail	5
33	Road	40
34	Other transport	40
35	Communication	5
36	Distribution	50
37	Business services	60
38	Professional service	60
39	Misc. services	60
40	Unallocated	60

SNA Account 4 (T) Commodity Taxes Net

1	Drink duty	5
2	Tobacco duty	5
3	M spirit & DERV duty	12
4	Other hydrocarbon duty	8
5	Protective duties	20
6	Purchase tax/car tax	10
7	Betting duties	2
8	VAT exemptions	40
9	Value added tax	40
10	Stamp duties	12
11	Export rebates	1
12	Expend. taxes n.e.s.	1
13	Commodity subsidies	15

SNA Account 5 Industries

See Account 3

SNA Account 6 (G) Government Services

1	Defence	5
2	NHS (central)	5
3	Other central exp.	5
4	Education (local)	5
5	Other local exp.	5

SNA Account 7 (Z) Other Activities

1	Ownership of dwellings	7
2	Sales by final buyers	20

SNA Account 8 (C) Household G & S

1	Bread & cereals	7
2	Meat and bacon	7
3	Fish	7
4	Oils and fats	7
5	Sugar preserves etc.	7
6	Dairy products	7
7	Fruit	7
8	Potatoes & vegetables	7
9	Beverages	7
10	Other manufactured food	7
11	Beer	7
12	Wines and spirits	7
13	Tobacco	2
14	Rent, rates etc.	7
15	Maintenance, etc.	20
16	Coal and coke	2
17	Electricity	2
18	Gas	2
19	Other fuel	7
20	Footwear	7
21	Other clothing	7
22	Motor cars etc.	7
23	Furniture etc.	15
24	Other durable goods	15
25	H'hold textiles etc.	15
26	Hardware	15
27	Matches, soap etc.	7
28	Books	7
29	Newspapers	2
30	Magazines	7
31	Chemists' goods	15
32	Misc. recr. goods	15
33	Other misc. goods	15
34	Running costs of m/vs	15
35	Rail travel	2
36	Bus	2
37	Other travel	7
38	Postal services	7

TABLE 2 (*cont.*)

39 T'phone t'graph	2
40 Entertainment etc.	7
41 Domestic service	20
42 Catering	7
43 NPMB wages	7
44 Capital consumption	7
45 Insurance	7
46 Other services	20
47 Income in kind	7
48 Foreign tourists exp.	7
49 Exp. abroad	7

SNA Account 9 (G)
Government Purposes

See Account 6

SNA Account 10
Other Consumption

No entry

SNA Account 11 (W)
Value Added

1	Empl. Cont. Personal	5
2	(private) I&C	5
3	Financial	5
4	Pub. Corp.	5
5	Cent. Gov.	5
6	Loc. Gov.	5
7	Empl. Inc. Personal	3
8	(W&S&ERS)I&C	3
9	Financial	3
10	Pub. Corp.	3
11	Cent. Gov.	3
12	Loc. Gov.	3
13	Surplus Personal	10
14	I&C	10
15	Financial	10
16	Pub. Corp.	10
17	Cent. Gov.	10
18	Loc. Gov.	10
19	Financial Adj.	25
20	Net. Comm. Taxes	20
21	Taxes on Imports	20
22	TV Contr. Levy	2
23	Petr. Royalties	2
24	SET/NIS	10
25	Rates	20
26	MVh Licences	20
27	Employment Subs	50
28	Other Subsidies	50
29	Balancing Taxes	10

SNA Accounts 12,14,23 (H) Institution:origin		A/cs 14,23
1 Households	1	10
2 Life assn. etc.	1	7
3 Ind. & comm. companies	1	7
4 Financial companies	1	7
5 Public corporations	1	2
6 Central government	1	2
7 Local authorities	1	2

SNA Account 13 (R)
Form of Income

1 Wages and salaries	5
2 Self-employment inc.	15
3 Gross trading profits	15
4 Employers' N.I. cont.	2
5 Employers' pvt. cont.	5
6 Imputed rent	5
7 Non-imputed rent	15
8 Employees' N.I. cont.	2
9 Employees' pvt. cont.	15
10 Ordinary dividends	25
11 Other divids. and int.	25
12 Consumer interest	25
13 Central govt. interest	2
14 Local auth. interest	2
15 Public corp. interest	2
16 Profits due abroad	40
17 Interest from abroad	40
18 Occupational pensions	25
19 Transfers to charities	25
20 Transfers paid abroad	40
21 Transfers from abroad	40
22 Retirement pensions	2
23 Child allowances	2
24 Unemployment benefits	2
25 Contributions to EEC	2
26 Other p.a. grants	2
27 UK taxes on income	2
28 Petroleum revenue tax	2

SNA Account 14 (H)
Institution: receipt

See Account 12

SNA Account 15 (S)
Industries' Stockbuilding

1 Basic materials	200
2 WIP	200
3 Finished goods	200

SNA Account 16
Government Stockbuilding

Total only	200

SNA Account 17 (V)
Industrial GDFCF

1 Buildings & works	4
2 Dwellings	7
3 Plant & machinery	7
4 Buses	15
5 Other road vehicles	10
6 Ships	2
7 Aircraft	2
8 Railway rolling stock	7
9 Legal fees	20
10 Land transfer	20

TABLE 2 (*cont.*)

SNA Account 18 (K) Social Capital Formation	
1 Education	7
2 Health	7
3 Roads	7
4 Other social investment	3
5 Dwellings	3
6 Legal fees	20
7 Land transfer CG	20
8 Land transfer LA	20
SNA Account 19 Other Fixed Capital	
Total only	4
SNA Account 20 Industrial Capital Formation	
Total only	7
SNA Account 21 (L) Capital Transfers	
1 CTT/death duties	2
2 Capital gains tax	2
3 Other capital taxes	2
4 Capital transfers	20
SNA Account 22 Financial assets	
Total only	50
SNA Account 23 (H) Institutions	
See Account 12	
SNA Account 24 Rest of the World	
1 Current transactions	20
2 Capital transactions	50

Note: The reliabilities are expressed as percentages of the mean.

and capital transactions. Since there were no capital transfers from the rest of the world in 1975, the capital account represents the finance of the current balance of payments deficit which is met entirely from within the framework of financial accumulation. The main entries in the row of the current account are of course payments to the rest of the world for imports, while the main debit items are purchases from the commodity accounts of exports.

