ON AGGREGATION METHODS FOR INTERNATIONAL COMPARISONS

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This paper brings together discussions of Geary–Khamis indexes now available only in scattered sources, and considers their application to a range of uses. The first section traces the development of the method from its initial proposal by Geary in 1953, with the aid of a numerical example illustrating differences among various formulations. The second section considers the least squares properties of Geary–Khamis indexes and some related variants. The final section considers adjustments to the method required for regionalization and spatio-temporal bilateral and multilateral comparisons, as well as to take account of the nature of available data.

INTRODUCTION

Prior to the launching of the International Comparison Project (ICP) in the late sixties (Kravis *et al* 1975, 1978, 1982), multilateral measurement of real gross domestic product (GDP) or its components and of purchasing power parities (PPPs) was generally made through the adaptation of national practices in compiling temporal national price and quantity indices and the conventional approximation to real GDP and other national accounting aggregates by expressing them in terms of constant price series. Use was also made of official exchange rates for international comparison, but this was well recognized to be unsatisfactory. It should be noted, however, that for bilateral inter-country comparisons, even the ICP methods are still confined to the adaptation of national practices used mainly in temporal national comparisons. The shortcomings and limitations of some of the national practices, although known, were not adequately taken into account when extending them to inter-country comparisons. Some of the major inadequacies of such extensions to multilateral measurements were recently very well elucidated in a report by Prof. T. Peter Hill prepared for the United Nations Statistical Office, the United Nations Economic Commission for Europe and the Statistical Office of the European Communities (EUROSTAT 1982). The report is a revision of an earlier draft by Prof. Hill and takes account of discussions which took place, in particular, at the United Nations Expert Group Meeting held in Bellagio in December 1980.

This paper, in addition to outlining the origin of the R. C. Geary definition of PPPs (Geary 1958) and its modification and amplification by the author (Khamis 1967, 1969, 1970, 1972), is mainly a follow-up of the Hill report, supplementing some of its main findings. A numerical illustration of the difference between the Geary proposals for measuring changes in regional and world agricultural production quantity indices and those of the author is given.

EARLY EVOLUTION OF THE GEARY PROPOSAL

In 1953, R. C. Geary, in his capacity as consultant to FAO, proposed that instead of FAO calculating national, regional and world temporal production indices using wheat based uniform weights[1] it would be advisable to keep unchanged the national Laspeyres quantity indices and aggregate the values in the numerators and denominators of the national indices through appropriate exchange rates referring to the base period. For this purpose, Geary proposed the use of exchange rates e_{jt} defined by

$$(1) \qquad e_{jt} = \sum_i p_{it}q_{ijt} \Big/ \sum_i p_{ijt}q_{ijt}, \qquad j = 1, 2, \ldots, M$$

where j refers to countries and t to time, p_{ijt} and q_{ijt} refer to the corresponding national average price and quantity produced during the period t and where p_{it} is an average regional or international price of the commodity i in the period t expressed in a uniform currency and defined by

$$(2) \qquad p_{it} = \sum_j e_{jt}p_{ijt}q_{ijt} \Big/ \sum_j q_{ijt}, \qquad i = 1, 2, \ldots, N.$$

The time subscript t is added by the author to facilitate later discussions in this paper. The Geary proposal was concerned with the exchange rates for a single time base period and hence the subscript t was not explicitly required. Geary pointed out that, because of the identity

$$(3) \qquad \sum_j \sum_i e_{jt}p_{ijt}q_{ijt} = \sum_j \sum_i p_{it}q_{ijt}$$

derived from each of the linear homogeneous set of M and N homogeneous equations (1) and (2) in the variables e_{jt} and p_{it}, at least one non-trivial solution exists (i.e. a solution other than that where all the e_{jt} and p_{it} are zero). The necessary and sufficient condition for the existence of a positive and unique solution (apart from a single arbitrary multiplicative parameter) for the e_{jt} and p_{it} of the Geary equations (1) and (2) was given by the author (Khamis 1970 and 1972) using properties of non-negative diagonally dominant matrices. This condition simply requires that, for non-negative quantities q_{ijt}, any proper subset of one or more countries of the set of M countries will have at least one commodity with positive quantities in common with the remaining subset of countries. This condition is generally satisfied in all cases where price and quantity comparisons are called for. The $M+N$ equations (1) and (2) are easily and meaningfully interpreted by comparing them with the usual methods of averaging national prices and defining PPPs (the reciprocal of the exchange rate) subject to the necessity for converting different national currencies into a uniform one to enable the aggregation of the values of the commodities over the different countries. Thus, the average Geary commodity i price of equation (2) for two or more countries is the total value of all transactions in the commodity divided by the sum of all the transacted quantities after converting the individual countries' values into a uniform currency through the exchange rates defined by equation (1). The definition of exchange rates proposed by Geary is, perhaps, the best that

[1]Recently FAO shifted to the use of *official* exchange rates to aggregate national Laspeyres indices for assessing changes in regional and world agriculture. The FAO is, however, reviewing the methodology used in calculating these indexes.

can be made in view of the relation amongst these rates and the average prices p_{it} expressed by identity (3). This identity, in addition to its enabling the existence of non-trivial solutions to the Geary equations, expresses the highly desirable property that, for a time period t, the sum of the transaction values over all countries and commodities is the same regardless of whether the average Geary prices p_{it} or the converted national commodity prices $e_{jt}p_{ijt}$ are used to value the transacted quantities. It is important to note that this identity holds for prices and quantities relating to the same period t.