3. A Closer Look at the Accounts

The Production Accounts

In the SNA the main accounts for production are those for commodities, commodity taxes, industries and government. Commodities are distinguished from industries so as to allow industries to produce not only their principal products but the products of other industries. These may be subsidiary production, such as own production of electricity, or they may be as by-products, such as the production of gas by the coke ovens industry. Commodity taxes are distinguished in order to derive producers' values (which include the taxes) from basic values (which do not).

The production accounts contain the main data needed to estimate input–output tables and other coefficient matrices used in multisectoral models of the economy. They provide the main data on the details of the economy's industrial structure. The input–output table is calculated from the make matrix which shows each industry's production of commodities and the absorption matrix which shows each industry's consumption of commodities. Different assumptions about technology determine the dimensions and estimates of the table (see Armstrong and Upton, 1969; U.K. CSO, 1973; UN, 1973).

The main sources for the production accounts are the 1974 input–output tables for the U.K. (U.K. CSO, 1978) updated to 1975 and kindly provided to us by the CSO. The updated tables are much more detailed than the disaggregation we have adopted (102 industries as opposed to 40 industries) so they have been aggregated and reclassified. Our classification for the production accounts is closely related to that of the CSO's own Production Accounts which distinguish 37 industries. The only changes we have made are to distinguish coke ovens and petroleum products and to divide up miscellaneous services into business services, professional services and the remainder. These differences reflect one of the uses of this accounting system; it provides the data base for the Cambridge Growth Project's multisectoral dynamic model of the U.K. economy (Barker *et al.*, 1980; Barker and Peterson, forthcoming).

The main problem with the official make matrix, SNA Block 5.3, is the suppression of entries in order to preserve commercial confidentiality. These entries are all off-diagonal elements shown as zero in the tables but included in row and column totals. Since we have no idea where these discrepancies should be allocated, we have put them into a special "unallocated" row and column.

The absorption matrix, SNA Block 3.5, is estimated from two official matrices of absorptions of domestic output and of imports. These are reclassified and aggregated, then added together with an allowance for protective duties on imports which are allocated across all commodity absorptions according to their import content.

The purchases of commodities by government, consumers and industry (for investment) present more difficulties. The body of the official tables is valued at purchasers' prices, with row totals for deductions of net taxes, distribution margins and imports being shown separately. Since SNA Blocks 3.6, 3.8, 3.17, 3.18 and 3.19 are valued at basic values, net taxes and distribution margins are allocated across the rows of the official tables before they are reclassified and aggregated. The consumers' expenditure matrix, SNA Block 3.8, was further disaggregated by column to correspond to the detailed time series available in the U.K. *National Income and Expenditure.* Again protective duties on imports are allocated to the absorptions according to import content.

The fact that we have had to allocate these margins and taxes across rows makes these final demand absorptions somewhat less reliable than the industrial absorptions and this has been taken into account in the balancing exercise (see section 6).

Stockbuilding data in SNA Blocks 3.15 and 3.16 are derived by assuming that industrial stocks of work in progress and finished goods are the principal products of the industry in question.

Exports and imports of goods and services are estimated from the CSO Production Accounts, rather than from the updated input–output tables, as this source has a more accurate allocation.

Commodity taxes are allocated to sectors purchasing taxed goods on the basis of data supplied by the CSO, supplemented by details of fuel oil purchases (Digest of Energy Statistics) and data on VAT and Miscellaneous Taxes from the Department of Customs and Excise.

SNA account 7 "private services" has been adapted in the U.K. version to include two activities not readily allocated to industries or government. These are the ownership of dwellings and sales and purchases of second-hand equipment by final buyers. The data for both categories come from the input–output tables.

Table 3 shows the main aggregates in the row and column for the construction commodity. The initial estimates show that most of the demand for the commodity comes from fixed capital formation with repairs and maintenance from industry, government and consumers being less important and some work-in-progress, imports and exports completing the picture. The small discrepancy of −£115m between supply and demand is mainly due to revisions to the import figures. The balanced estimates are discussed in section 7 below.

TABLE 3

DEMAND AND SUPPLY OF THE CONSTRUCTION COMMODITY, U.K. 1975 (ACCOUNT 3-28)
(£ million)

SNA Block		Initial Estimate	Balanced Estimates: Neutral Reliability	Balanced Estimates: Subjective Reliability
	Demand			
3.5	Industrial input	4,008	4,012	4,019
3.6	Government current spending	959	959	893
3.8	Consumers' expenditure	1,605	1,596	1,565
3.15	Work in progress	−348	−346	−324
3.17	Industrial fixed investment	4,413	4,608	4,452
3.18	Social capital formation	4,023	4,050	4,038
3.19	Dwellings	1,982	1,816	1,968
3.24	Exports	134	134	134
	Total	16,777	16,829	16,746
	Supply			
5.3	Industrial output	16,415	16,358	16,274
24.3	Imports	477	471	472
	Total	16,892	16,829	16,746
0.3	Discrepancy	−115	0	0

Income–Outlay and Capital Accounts

The National Accounts of the United Kingdom provide much of the detailed information needed for the presentation of income–outlay and capital accounts in an SNA framework. For reasons connected with convenience in modelling, value-added in SNA account 11 contains a breakdown of factor income by originating sector. This split is achieved using data supplied by the Inland Revenue and is important, since it determines the extent to which profits flow directly into personal income and thus affect private consumption. The split is then reconciled with aggregate estimates published in the National Accounts as part of the balancing procedure. Data on factor taxes paid and subsidies received by industries were kindly provided by the CSO. Estimates of those falling elsewhere are based on a variety of diverse sources of information.

The matrices of institutional receipts and payments of various types of income in SNA accounts 12, 13 and 14 can be considered as direct analogies of the make and absorption matrices of accounts three and five, for GDP can be identified either in the Production Accounts netting out intermediate flows of goods or in the institutional accounts netting out transfers within and between sectors.

In our treatment, institutional sectors are identified at a finer level than is usual. In particular Life Assurance and Superannuation funds are distinguished from the rest of the personal sector, reflecting the increasing attention paid to the possibility that their aggregation in, for example, the estimation of consumption and savings functions, may be unjustified (Threadgold, 1978). It is however unfortunately not possible to separate small businesses from the "Household" sector. Financial Companies are distinguished from Industrial and Commercial Companies reflecting the need to distinguish their operations in model construction, and the Public Sector is identified in its three component parts, Public Corporations, Central Government and Local Government.

The classification of transfer payments is again pursued in detail. Different types of state benefit are distinguished and property income is broken down into various types of dividend and interest. This latter split has required the allocation of some published items on rather simple principles, with a low degree of accuracy. Inconsistencies are reconciled in the balancing process and the information contained in the marginal totals ensures that the balanced estimates are more reliable than the unbalanced ones.

The Capital Accounts are again derived from the National Accounts. Capital transfers are published and are of course mainly flows to and from the Government. The classification used here distinguishes three types of tax from other transfers; the dominant feature of these transfers is their great instability over time, a fact which adds to the difficulty met in forecasting financial saving. At present the financial accounts are not disaggregated. Only a consolidated financial asset is identified, but parallel work is proceeding on constructing a time series of data on financial acquisitions and holdings at a detailed level for use in an analysis of the financial system and the construction of a portfolio allocation model. It will eventually become possible to treat the financial sector with the same degree of detail as the rest of the economy.

Table 4 illustrates the capital account of the Household Sector cast in SNA format. Saving is measured net of depreciation, which therefore appears as a negative entry on the debit side. It can be seen that the unadjusted data published by the CSO cumulate the discrepancy involved in measurement of household sector accounts into the capital account, where it is entered as unidentified financial accumulation, and has a magnitude substantially larger than some of the identified elements in the accounts.

4. Discrepancies and Unidentified Items

We have identified four types of data problems in the initial estimates of the accounts:

TABLE 4
THE CAPITAL ACCOUNT OF THE HOUSEHOLD SECTOR, U.K. 1975 (ACCOUNT 23-1)
(£ million)

SNA Block		Initial Estimate	Balanced Estimate: Neutral Reliability	Balanced Estimate: Subjective Reliability
	Demand			
23.14	Savings (net)	1,997	2,307	3,119
23.21	Capital transfers received	423	438	427
	Total	2,420	2,745	3,546
	Supply			
11.23	Depreciation	−2,306	−2,759	−2,453
18.23	Social investment: education	64	63	64
	health	39	39	39
19.23	Investment in dwellings	2,146	1,979	2,132
20.23	Industrial investment	573	540	567
21.23	Capital taxes: CTT/Death duties	307	303	307
	Capital gains tax	400	390	400
	other	2	2	2
	Capital transfers paid	19	19	19
22.23	NAFA by households	2,680	2,169	2,469
	Total	3,924	2,745	3,546
0.23	Discrepancy	−1,504	0	0

1. Those data items which cannot be allocated, for example because of the need for confidentiality. We have these in SNA block 3.5 and they are simply left in the "unallocated" row and column (number 40).

2. The official residual errors. The input–output table puts the residual error between expenditure and output measures of GDP into services output. This has been taken out and put into the special account 0 for discrepancies (SNA block 0.3). The aggregate figure in Table 1 of −£306m is somewhat misleading, since many of the errors in the individual commodity accounts are substantially larger. The unidentified domestic acquisition of financial assets also appear as −£356m (SNA block 0.23).

3. Differences between sources of data. These have been kept to a minimum because we have used the 1980 edition of *National Income and Expenditure* as our source for incomes and outlays, and these were consistent with the input–output tables for 1975, the source of the production data. The most serious discrepancies of this type are in the accounts for commodity taxes where we had estimates of total tax revenues by type of tax from the National Accounts and the allocation of total taxes by industry and final demand sector (SNA blocks 0.4, 0.5, 0.6, 0.8 and 0.11).

4. Allocation errors. In several instances no information was available to us on the allocation of a total among its components. Even when some information is available, as in the case of the distribution margins on consumers' expenditure, the allocation often follows a simple proportional rule. We have set the subjective

reliabilities on individual accounts and on blocks of accounts to reflect these errors and their sources (see section 6).

Although in the accounts presented by the CSO the residual error of the real economy balances the sum of the unidentified net acquisitions of financial assets, our data construction contains many discrepancies in the other data accounts. Nevertheless the accounting constraint, that the sum of all discrepancies is zero, is retained by virtue of the accounts being a closed system.