The importance of identity (3) becomes apparent when one inspects the Geary equations as originally evolved by him for FAO and published much later (Geary 1958). Geary expressed no concern then with his average prices p_{it} except as a means to obtain exchange rates or purchasing power parities more appropriate for international production indices. As he pointed out, the national Laspeyres indices of agricultural production (assuming $i = 1, 2, \ldots, N$ refers only to agricultural commodities) defined for country j and period t with respect to the base period 0 by

$$(4) \qquad Q_{0t}^{L_j} = \sum_i p_{ij0} q_{ijt} \Big/ \sum_i p_{ij0} q_{ij0}, \qquad t = 1, 2, \ldots$$

would be aggregated over countries through the formula

$$(5) \qquad Q_{0t}^{LG} = \sum_j \sum_i e_{j0} p_{ij0} q_{ijt} \Big/ \sum_j \sum_i e_{j0} p_{ij0} q_{ij0},$$

where L refers to Laspeyres and G to Geary. Identity (3) obviously holds *only* for the base period because only base period exchange rates and average prices are proposed by Geary for aggregation. The rationale behind Geary's proposal to FAO to use equation (5) for aggregation is to ensure that the national series of quantity indices (4) are kept intact. This rationale is well appreciated for political reasons because national statisticians may face unenlightened but embarrassing questions from national authorities as to why country indices calculated by an international agency differ from their national indices. The proposed Geary formula (5) may be expressed in the equivalent form

$$(6) \qquad Q_{0t}^{LG} = \sum_j Q_{0t}^{L_j} \left(\sum_i e_{j0} p_{ij0} q_{ij0} \right) \Big/ \sum_j \sum_i e_{j0} p_{ij0} q_{ij0}$$

which is a weighted average of the national Laspeyres quantity indices using *the base period* national value ratios as weights where the values are expressed in a common currency unit through the base period Geary exchange rates.

While equation (6) defining the Geary index of world or regional agricultural production might be useful for certain analytical purposes, and is certainly superior to the use of official exchange rates, the averaging of Laspeyres indices of production using for a second time a Laspeyres type formula (i.e. with base period weights) may result in the accumulation of the effect of the Laspeyres formula in so far as it *usually* (but not necessarily always) overstates the index number as compared with e.g. a Paasche type formula. The author (Khamis 1967, 1972) pointed out that the average prices p_{it} (not utilized by Geary except for defining the exchange rates e_{jt}) are at least equally important for international comparisons work, especially for the valuation of national aggregates at international prices. Instead of using the Geary aggregation of equation (5) to calculate regional and world production indices, the author proposed the alternative quantity or production index

$$(7) \qquad Q_{0t}^{LK} = \sum_j \sum_i p_{i0} q_{ijt} \Big/ \sum_j \sum_i p_{i0} q_{ij0}.$$

This index may be expressed in the equivalent form

$$(8) \qquad Q_{0t}^{LK} = \sum_i (q_{it}/q_{i0}) w_{i0}$$

where

$$(9) \qquad q_{i0} = \sum_j q_{ij0} \quad \text{and} \quad q_{it} = \sum_j q_{ijt}, \qquad t = 1, 2, \ldots$$

and where

$$(10) \qquad w_{i0} = p_{i0} q_{i0} \Big/ \sum_i p_{i0} q_{i0}.$$

The summation over countries of the quantities q_{ijt} in equation (9) is consistent with Geary's summation in the denominator of his equation (2), especially because of the assumption that the commodity i is of identical specification in all countries. The advantage of the author's alternative index of equation (7) as compared with Geary's index of equation (5) or (6) may be illustrated by the property that the former yields the correct index when the quantity relatives q_{it}/q_{i0} of the regional or world total production of each commodity i is a constant c independent of i. In this case we do know that the correct index is c and this is obtained using the author's index (7). This property is not shared by the Geary index (5) nor by the use of official exchange rates.

The difference between the indices (5) and (7) was illustrated numerically by the author (Khamis 1972, page 110) by a hypothetical set of data. It is useful to illustrate this difference in a somewhat more realistic situation and at the same time make possible the comparison with indices based on the use of official exchange rates. We quote here the results of calculations limited to three countries and five commodities, where all the base year price data and the quantity data for the last three commodities correspond to actual figures and the quantity data for the first two commodities have been slightly adjusted to satisfy the relation $q_{1t}/q_{10} = q_{2t}/q_{20} = 1$ (see equation (9) above). The data used in the calculations are given in the following table, with the prices referring to those at the farm gate or the nearest point of sale and the quantities to commodity production. These calculations were made by Messrs. Sami Zarqa and Josef Krane of FAO.

When the Geary base year exchange rates and average prices are calculated using only the data shown for the first two commodities, the following index numbers of aggregate agricultural production of the three countries for the current

	Commodity	Average National Price (Base Year)	Quantity (Base Year)	Quantity (Current Year)
Country 1	1	842	20,093	19,093
	2	728	63,338	63,000
	3	609	2,716	2,916
	4	572	7,486	7,986
	5	619	8,105	8,805
Country 2	1	840	2,676	3,376
	2	1,190	405	743
	3	840	237	200
	4	900	8,879	9,179
	5	650	2,747	2,500
Country 3	1	3,781	115	415
	2	5,942	6	6
	3	—	0	0
	4	1,410	42	42
	5	1,437	1,534	1,284

year with respect to the base year are obtained:

Using base year official exchange rates	98.46 percent.
Using Geary's exchange rates (formula (5))	101.50 percent.
Using average prices (author's index, formula (7))	100 percent.

Obviously the correct index is 100 as the total production over the three countries of each of the first two commodities for the current period is the same as the corresponding total for the base period. When the same calculations are repeated using the whole set of data for the five commodities and the three countries, the following corresponding indices are obtained:

Using base year official exchange rates	101.45 percent.
Using Geary's exchange rates (formula (5))	101.95 percent.
Using average prices (author's formula (7))	100.86 percent.

It is expected that the small differences amongst the indices obtained by the three alternative aggregation methods illustrated above may become somewhat smaller or larger in absolute terms when more countries and commodities are included. Such small differences may, however, be *relatively* high when the basic concern is to measure, forecast or analyse the *rates* of change over time in aggregate production or real GDP. It is usually necessary to ensure obtaining the most accurate estimates of changes and rates of change.