5. Different Approaches to the Data Reconciliation Problem

In the previous section we observed that the income, expenditure, production and financial estimates of data are typically inconsistent. The presence of such accounting inconsistencies and conflicting views on the same pieces of data emphasises the unreliable nature of economic data. Some statisticians are prepared to assign relative degrees of reliability to the various items of data collected (e.g. Maurice, 1968) and one of the objectives of this paper is to show how such information may be used to reconcile the initial estimates of a system of data. This is achieved by trading off the relative degrees of uncertainty of the various data items in the system in order to adjust the prior data to fit the accounting identities. This is essentially what national income accountants do during the last stages of compiling the accounts when faced with major discrepancies between data from different sources which need to be reconciled within the accounting framework. Here it is done in a more formal manner so that the reliabilities assigned by those who know the data and its derivation are used systematically in a procedure to balance the whole system of accounts. The technique is not a replacement for knowledge of the data and its sources but an enhancement of it, allowing us to produce fully balanced accounts with the adjustments reflecting the quality of the data.

The technique has firm Bayesian foundations (see van der Ploeg, 1982b, c) but may also be based on a generalised conditional least squares methodology. The result is a posterior set of balanced accounts with typically an improved level of accuracy, where the magnitude of the adjustment increases with the degree of constraint violation.

In practice some of the data may not be collected at all. Obvious examples are the items which follow from definitions (such as the balance of payments, savings, or profits net of depreciation). Less obvious examples are the unidentified items of net acquisition of financial assets by the various institutional sectors, since sometimes it is known that changes in certain financial assets and liabilities are simply not measured. One possible solution to the definitional items is to consolidate the accounts in order to remove these items from the accounts without suppressing any of the other items. A more general solution is to exploit some of the accounting restrictions in order to obtain an expression for some of the unknown items in terms of the known items. This solution suggests an identification condition: the number of unknown items must not exceed the total number of accounts which are linearly independent in these items, since the identification of each uncollected item requires the elimination of a separate accounting constraint.

Below we give a more formal procedure of the above discussion, based on the conditioned least squares technique, for removing inconsistencies in the national accounts and recovering unobserved items and sections 6 and 7 discuss a large-scale application.

Collect all the data items in the system of national accounts in the vector $\mathbf{p}$, which may be interpreted as a row-vectorisation of a social accounting matrix. Partition the vector of data such that $\mathbf{p}' = (\mathbf{p}_1', \mathbf{p}_2')$ where $\mathbf{p}_1$ denotes the unobservable items for which no prior estimate is available and $\mathbf{p}_2$ denotes the observed component of the data vector. Let the prior distribution of the measured account items be given by $\mathbf{p}_2 \sim D(\mathbf{p}_2^0, \mathbf{\Sigma}_{22}^0)$, where $\mathbf{p}_2^0$ and $\mathbf{\Sigma}_{22}^0$ denote the mean and covariance matrix of the initial estimates. The accounting constraints may be summarised by the system of simultaneous equations

$$\mathbf{H}_1\mathbf{p}_1 + \mathbf{H}_2\mathbf{p}_2 = \mathbf{0}, \tag{5.1}$$

where $\mathbf{H}_1$ and $\mathbf{H}_2$ are known matrices. There will be $n-1$ constraints, where n is the total number of accounts, since the n-th account follows from the first $n-1$ accounts by adding them up and is therefore unnecessary. A generalised least squares procedure for adjusting and recovering the national accounts proceeds by minimising the distance between the initial and final estimates or by minimising the weighted sum of squares

$$S_2 = (\mathbf{p}_2 - \mathbf{p}_2^0)'\mathbf{\Sigma}_{22}^{0^{-1}}(\mathbf{p}_2 - \mathbf{p}_2^0) \tag{5.2}$$

subject to the accounting restrictions (5.1). It can be shown that the optimal revised estimates of the mean values of the observed components of the data vector are given by

$$\mathbf{p}_2^* = \mathbf{p}_2^0 - \mathbf{\Sigma}_{22}^0\mathbf{H}_2'(\mathbf{H}_2\mathbf{\Sigma}_{22}^0\mathbf{H}_2')^{-1}(\mathbf{H}_1\mathbf{p}_1^* + \mathbf{H}_2\mathbf{p}_2^0) \tag{5.3}$$

and of the unobserved components of the data vector are given by

$$\mathbf{p}_1^* = -\{\mathbf{H}_1'(\mathbf{H}_2\mathbf{\Sigma}_{22}^0\mathbf{H}_2')^{-1}\mathbf{H}_1\}^{-1}\mathbf{H}_1'(\mathbf{H}_2\mathbf{\Sigma}_{22}^0\mathbf{H}_2')^{-1}\mathbf{H}_2\mathbf{p}_2^0. \tag{5.4}$$

The corresponding posterior covariance matrices are given by

$$\mathbf{\Sigma}_{11}^* = \text{cov}\,(\mathbf{p}_1) = \{\mathbf{H}_1'(\mathbf{H}_2\mathbf{\Sigma}_{22}^0\mathbf{H}_2')^{-1}\mathbf{H}_1\}^{-1} \tag{5.4}$$

$$\mathbf{\Sigma}_{12}^* = \text{cov}\,(\mathbf{p}_1, \mathbf{p}_2) = \mathbf{\Sigma}_{11}^* - \mathbf{H}_1'(\mathbf{H}_2\mathbf{\Sigma}_{22}^0\mathbf{H}_2')^{-1}\mathbf{H}_2\mathbf{\Sigma}_{22}^0 \tag{5.6}$$

and

$$\mathbf{\Sigma}_{22}^* = \mathbf{\Sigma}_{22}^0 - \mathbf{\Sigma}_{22}^0\mathbf{H}_2'(\mathbf{H}_2\mathbf{\Sigma}_{22}^0\mathbf{H}_2')^{-1}(\mathbf{H}_1\mathbf{\Sigma}_{12}^* + \mathbf{H}_2\mathbf{\Sigma}_{22}^0). \tag{5.7}$$