The above mentioned considerations show the importance of the Geary average commodity prices. When one is concerned only with national prices and quantities pertaining to the same period, identity (3) ensures that the use of Geary's exchange rates in conjunction with national prices leads to the same aggregate values over countries as those obtained through the use of Geary average prices in any *direct application* of equations (1) and (2). *When one deviates from this condition* there is need for much caution as to what one obtains, particularly as some of the desirable properties enjoyed by the Geary–Khamis average prices and exchange rates in the calculation of multilateral index numbers may be lost and/or the meaningfulness of the resulting aggregates or index numbers based on modifications of equations (1) and (2) may have to be appropriately interpreted. However, under this condition the quantity index proposed earlier by the author and defined directly for country k with respect to country s by

$$(11) \qquad Q_{sk,t} = \sum_i p_{it} q_{ikt} \Big/ \sum_i p_{it} q_{ist}$$

will be the same as that obtained through the application of the converted national prices, i.e.

$$(12) \qquad Q_{sk,t} = \sum_i e_{kt} p_{ikt} q_{ikt} \Big/ \sum_i e_{st} p_{ist} q_{ist}.$$

The corresponding exchange rate index or reciprocal of the PPP index also defined directly (Geary 1958, Khamis 1967, 1970, 1972) is given by

$$(13) \qquad E_{sk} = e_k / e_s = \mathrm{PPP}_s / \mathrm{PPP}_k = 1/\mathrm{PPP}_{sk}.$$

These indices together with those of equations (5) and (7) are free of the arbitrary parameter. When one is interested in the aggregates of the numerators and/or denominators of these indices the arbitrary parameter may be assigned any desired value for the purpose at hand. For many such purposes any one of the exchange rates e_{jt}, say for $j = k$, may be taken as unity or 100 and the country k then serves merely as a numeraire with no effect on the ratios of the e_{jt} and p_{it} for different j and i. Alternatively, for some puposes the international or average price of one of the commodities may be taken as unity or 100 and this commodity will then serve merely as a numeraire commodity. In any case, all the indices based on equations (11) or (12) and (13) satisfy the reciprocity, transitivity, restricted factor test and other properties of general concern to economic analysis, especially for multilateral comparisons and aggregations of GDP and its components over countries as already established through the ICP.

It should be noted that these properties were not sought on purpose by Geary in devising his ingenious definitions of exchange rates and international or average prices across countries. It just turns out that the Geary definitions through the meaningful equations (1) and (2) lead to indices satisfying these properties. This distinguishes the Geary definitions from other attempts which in most cases either artificially devise definitions of index numbers which satisfy some of these properties or which minimize a certain sum of squares and, to the extent possible, meet some of the desirable properties. As indicated above, assuming that all the commodity, quantitiy and price data are available, the Geary equations (1) and (2), or some variations thereon, lead to *direct* definitions of quantity and exchange rate or PPP indices and to aggregation methods which happen to satisfy the factor test in its restricted form. The other methods devised for the same purpose either derive one index from the other through the factor test and hence the derived index is an indirectly calculated one or result in indices which do not satisfy the factor test. However, in the ICP choice of the so-called Geary–Khamis method as the preferred aggregation methodology in comparison with other available techniques, adjustments and adaptations had to be made to take account of the nature of the national data available for measuring GDP. Such adaptations preclude the applicability of the direct derivation of the two indices and as a result one of them is generally derived indirectly. Perhaps this was behind the description of the Geary–Khamis method as leading to an indirect measure of one of the indices (EUROSTAT 1982, page 49). This does not, however, invalidate the property of the Geary–Khamis method as leading to the *direct* calculation of the two indices provided all the required data are available as entered for each commodity in equations (1) and (2).

Least Squares Properties of the Geary–Khamis Method and Variants and Alternatives

The reports on phases I, II and III of the ICP (Kravis *et al.* 1975 and 1978 and also 1982, pp. 71–94) contain most of the details on the characteristics of binary (bilateral) and multilateral aggregation and related methods for international comparisons of GDP and PPPs. The phase III report incorporates many of the additional clarifications contained in the Hill report (EUROSTAT 1982) which favour the use of the Geary–Khamis aggregation method. We confine ourselves in this section to the brief mention of some variants of the Geary–Khamis (henceforth referred to as GK) method, to its least squares properties and to a few remarks about some alternatives to it.

Some of the variations of and alternatives to the GK method are characterized by a least squares property which is considered as a major advantage. Least squares techniques are certainly important in statistical theory and practice when applied to data which are the result of observations on a randomly selected sample of the population. This is an essential qualification for the proper use and justification of the least squares methods of estimation in applied statistics. Such a justification is often not observed in numerous applications of the least squares method. However, a least squares property, with or without its statistical justification, may sometimes add to the understanding of the subject to which it relates. Also, in some cases, the least squares property of some theoretical or applied conclusions is retrospectively derived to further justify the conclusion reached by other means. There is nothing basically wrong with such retrospective application of least squares provided the theoretical or applied conclusion has some other deeper justification. The GK method based on equations (1) and (2) has its own merit in so far as it corresponds to usual practices in defining average prices and exchange rates or PPPs. However, it may be useful for further analysis to give below its least squares properties derived by the author as early as 1968.[2]

We consider first the least squares property of the Geary average prices. Assuming the prior choice of known and appropriate exchange rates e_{jt} (defined independently in any meaningful manner), we may assume that the unknown

[2]The author mentioned these properties orally on many occasions including the Ronneby 1971 conference of the IARIW. They are given here to make them known to a wider audience in response to enquiries from colleagues.

international commodity prices are characterized by the model

$$(14) \qquad e_{jt}p_{ijt} = p_{it} + u_{ijt}, \qquad i = 1, 2, \ldots N \text{ and } j = 1, 2, \ldots, M$$

where $e_{jt}P_{ijt}$ is the converted national price and u_{ijt} is a random variate with zero expectation and variance σ_1^2. It is interesting to note the intuitive meaning of model (14) which simply requires that the average price of a commodity differs from its converted national price by a random variable of zero expectation. This model, of course, implies important and intuitive restrictions on the choice of appropriate exchange rates e_{jt}. We then minimize with respect to the variates p_{it} the function

$$(15) \qquad f_1 = \sum_i \sum_j (p_{it} - e_{jt}p_{ijt})^2 q_{ijt}$$

where q_{ijt} are quantity weights. Equating to zero the derivative of f_1 with respect to each p_{it} we obtain the relation

$$p_{it} \sum_j q_{ijt} = \sum_j e_{jt}p_{ijt}q_{ijt}, \qquad i = 1, 2, \ldots, N,$$

which is the same as Geary's equation (2). Similarly, we assume that appropriate international prices p_{it} are chosen (defined independently in any meaningful way one desires) and that the unknown exchange rates are governed by the model

$$(16) \qquad p_{it}/p_{ijt} = e_{jt} + v_{ijt},$$

where v_{ijt} is a random variate with zero expectation and variance σ_2^2. Again, model (16) implies important and intuitive restrictions on the choice of the p_{it}. We then minimize with respect to e_{jt} the function

$$(17) \qquad f_2 = \sum_i \sum_j (e_{jt} - p_{it}/p_{ijt})^2 (p_{ijt}q_{ijt})$$

where the values of the commodities in national currencies are used as weights. Equating the derivative of f_2 with respect to each e_{jt} to zero we obtain

$$e_{jt} \sum_i p_{ijt}q_{ijt} = \sum_i p_{it}q_{ijt} \qquad j = 1, 2, \ldots, M$$

which is the same as Geary's equation (1). It is interesting to note that the Geary definition of exchange rates derived from (16) has the intuitive meaning that the exchange rate for a country is more or less determined by the ratios of the assumed average prices p_{it} of each commodity to their corresponding national prices and that these ratios would approximately be equal, apart from a random fluctuation whose expectation is zero. The use of value weights in defining the function f_2 and the value weights implicit in Geary's equation (1) are analogous to Walsh's use of value weights in his definition of PPPs (cf. Kravis *et al.* 1975, pp. 66 and 70). The use of value weights in defining the Geary e_{jt} and in the function f_2 and of quantity weights in defining the Geary average prices p_{it} and in the function f_1 are analogous to national practices when dealing with similar types of problems for national temporal comparisons. In fact, the use of two different functions f_1 and f_2 for the application of the weighted least squares technique is necessitated by the intuitive desiderata that prices are averaged by transaction quantity weights and PPPs are averaged through transaction value weights.

The models (14) and (16) are, however, almost identical and hence it appears most appropriate to use them together. This leads to the Geary equations (1) and (2). These considerations lead to another way of looking at the rationale of the GK method. Under the assumption that the necessary and sufficient condition established by the author for the existence of a unique solution is satisfied (as it almost always is) iterative procedures starting with any approximate set of average prices or of purchasing powers can be used successfully to solve the Geary equations as the convergence to the unique GK solution is then assured. Also, considerations of this kind, from another point of view, may have possibly been behind the remarks in the Hill report (EUROSTAT 1982, p. 61) as to other ways of looking at the GK method. We note here that the least squares properties based on models (14) and (16) together with their implication of identity (3) may be amongst the more important factors distinguishing the GK method from other existing techniques. However, it will be useful to consider some variants of the GK method obtained by using only one of the two minimization techniques or the corresponding set of Geary equations (1) and (2).

Once the Geary equations (1) and (2) and related mathematical properties and practical applications became known, it was expected that researchers would investigate the effect of some variations in these equations. The basic idea has, of course, been laid down by Geary himself and the variations involved are usually simple to envisage, although sometimes involving ingenious ideas. Three such types of variants of the Geary equations are almost obvious. One type, as already indicated, may be by adoption of only one of the two sets of equations (1) and (2) in conjunction with another set of equations different from the one not adopted. A second type of variant may be through the introduction of a different set of weights in the Geary equations themselves and/or the use of averaging procedures different from arithmetic averages. A third type of variant is to apply to the second type the corresponding variation envisaged in the first type. This may be considered as a mixture of the first two types. These three types do not necessarily exhaust all possible variants.

Examples of the first type are to accept the Geary set of equations (1), possibly with some minor modifications e.g. using notional quantities and/or ICP prices, for defining the exchange rates (and determining PPPs) and to define the corresponding average prices p_{it} through a geometric average of commodity prices, possibly after conversion to price relatives using one of the countries as a numeraire. The geometric average may be with equal or unequal weights. This includes the so called Gerardi method used in 1975 by the Statistical Office of the European Communities (EUROSTAT 1982) where average prices are defined for each commodity as the simple geometric mean of its national prices expressed in their own national currency units. Other examples are to accept Geary's definition of average prices as in equation (2), possibly with minor modifications, and use a geometric or other type of average different from Geary's, with equal or unequal weights, to define the corresponding exchange rates. Apparently this type of variant has not so far been utilized. It will be worthwhile to investigate the various kinds of such variants which may be developed to meet some desired properties. Examples of the second type of variant are obtained by replacing the arithmetic means involved in each of Geary's sets of equations (1) and (2) by geometric or other types of means (different from Geary's). An example of this type is a weighted geometric mean set of $M + N$ equation analogous to the Geary equations (1) and (2) proposed by D. S. Prasada Rao in his 1972 Ph.D. thesis. The so-called Walsh type of index also falls into this category (Kravis *et al.* 1975, 1982), some of which have been used in one form or another in earlier studies. Other variants of this type or of the mixed (third) type mentioned above may be derived. We do not propose to discuss such or other types of variants here apart from remarking that, whatever their merits, they may not satisfy all the desiderata for a consistent system of international comparisons of GDP and PPPs[3] (Kravis *et al.* 1982). This last reference and the reports on phases I and II of the ICP discuss, *inter alia*, the more important types of such indices. Some of the categories of variants of the GK method possess least squares properties of the logarithmic type which may be difficult to interpret or justify in terms of economically meaningful or acceptable models.