From the above; it is clear that a necessary and sufficient condition for the identifiability of the unobserved account items is invertibility of the matrix $(\mathbf{H}_2\ \mathbf{\Sigma}_{22}^0\ \mathbf{H}_2')$ and

$$\text{rank}\,\{\mathbf{H}_1'(\mathbf{H}_2\mathbf{\Sigma}_{22}^0\mathbf{H}_2')^{-1}\mathbf{H}_1\} = \dim\,(\mathbf{p}_1) \tag{5.8}$$

and a necessary condition for identifiability is $\dim\,(\mathbf{p}_1) \leq n-1$. In other words there must be a sufficient number of linearly independent accounting restrictions present in order to recover the unknown items.

The case of no unknown items corresponds to $\mathbf{p}=\mathbf{p}_2$ and yields

(5.9) $$\mathbf{p}^*=\mathbf{M}\mathbf{p}^0=\mathbf{p}^0-\boldsymbol{\Sigma}^0\mathbf{H}'(\mathbf{H}\boldsymbol{\Sigma}^0\mathbf{H}')^{-1}\mathbf{H}\mathbf{p}_0$$

and

(5.10) $$\boldsymbol{\Sigma}^*=\mathbf{M}\boldsymbol{\Sigma}^0=\boldsymbol{\Sigma}^0+(\mathbf{M}-\mathbf{I})\boldsymbol{\Sigma}_0$$

where $\mathbf{M}$ is an idempotent matrix defined by

(5.11) $$\mathbf{M}=\mathbf{I}-\boldsymbol{\Sigma}^0\mathbf{H}'(\mathbf{H}\boldsymbol{\Sigma}^0\mathbf{H}')^{-1}\mathbf{H}$$

and $\mathbf{H}=\mathbf{H}_2$. It follows from (5.9) that the adjustment $(\mathbf{p}^*-\mathbf{p}^0)$, depends on the magnitudes of the initial violations of the accounting identities $(\mathbf{H}\mathbf{p}_0)$. Because the matrix $(\mathbf{M}-\mathbf{I})$ is negative semi-definite, the adjustment procedure typically reduces the variance although this result is contingent on the initial variance matrix $\boldsymbol{\Sigma}^0$ being correct. It is expensive to allow for the treatment of unmeasured account items with the aid of (5.3) and (5.4). Instead Byron (1978) and van der Ploeg (1982a) show that a good approximation is to treat the unknown items as observed items by including them in $\mathbf{p}=\mathbf{p}_2$, assuming $\mathbf{p}_1\sim D(\mathbf{0},\psi\mathbf{I})$, where ψ is a large positive scalar, and applying (5.9) and (5.11) (with $\mathbf{H}=(\mathbf{H}_1\mathbf{H}_2)$ and $\mathbf{p}'=(\mathbf{p}_1',\mathbf{p}_2')$) instead of (5.3) and (5.4). This approximation becomes perfect as ψ tends to infinity and simplifies the computations considerably.

It can be shown that $\mathbf{p}^*$ is the best (minimum-variance) linear unbiassed estimator (BLUE) of $\mathbf{p}$, which corresponds to a version of the Gauss–Markov theorem. If one is prepared to assume that the initial estimates are normally distributed $(D=N)$, then a version of the Rao–Blackwell theorem suggests that $\mathbf{p}^*$ is the best general unbiassed estimator of $\mathbf{p}$.

Further details on the theory of the adjustment of prior data with subjective estimates of reliability and applications may be found in Byron (1978), van der Ploeg (1982a, b, c) and Stone (1982), which are extensions of the pioneering article by Stone *et al.* (1942).

The U.K. system of accounts discussed in this paper implies 26 independent accounting constraints. This implies that 19 K is needed for the storage in single precision on a digital computer of the matrices $\mathbf{H}$ and $\boldsymbol{\Sigma}^0$ alone. Even if $\boldsymbol{\Sigma}$ is diagonal and the system of accounts (and therefore the vector of variances) is only 15 percent full, one requires almost 2000 K storage for the matrices $\mathbf{H}$, $\boldsymbol{\Sigma}^0$ and $(\mathbf{H}\ \boldsymbol{\Sigma}^0\ \mathbf{H}')$ alone and the inversion of a 261×261 matrix. These storage and computational requirements are incompatible with current computer technology, hence the appendix develops an alternative formulation that is feasible and solves our problem.

6. Reliability of the Data

This section discusses a parsimonious means for assigning the reliability of a data system of the type discussed in sections 2–4. Consider any submatrix of the system of national accounts, say $\mathbf{A}_{ij}$, where i and j denote the row and column block-account respectively. When covariances are ignored, the variances of $\mathbf{A}$ may be written in the matrix $\mathbf{V}$ (the diagonal of $\boldsymbol{\Sigma}$). A reasonable choice of $\mathbf{V}$ is perhaps (cf. van der Ploeg, 1982a)

$$\mathbf{V}_{ij} = \theta_{ij}\hat{\boldsymbol{\rho}}_i(\mathbf{A}^0_{ij} \cdot \mathbf{A}^0_{ij})\hat{\boldsymbol{\rho}}_j \qquad (6.1)$$

where "·" denotes element by element multiplication, the vector $\boldsymbol{\rho}_i$ denotes the "standard deviation" of the accounts in block i expressed as a proportion of the mean values, $\mathbf{A}^0_{ij}$, and θ_{ij} is a scalar included to increase or decrease the reliability of the SNA Block (i, j). In other words the basic standard deviation of any account item is a geometric average of the corresponding row and column standard deviation, although it is possible to modify this basic rule in order to allow for differential block reliabilities. The bi-proportional nature of (6.1) reminds one of the RAS-technique (Bacharach, 1970), although we hasten to add that our method (in contrast to RAS) allows for differential bi-proportional reliability *based upon subjective views.*

Tables 2 and 5 give our estimates of $\boldsymbol{\rho}_i$ and θ_{ij}, which are partially based on Maurice (1968) and discussions with the U.K. CSO reported in van der Ploeg (1982a). Considering the block-reliabilities first, these reflect the disparity of sources from which the data are derived. Thus commodity absorptions by final demand are collected differently from estimates of intermediate flows and are regarded as being less reliable (with an index of 2). Taxes paid on intermediate demand are estimated by the CSO who kindly supplied us details, while those falling on final demand reflect our own allocation and are therefore more unreliable ($\theta_{4,j} = 4, j \neq 5$). Total commodity taxes are accurately known from details of Government revenue, hence $\theta_{i,4} = 0.1$. Savings, net profits and balance of payments contribute no information to the system since they are derived as residuals. They are therefore conceptually equivalent to unknown data (a priori) and have been given an infinite variance.

TABLE 5

SUBJECTIVE RELIABILITIES FOR BLOCKS IN THE U.K. SYSTEM OF NATIONAL ACCOUNTS (θ_{ij})†

SNA Block			
Row	Column	Description	Unreliability Index
3	6	Commodity absorptions by final demand	2
3	8		2
3	15		2
3	17		2
3	18		2
3	19		2
4	6	Commodity taxes paid by govt.	4
4	8	Commodity taxes paid by h'holds	4
4	17	Commodity taxes paid on GFCF	4
5	4	Total commodity taxes	0.1
11	4		0.1
12	11	Net profits	∞
23	14	Savings	∞
24	24	Balance of payments	∞

† $\theta_{ij} = 1$ for all other blocks.

As an illustrative example of the construction of an individual standard deviation, demand for dwellings (Table 3) is considered. This appears in SNA Block 3.19 and is constructed from construction or commodity 28 (Table 2). Thus:

$$\left.\begin{aligned}\boldsymbol{\rho}_3(28) &= 0.15\\ \boldsymbol{\rho}_{19}(1) &= 0.04\end{aligned}\right\} \quad \text{Table 4}$$

$$\theta_{3,19} = 2 \qquad \text{Table 5}$$

$$\mathbf{A}_{3,19}(28,1) = £1{,}982\text{m} \qquad \text{Table 2.}$$

The standard deviation of this estimate is therefore from (6.1) given as

$$1982 \times \sqrt{2 \times 0.15 \times 0.4} = £217.1\text{m}.$$

Comparing different accounts, stockbuilding (accounts 15 and 16) and the financial accounts (22) are taken to be much more unreliable than the other accounts. There are also considerable differences between the reliabilities within individual accounts. For example services are in general measured with much greater error than goods in the production accounts. The indices of unreliability for blocks in the SNA given in Table 5 allow us to modify the reliabilities obtained from row and column factors. These take into account the lower precision of some commodity flows to final demand, the higher precision of total tax revenues (SNA Blocks 5.4 and 11.4) and the fact that certain blocks are estimated as residuals and should be treated as without prior information content in themselves.

We may contrast the subjective choice of reliabilities (6.1) with a neutral variant defined by

$$\mathbf{V}_{ij} = \theta(\mathbf{A}_{ij}^0 \cdot \mathbf{A}_{ij}^0) \tag{6.2}$$

where the magnitude of the common reliability parameter, θ, does not affect the adjustment of the accounts as only relative reliabilities matter.

It should be realised that in collecting the data one should make use of multiple prior estimates of the same item of data, say the K estimates $\{A_{ij}^{0,1}, \ldots, A_{ij}^{0,K}\}$, of A_{ij}. This may be done by constructing the weighted prior (see van der Ploeg, 1982b)

$$\mathbf{A}_{ij}^0 = \sum_{k=1}^{K} (\mathbf{A}_{ij}^{0,k}/\mathbf{V}_{ij}^k)\mathbf{V}_{ij} \tag{6.3}$$

where the variance of the weighted prior is given by the harmonic mean

$$\mathbf{V}_{ij} = \left[\sum_{k=1}^{K} (1/\mathbf{V}_{ij}^k)\right]^{-1} \tag{6.4}$$

The advantage of multiple prior estimates is that they provide an opportunity to reduce the variance of the item concerned, since $V_{ij} \le \min_k (V_{ij}^k)$.

An approximation to the posterior variance may be derived as

$$\boldsymbol{\Sigma} = \boldsymbol{\Sigma}^0(\mathbf{I} - \mathbf{H}\boldsymbol{\lambda}\mathbf{p}^{0\prime})$$

since

$$\boldsymbol{\lambda} = (\mathbf{H}\boldsymbol{\Sigma}\mathbf{H}')^{-1}\mathbf{H}\mathbf{p}^0$$

and

$$\mathbb{E}(\mathbf{p}^0\mathbf{p}^{0\prime}) = \mathbf{\Sigma}^0$$

where the t-ratios so derived have a Cauchy distribution. This approximation suffers from the drawback of not being symmetric although its expected value of course is. The more computationally expensive approach of obtaining posterior standard deviations from a likelihood-ratio test, that is the posterior standard deviation for item k is calculated from $\{\Delta p(k)/\sqrt{S^c - S^*}\}$ where S^c is the sum of squares obtained by fixing item k at $\{p^0(k)+\Delta p(k)\}$ and $\Delta p(k)$ is an infinitesimal change, is preferred and used in section 7.