In addition to the variants of the GK method mentioned above, the report on phase III of the ICP (Kravis *et al.* 1982) discusses, within a useful framework, some of the better known alternatives to the GK method for aggregation purposes. We limit ourselves here to brief remarks on the alternatives mentioned therein which relate to binary (bilateral) comparisons and their generalizations to ensure transitivity and base country invariance when applied to multilateral comparisons. The so-called ideal bilateral Fisher type index, extensively used in the ICP, is the one alternative most applied by many practitioners. The index is usually used for multilateral comparisons with one country (or more when a star pattern is introduced (Kravis *et al.* 1982)) serving as a base for comparing all other countries, even though the resulting comparisons are not base country invariant. The lack of rationale or meaning and the serious drawbacks of the so-called ideal Fisher index itself were dealt with by the author in other papers (Khamis 1967, 1968 and 1972). The author is of the opinion that the persistent use of the Fisher index by many practitioners and official agencies is one of the most serious drawbacks in the history of index numbers and their application. This does not in any way reflect on the good intentions of the persons involved. This also does not imply that the separate use of the Laspeyres and Paasche components of the Fisher index for *national temporal* comparisons and for purposes of deflating or inflating economic aggregates have no pragmatic justification (cf. Khamis 1968). The main error lies in the use of their geometric mean as a price or quantity index which is unjustifiably claimed to be better than either of them. When national temporal practices are adapted for bilateral comparisons, with time being replaced by country, the resulting Paasche and Laspeyres type indices lose their pragmatic justification and their geometric average becomes a most deceptive conception with highly dubious validity. Any generalization of the ideal Fisher index for multilateral comparisons purposes, however ingenious and brilliant the devices

[3]It is stated in the phase III ICP report (Kravis *et al.* 1982, p. 74) that economic theory indicates that all the desirable properties cannot be possessed by any single set of index number formulae. While such a statement may ultimately be borne out by further theoretical advances, the author is not aware of a satisfactory proof of this statement.

behind it are, is bound to result in a much less meaningful aggregation method. In these circumstances, least squares properties do not provide a sufficient justification. This applies to the Elteto–Koves–Szulc (EKS) and Van Yzeren methods as well (cf. Kravis 1975 for details of these methods and related references). Also, in spite of the many reminders in the literature on index number methodology calling attention to the misconception that the Laspeyres and the Paasche indices provide respectively upper and lower bounds to the "true" index (if amenable to measurement) many proponents of the ideal Fisher formula and/or its extensions still commit this error. All we know is that the Laspeyres type index *tends* to give higher numerical results than the Paasche type index and that in certain practical situations the opposite numerical relationship has been noticed by practitioners. Under such circumstances, any meaningful index number formula which *tends* to give numerical results that lie between those of the Paasche and Lespeyres formulae but allows for results that lie, though infrequently, outside the range of those obtained by the two classical indices is apparently to be preferred to any index number which leads to results strictly within that range. The author established that the GK PPPs index number formula in its binary form is of the former type (Khamis 1972).

Most of the arguments in the ICP report on phases I, II, and III favouring the choice of the GK method for multilateral comparisons apply equally for the case of two countries. Hence, bilateral comparisons, if desired, may be better based on the GK binary method. It is also possible, but not generally desirable, to extend the GK method for $N = 2$, in a manner analogous to the EKS device, to base multilateral comparisons on a system of binary comparisons which satisfies the transitivity criterion. In much of the discussion of the Gerardi method, the desideratum to have binary comparisons which do not give an unduly higher weight for one of the countries than the other is generally implied. The ideal Fisher and the binary form of the Gerardi methods give equal weights to each of the two countries and this principle is preserved in the extension of the former and in the multivariate use of the latter method. The argument against this desideratum was well presented in the Hill report (EUROSTAT 1982). If in certain binary comparisons this desideratum is required for one reason or another, the GK method in its binary form may still be applicable with some minor modifications. However, the conclusions of the Hill and ICP reports in the case of multilateral comparisons are quite convincing to discourage the mechanical extension of binary indices to multilateral ones or the use of averages which do not give equal importance to commodity transactions.

When meaningful binary comparisons which in themselves lead to multilateral comparisons not much different from the GK or other base country invariant methods are available, one may perhaps be justified in using them as approximations to the multivariate case. This situation does not exist in the case of the ideal Fisher index, although its individual components are justifiable on pragmatic grounds in national temporal comparisons provided the base period is changed rather frequently, especially when basic important changes are noted in the national price and/or production structures. The relatively small differences noted between the two components of the ideal Fisher index in *national temporal comparisons* aggregated over a large number of commodity categories may not lead to significant errors when a simple geometric average of the components is used even though this average is meaningless. The situation is radically different when the two components refer to binary inter-country comparisons. Inspection of the binary calculations reported in the ICP publications show extremely large differences at the GDP level (cf. Kravis *et al.* 1982, p. 75). This situation is usually much more pronounced at the more detailed category levels. It may be instructive to quote here data from the results of the phase I report (Kravis *et al.* 1975) for Colombia and U.S.A. binary comparisons:

QUANTITY INDICES OF CONTRIBUTION TO PER CAPITA GDP AND SOME OF ITS COMPONENTS, 1970

(Colombia with U.S.A. = 100)

	U.S.A. Weights	Colombia Weights	Fisher Index	GK Index
Food, beverages and tobacco	29.7	20.7	24.8	28.4
Fruits and vegetables	56.7	17.2	31.3	39.8
House furnishings, operation	44.8	11.4	22.6	17.5
Consumption	23.3	12.2	16.8	16.8
Capital formation	22.7	9.8	14.9	18.0
Government	11.7	6.0	8.4	7.2
GDP	21.5	11.1	15.4	15.9

The corresponding indices from the multilateral application of the GK method are added in the last column.[4] The sheer magnitude of the differences between the components of the Fisher index noted in, for example, the ICP phase I in GDP and its components is sufficient to make one shudder at the argument that the index is a compromise between its two components. In such circumstances and on the basis of theoretical considerations, such a compromise is nothing less than a confounding of errors. A more acceptable alternative to the ideal Fisher index is, therefore, called for. The limited resources available to the ICP and other agencies will certainly be better utilized by completely abandoning the use of the Fisher index for binary or other comparisons. The same conclusions apply to any of its generalizations.

The discussions of alternatives to the GK method will not be complete without referring to two other rather recent methods. A somewhat comprehensive approach by Professor K. S. Banerjee, which has the advantage of providing a framework embracing a number of the traditional index number formulae, has been developed by adopting the terminology and results of the theory of statistical design and analysis of experiments. In particular the 2^2 factorial experiments for binary comparisons and the 2^n factorial approach for multilateral comparisons (Banerjee 1961, 1980) should be noted. Another approach for binary comparisons involving brilliant ideas was developed (D. M. Iklé 1972) which has some promise when some of the underlying ideas are complemented with more satisfactory

[4]Note that the GK results are not all higher than the Fisher index results and that the official or market rate conversion of national GDPs leads to an index of 6.85 for per capita GDP in 1970.

justifications to enable the construction of unique indices. We will not deal with these two alternatives in this paper but they deserve attention for future research.

It was not the intention of this section to discuss all variants or alternatives for binary and multilateral methods. Only some of the more important methods relevant to the current ICP work have been briefly referred to and to the extent they shed some light on the main issues relating to aggregation methods. In the following section we concentrate on some aspects of the application of the GK method itself including its possible adaptation for other purposes. On the other hand, the unfavourable remarks on the ideal Fisher index included in this section should not be interpreted as renouncing the use of geometric averages which are useful in some pragmatic situations, especially for linking (splicing) two time series of index numbers or similar indicators. It is well recognized now that arithmetic averages are more useful and meaningful for index number purposes and these are used in temporal price and quantity indices by almost all official and international or regional and other agencies (cf. for example, Stafford 1951 p. 452). Even then, geometric averages including temporal Fisher indices, are still useful pragmatic devices used for special purposes in conjunction with the more meaningful arithmetic average types of index number formulae.

Selected Aspects of the GK Aggregation Methods

There are various ways by which the GK method may be applied to different subject matter. Some of these were referred to by the author (Khamis 1972). Furthermore, problems related to regionalization and spatio-temporal bilateral or multilateral comparisons require adjustments in the application of the GK method. Also, adjustments are almost always needed to take account of the nature and type of the available data for each application. We confine ourselves here mainly to applications for the multilateral measurement of GDP and its components and of global PPPs. It is essential, however, to state that one can hardly overemphasize that the accuracy and usefulness of the application of the GK or any other aggregation method are mainly dependent on the nature of available data and their coverage, comparability and error margins, and especially on the availability of comparable and reasonably accurate price, quantity and expenditure data. The errors introduced by meaningful aggregation methods are generally expected to be smaller than the errors caused by the deficiencies of the basic data to which the methods are applied. Also, the collection and processing of the required basic data is a much more demanding and difficult task than the development of appropriate aggregation methods.

It will be useful for the purpose of what follows to recall that national practices in comparing volume measures of GDP or national income over time involve valuation at base period average prices. Although earlier the resulting time series were referred to as "real" GDP or "real" national income (in the same way as early national consumer price indices were referred to as cost of living indices), it is well recognized now that the base period price structure has significant effects on the numerical values of the resulting time series and their ratios, and the term "real" has been progressively, and now almost universally, replaced by the terminology "at constant prices." National accounting aggregates at constant prices are currently prepared by many countries and are being utilized for preparing an integrated system of related price and quantity indices. For our purpose, two questions arise. The first is the extent to which the GK method, with appropriate modification as required, can be usefully applied in current work on measuring real *national* GDP and its components and to other types of price and quantity indices. The second question relates to the possible ways in which the GK method is applicable to multilateral measurements of real GDP and its components with special attention to preserving, to the extent possible, the ease of comparability with national series of GDP at constant prices or other possible measures of real national GDP. The former question, although relevant to the subject under consideration, is outside the scope of this paper.

We give here a brief summary of the application of the GK method in the ICP. The details are given in the ICP reports on its three phases and other related matters are given in the Hill report (EUROSTAT 1982). Because most available data relate to national expenditure on detailed categories of final product, highly commendable extensive and intensive efforts have been undertaken by the ICP staff and all collaborating national and regional bodies to collect average item (specification, commodity or service) prices for each category which are comparable, to the extent possible, amongst countries or groups of countries. Special techniques and/or imputations are used to supplement or complete the average price tables for the selected items within each detailed category for all countries or groups of countries or to deal with those categories which are not homogeneous or for which no specific items were priced or which require special treatment. The so-called country-product-dummy (CPD) method involving regression techniques was extensively used in the ICP to estimate missing item price data and PPPs for categories for which no items were priced in a country. Some expenditures relating to certain heterogeneous categories were distributed amongst expenditures relating to other categories according to pragmatic knowledge of their relative representativeness. Special treatment of the net foreign trade balance was made. For some complex items which are difficult to compare amongst countries hedonic price indices were utilized. Known or estimated category expenditures for countries not included in a particular phase of the ICP were imputed to the related expenditure data for categories of the included countries (the so-called supercountry device) to ensure that the resulting average commodity prices and PPPs at the GDP level are not significantly affected by the countries excluded from the ICP in any particular phase. Subject to some variations, such and other devices enabled the calculation of category PPPs, average prices and 'notional' quantities, with the U.S.A. being used as a numeraire. These constituted the input into the GK method, thus resulting in *global* (relative to GDP as a whole) PPPs and average category prices relative to the U.S.A. as a numeraire (cf. Kravis *et al.*, 1975 pp. 69–70). These results related to each benchmark year. The average category prices and global PPPs are therefore expressed in terms of an international U.S.A. dollar for each benchmark year of the ICP. It is essential to stress that the ICP adaption of the GK method assumes that the purchasing power of the international dollar applies to total GDP of a country but differs from its purchasing power for any of the GDP categories or components at a lower level of aggregation and that the international dollar purchasing power for a benchmark year differs from its purchasing power for another benchmark year. The resulting global PPPs are shown in the detailed tables in the ICP reports.

The details and characteristics of the methods used by the ICP in calculating "notional" category quantities and average prices relative to the U.S.A. as a numeraire are of fundamental importance and may significantly affect the final results at the GDP level. The same may apply to the corresponding aspects of EUROSTAT practices at the category (or basic heading) level. Reference to the ICP reports and the Hill report may be made for such details. We shall not deal here with these aspects. However, the author is of the opinion that it might also be worthwhile to consider applying the GK method first to categories across countries and then combining the results over categories. This alternative might enable the use of other useful devices to meet data requirements. Such an alternative may have different economic implications. The GK method may be adapted to such an approach without losing its desirable properties.