7. A Balanced System of National Accounts

The aggregated SNA in Table 1 above shows two sets of balanced estimates besides the original, unadjusted ones. The first set assumes no information on the relative reliability of the estimates and they are all given the same (neutral) measure of the reliability (equation (6.2)). The second set is calculated using the estimates of subjective reliability (equation (6.1)) given in Tables 2 and 5 (see section 6). The sources of the original estimates are described in section 3 above: here we look at the results of the two balancing exercises.

The largest single residual in the 262 accounts of the system is the unidentified net acquisition of financial assets (NAFA) in the household sector. This residual is in the financial capital account (SNA account 23) where savings plus transfers for households are £1,504m less than finance of dwellings, identified NAFAs and other capital finance (see Table 3). This account and its adjustment is discussed in detail below, but it is relevant to the general picture because many of the larger balancing adjustments in the rest of the system follow from the removal of this discrepancy.

The first point to note is that, although in many accounts the discrepancy is quite small, the adjusted estimates might change by much more. For example, in the commodity accounts, the discrepancy is only −£306m, yet the subjective balanced estimate of industrial output is £1,404m below the original figure (see SNA block 5.3). A large part of the adjustment can be traced back to a downwards revision of £889m in consumers spending on commodities (block 3.8), which is in turn due to the removal of the unidentified household NAFA mentioned above.

A second point is that the differences between the neutral estimates and the subjective ones are substantial. This shows the potential value of including information on the likely magnitude of the errors in the data. It also turns out that putting no reliability at all on residuals, such as savings, included in the system has an important effect on the results. Indeed it is this feature which is responsible for the much greater movement in the subjective estimates for the commodity aggregates compared to that in the neutral estimates.

In order to follow through these points and give a flavour of the detail available in the accounts we have taken two examples: first the demand and supply of the construction commodity presented in Table 3 and second the capital accounts of the household sector presented in Table 4. Let us now consider the effects of the balancing exercise in these examples.

The balanced estimates for construction in Table 3 illustrate three features of the balancing method. First, since the supply measure is taken to be less reliable than the demand measures, the balanced estimate with subjective reliabilities puts most of the discrepancy between supply and demand into the supply estimate. This falls from £16,892m to £16,746m whereas the demand estimate is hardly affected.

Second there are upward movements in the balanced estimates for investment and stockbuilding and downward movements in those for current spending by consumers and government. Tracing these back through the accounts, it turns out that they are caused by the substantial discrepancies in the capital accounts of institutions (SNA block 0.23). These are due to unidentified transactions for the personal sector (£1,504m) and for industrial and commercial companies (£791m). Balancing the accounts means that personal and government sector savings both increase whilst current spending falls to compensate. The reduction in spending on construction goods is part of this adjustment. Note that in the balanced accounts with neutral reliability, much more adjustment has come through a downward reduction in dwellings. This is due to the fact that savings have been given the same degree of reliability as all the other estimates; with subjective estimates, savings have been given an infinite variance reflecting its status as a residual.

A third feature of the balancing method brought out by the construction table is the greater movement in the final demand estimates than in intermediate demand. This is due to the higher reliability of intermediate demand in that it required fewer alterations for changes in definition when the initial estimates were compiled.

Table 4 illustrates the effects different reliabilities can have on the various estimates for the household capital account. In this framework the reconciliation of the account, removing the unidentified net acquisition of financial assets (which is treated as the residual), is achieved by an increase in saving on the credit side, and substantial reductions to depreciation, investment in dwellings and net acquisition of financial assets on the debit side.

However this neutral variant does not satisfactorily take into account the manner in which the data were originally derived. Savings are the balance between income and expenditure and are not independently observed. Thus the variance of savings is in effect infinite and is constrained *ex post* by the fact that income and expenditure have finite variances. In the same way net profit is derived from gross profit and an estimate of depreciation. It is not independently observed. Its *ex ante* estimate too must be given infinite variance.

These observations imply that much of the adjustment will be borne by savings. Relatively small percentage changes in income and expenditure can together imply a large adjustment to savings. A comparison of the unbalanced with the subjective variant shows that the adjustment to household saving required takes it from an initial estimate of £1,997m to a neutral balanced estimate of £2,307m and to a subjective balanced estimate of £3,119m. This is achieved by a change of about $1\frac{1}{2}$ percent to consumption and about 1 percent to transfers paid: neither of these are particularly large but they do illustrate clearly the uncertainty of savings estimates.

A substantial adjustment is borne by the net acquisition of financial assets (SNA block 22.23), although not of the same magnitude as that made to saving. In this example it was assumed that the percentage standard error to net acquisitions of financial assets was 50 percent. This is of course large compared with almost every other non-residual item in the matrix. But the assumption that the estimates of the net acquisition of financial assets contain some information and therefore has finite variance implies that, in this example, the bulk of the adjustment will be borne by the residual item, savings. The net acquisitions of the household sector change from £2,680m to the neutral balanced estimate of £2,169m and to a subjective balanced estimate of £2,469m.

These are the dominant differences between the neutral and the subjective results. Although it has also been assumed that household sector accounts are less reliable than corporate sector accounts, the influence of this assumption is not clearly visible in a comparison of the three sets of accounts, for the adjustment arises not only from the variance but also from the magnitude of the discrepancy in the relevant account. Capital Transfers (SNA blocks 21.23 and 23.21) are believed to be unreliable. The fact that they are scarcely changed in the adjustment process does not alter this belief, although a large adjustment can on the other hand be regarded as providing or confirming evidence of a large variance.