Another important question arises as to whether, within the framework of multilateral comparisons, it is preferable not to consider the global PPP of a national currency as different from its PPPs with respect to GDP components. The original intention of the GK method was to obtain unique global exchange rates, or PPPs, and average prices in a uniform currency to enable the calculation of different types of international indices, and to aggregate over different commodities and countries for any meaningful economic flow, especially those classified by industry of origin. Only at the GDP level are the GK global exchange rates or PPPs (and the resulting regional or world prices) essentially amenable to being described as PPPs, regional or international average prices, etc. For special economic flows, e.g. those relating to agricultural products, the GK exchange rates and average prices may be interpreted as means to obtain more adequate weighting patterns or aggregations over different commodities and countries. While they may be termed *partial* exchange rates or PPPs and *industry* average prices, etc., they need not be viewed as actual measures of national currency PPPs nor of commodity regional or world prices. This view was implied by the author in the earlier discussions of his and Geary's proposed indices when applied to indices of agricultural production. This should not be construed, however, to mean that the GK method applied at the GDP level would not result in more acceptable global PPPs of national currencies and related international prices in relation to one of the countries being chosen as a numeraire. All that is intended is to assert the applicability of the GK method as a more meaningful alternative to other methods in current use for different types of purpose such as indices of agricultural production, consumer price indices amongst countries for special population groups (currently based on the indefensible use of the ideal Fisher index with or without incorporation of a bridge country technique), etc.

As an immediate consequence to the preceding remarks a relevant question may be raised. After the calculation of the GK average category prices and global PPPs for a particular year, would it not be legitimate also to apply these in a revaluation of the components of GDP? This procedure keeps the relative structure of the multilateral measures of GDP components for each country in terms of the "international dollar" the same as the relative structure when current prices in national currencies are used. For the same benchmark year, equation (1) ensures that it will make no difference whether the global PPPs or average prices are used for this purpose. Some national authorities, and no doubt other users, will find such revaluations and the resulting tables quite useful for many purposes. This was, in fact, the original intention of the GK method. It may be noted that the Hill report (page 61) also proposes such revaluations and tables. The arguments in the ICP reports for the valuation of the components of GDP at their respective derived PPPs apparently have merit for certain international analytical purposes. On the other hand, the alternative revaluation proposed above and earlier in the author's papers referred to herein also has its own applications for analytical and other purposes. Accordingly, it is worth considering further the economic and other merits of each of the two corresponding sets of national GDP components amongst countries. Both methods of valuation of "real" GDP components (at global and at lower level PPPs) enable aggregation over countries of GDP and its components expressed in a numeraire currency to prepare totals for any desired groupings of countries. The resulting tables from the two approaches will show different values for the components of GDP and their totals but the same values for GDP totals by country or their aggregation over country groups. Attempts to obtain aggregates over countries of both types for the ICP were considered in Phase III and some indicative results are indicated in the report on this phase. Also, the Statistical Office of the European Communities produced tables in accordance with the second alternative (EUROSTAT 1982, p. 61). For each of the two methods of aggregation one obtains a set of tables "at current international prices" of the benchmark year for each country as well as a set of tables of totals of GDP and its components for each desired grouping of countries.

In addition to the two possible methods of aggregation discussed above, multilateral tables of real GDP at constant international prices may also be obtained. This is achieved by choosing a suitable benchmark year as a base period and using its average commodity or category prices (not PPPs) in the valuation of each national GDP and its components for the following and/or preceding years in terms of the numeraire country's currency of the base period. These tables are analogous to individual countries' tables of national GDP and its components at constant prices and the relationships of the two types of tables are useful for analytical purposes. The international counterpart of the national tables of GDP at constant prices can be used to calculate international types of production indices for GDP as a whole and for its components and for corresponding aggregrations over countries. This will also result in a new set of national indices, if desired, based on international price weights. When average prices for a new benchmark become available the base period may then be shifted to the new benchmark year and the series linked or spliced using methods analogous to those applied in the case of national temporal comparisons. Alternatively, the linking may be achieved by making use, *inter alia*, of another application of the GK method to the data for the two successive benchmark years concerned.

It should be noted that the use of global PPPs of the base period will not lead to the same set of tables of multilateral estimates of GDP or its components at international constant prices, as already shown by the author (cf. Khamis 1967, 1970, 1972 and the earlier section of this paper on the evolution of the GK method). This is, of course, due to the fact that identity (3) does not hold any more between the use of average prices and of PPPs pertaining to one period in the valuation of production pertaining to another period. This is the main reason for preferring the use of international prices to category or global PPPs. The main difficulty in applying this valuation method to GDP and its components is the lack of adequate item price and quantity data. The possible derivation of appropriate notional category average prices may enable an approximation to such tables. The suggested tables of multilateral GDP at constant prices using base period regional or international average prices discussed in the preceding paragraph will lead to Laspeyres type international temporal quantity indices of production for any desired groupings of countries, as easily shown by reference to equation (7). While a set of national temporal quantity indices based on international prices can be also obtained by this method and these may be useful for some international analytical purposes, the national temporal indices can also be kept intact. If one considers a national entity in isolation from other countries, the national temporal indices or GDP at constant prices series using only national prices of a base year may be considered better measures of national growth rates than indices or GDP series based on international or regional prices. However, the increasing state of interdependence of the various national economies also renders the use of at least some types of average regional commodity prices, if not world prices, in calculating national quality indices a useful tool for some analytical economic studies.