The adjustment procedure typically reduces the standard error. For example, the prior standard error of the demand for dwellings, item (3-28, 19-1), is £217.1m (or 11.0 percent) and the posterior standard error is £122.3m (or 6.2 percent). Hence, the enforcement of accounting identities typically improves the reliability of the data.

8. Concluding Remarks

The application of balancing procedures to large systems of accounts provides a systematic means of removing discrepancies and enhancing data accuracy. We hope that the development of such techniques will encourage official statisticians to publish their unbalanced data, if possible with some estimate of the relative accuracy of the data, and that economists will pay greater attention to the reliability of the data they analyse.

Future efforts are directed at four extensions of the techniques discussed in this paper. Firstly, the construction of a time-series of detailed accounts poses new conceptual problems. Van der Ploeg (1982a) shows that the presence of systematic, cyclical and/or seasonal discrepancies requires the simultaneous adjustment of the accounts for all years of the sample period. In any case there are clear economic linkages between the accounts of subsequent years, since the closing assets of one year plus financial accumulation (column accounts 23) and revaluations yield the opening assets of the following year.

Secondly, the simultaneous balancing of constant and current price accounts is necessary when independent price data are available (Weale, 1982). If one is prepared to assume that the independent price data are perfectly known, the adjustment problem remains linear quadratic although strong covariances are introduced. The general case of unreliable price data makes the simultaneous balancing problem nonlinear and therefore iterative quasi-Newton techniques will be required.

Thirdly, we assumed in the present exercise that no further revisions of the data will occur. This is not too bad an approximation, since we considered a seven year time-lag between the relevant data used and the date of publication. However with shorter time-lags revisions may well be important and, if the revisions follow a systematic pattern, the data should be modelled as a dynamic time-series model and then filtered with the aid of an optimal observer (e.g. Harvey *et al.*, 1981). The problem with such previous studies is that they ignore the economy-wide interactions in the interactions, since they model the revisions as univariate time-series models. Future work might look at the interactions between balancing and filtering of revisions.

Finally, the fact that economic data are unreliable forces econometricians to develop a new methodology for taking account of the influence of measurement errors on parameter estimates (van der Ploeg, 1982c). At the same time as estimating the parameters the econometrician is forced to estimate data and the final estimates of the data as well as the parameters will therefore depend upon implicit or explicit economic models.

APPENDIX: THE ADJUSTMENT ALGORITHM

This appendix reformulates the least squares adjustment algorithm (5.9)–(5.11) to exploit constraint sparsity and the specific structure of the economic accounting matrix. The reformulation avoids the specification of **H** and the storage of **H**, **Σ** and (**HΣH**′), so that storage (and time) requirements are minimal. Section 6 defined the variances of the prior estimates as (6.1). The matrices $\mathbf{A}^0$ and **V** are very sparse, which is mainly due to the block-structure of systems of accounts (see Table 1), although elementary sparsity in the submatrices $\mathbf{A}^0_{ij}$ and $\mathbf{V}^0_{ij}$ may also be important. Let the structure of the matrices **A** and **V** therefore be defined by a set of indices, say I_A, such that $\mathbf{A}_{ij}$ and $\mathbf{V}_{ij}$ are zero when $i, j \in I_A$. The accounting identities (5.1) may now be rewritten as

$$\sum_j \mathbf{A}_{ij} = \sum_j \mathbf{A}'_{ji}, \forall i, \qquad \forall i, j \in I_A.$$

The optimal adjusted system of accounts is given by

$$\mathbf{A}^*_{ij} = \mathbf{A}^0_{ij} + \hat{\boldsymbol{\lambda}}_i \mathbf{V}_{ij} - \mathbf{V}_{ij} \hat{\boldsymbol{\lambda}}_j$$

and the Lagrange multipliers, $\boldsymbol{\lambda}$, follow from solving the symmetric system of equations $\mathbf{B}\boldsymbol{\lambda} = \mathbf{b}$, where

$$\mathbf{B}_{ij} = \mathbf{V}_{ij} + \mathbf{V}'_{ji} = \mathbf{B}'_{ji}, \qquad \forall i \neq j \in I_A$$

$$\mathbf{B}_{ii} = \sum_{j \neq i} (\mathbf{V}_{ij} + \mathbf{V}'_{ji}), \qquad \forall i, j \in I_A$$

and the initial constraint violation is defined by

$$\mathbf{b}_i = \sum_j (A^0_{ij} - A^{0\prime}_{ji}).$$

To ensure a positive definite **B**, the final row and column of **B** are deleted and the final Lagrange multiplier is set to zero. This is necessary, since the final account is linearly dependent upon the others. Observe that for account items with a zero variance, for diagonal items, or for zero constraint violations, the adjustment is zero. Van der Ploeg (1982b) shows how the above can be modified in order to take account of prior information on certain totals (see section 6).

Normally one would obtain $\boldsymbol{\lambda}$ as $\boldsymbol{\lambda} = \mathbf{B}^{-1}\mathbf{b}$, but this is infeasible from a computational and storage point of view. Instead **V** is stored implicitly in terms of $\mathbf{A}^0$ (see (6.1) or (6.2)), **B** is not stored but derived from **V**, and the system $\mathbf{B}\boldsymbol{\lambda} = \mathbf{b}$, is solved by means of the conjugate gradient algorithm with a suitable scaling procedure (see van der Ploeg, 1982b). A fast computer program, documented in a Rocket Program Paper entitled "An Algorithm for Balancing Large Systems of National Accounts" by van der Ploeg, may be made available upon request.

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