Nonetheless, the use of constant base period GK average prices for aggregation purposes and the multilateral temporal quantity comparisons thus obtained results in measures analogous to the use of constant prices of a base period in national temporal aggregates or indices of quantity. The question therefore arises as to whether the GK method can be modified so as to at least reduce the effects of the prices of a base period. It may be noted that the GK method in its present form is applicable to data pertaining to one single year and leads to quantity indices for one country with respect to another which are almost independent of the corresponding national prices of the year under consideration. Again this is so because of identity (3) and can be observed by comparing the equivalent equations (11) and (12) defining the quantity index $Q_{sk,t}$. For this purpose equation (12) may be viewed as equivalent to a value index of country k (with valuation of its quantities being made at the international prices $e_{kt}p_{ikt}$) as compared with country s (with valuation of its quantities being made at the international prices $e_{st}p_{ist}$). Such a property characterizes the GK aggregation method as compared with the variants and alternatives such as those discussed in the preceding section. Indices of the type defined by equation (11) or (12) can be calculated for each benchmark year for which the e_{jt} and p_{it} are determinable in terms of the currency of a numeraire country. Such indices are obviously transitive and their weighting coefficients are base country invariant. Similarly, the two analogous GDP or other production aggregates for two different or overlapping groups of countries with valuations at the regional or international prices p_{it} and $e_{jt}p_{ijt}$ for the same period t will be the same because of equation (1). Thus for $S=(a_1, a_2, \ldots, a_s)$ and $K=(b_1, b_2, \ldots, b_k)$ where S and K denote subsets of the set of countries $j=1, 2, \ldots, M$, the quantity index of total GDP or production of subset K with respect to subset S is given by

$$Q_{SK,t} = \sum_{j\epsilon K} \sum_{i} p_{it} q_{ijt} \Big/ \sum_{j\epsilon S} \sum_{i} p_{it} q_{ijt} \tag{18}$$

which is equivalent to

$$Q_{SK,t} = \sum_{j\epsilon K} \sum_{i} e_j p_{ijt} q_{ijt} \Big/ \sum_{j\epsilon S} \sum_{i} e_j p_{ijt} q_{ijt} \tag{19}$$

as in the case of equations (11) and (12).

Thus, comparisons of different types for the same period t can be achieved because the arbitrary multiplicative parameter in the solution of equations (1) and (2) cancels out (or a numeraire country may be used) and it makes no difference whether valuation is made throughout by the average price p_{it} or the converted national prices $e_j p_{ijt}$.

Problems will, of course, arise when one wishes to compare the total real product in one period of time t_1 with the corresponding total for another period of time t_2 for two or more countries or for the same country within a multilateral framework. In such cases, ideally, one has to determine the relationship between the two arbitrary multiplicative parameters resulting from the solution of equations (1) and (2), obtained once for $t = t_1$ and once for $t = t_2$. Of course, it will be sufficient to determine only the ratio of the two arbitrary parameters. An approximate way to do this when t_1 is the base period is already provided by utilizing the Geary–Laspeyres index of equation (5) or the author's Laspeyres index of equation (7). As noted in the discussion of these two indices, the latter is to be preferred to the former, although the numerical difference between them may be small in particular situations. This problem is, of course, one of those met in making spatio-temporal comparisons and will be dealt with by the author in a subsequent paper, where other methods will also be discussed.

Another alternative way of using the GK aggregation method consists in applying it once to each of some meaningful groupings of countries (by regions, economic criteria, etc) and then combining the results in a suitable way to maintain, to the extent possible, intragroup comparability. This is known as the regional approach to multilateral comparisons and it is currently commanding attention. There are advantages to be gained from this approach, especially the possibility it offers for obtaining a better and more comprehensive list of adequately comparable items to be priced in each country within the group. It is also believed by some that better intra-regional comparability amongst countries will result, although inter-regional comparability will be affected. The advantages and disadvantages of regionalization are discussed in the ICP report on Phase III (Kravis *et al.* 1982) and in the Hill report (EUROSTAT, 1982). The subject is also considered in other papers in this issue. The difficulties involved in combining the results of the separate applications of the GK method to each grouping are analogous but not fully equivalent to those involved in spatio-temporal comparisons and the author will deal with both problems in the subsequent paper referred to earlier. It will be instructive, however, to refer here to the similar but less difficult situation arising when aggregations or indices are made for different administrative or other divisions of a single country. One method is to use country-wide average prices and total production of items for each of a base year and another year in calculating a Laspeyres (or Paasche) quantity index. Another method is to calculate separate Laspeyres (or Paasche) quantity indices for each division of the country using average prices and production data pertaining to each division separately and then combine the resulting divisional indices to obtain a national index. It is well known that the two national indices thus obtained will generally be different, although the use of some devices may reduce this difference. This situation holds regardless of whether a Laspeyres, Paasche, ideal Fisher or other traditional index number formula is utilized. In this particular one country case, the author is of the opinion that the GK method can be meaningfully applied and it avoids the problems associated with the choice amongst the competing traditional index number formulae. This is so because the same national currency unit is used and its relative PPP between the two time periods concerned can be uniquely determined.[5]

It may be noted that a satisfactory solution of the regionalisation problem will result in a more comprehensive and comparable list of items for each region for pricing purposes. The efforts currently being made in the ICP to price a satisfactorily *representative* list of items are expected to enlarge the number of items and to include as many items as feasible which are common amongst regions. If these efforts are extended also to obtaining corresponding item quantity data it might be possible to apply the GK method in a more direct manner making better use of the available expenditure data coupled with proper imputations of weights to obtain improved estimates of PPPs and the average prices, etc. The author is of the opinion that the example set by the existing international co-operation in the ICP in item pricing and other aspects can be duplicated to obtain also item quantity data wherever feasible. This should enable a more direct application of the GK method, almost as originally envisaged, with appropriate modifications. This will most likely result in better estimates of global PPPs to apply to national GDP data and their components. The large number of assumptions, special techniques and approximations currently involved in the present form of the ICP application of the GK method (or required for other aggregation methods) may result, in the opinion of the author, in larger margins of error in the final results than the more direct application of the GK method proposed herein. This is also closely related to future ICP plans for the multilateral measurement of GDP by industry of origin where item producer prices and national quantities will be required to a greater extent. Also the more direct application of the GK method suggested here will result in a single set of global PPPs and average item prices applicable to whatever method is used for the multilateral measurement of GDP and its components. This is an essential advantage of the collection of quantity as well as price data. National statistical agencies in most countries are engaged in the collection of producers and

[5]For the details of such possible application reference may be made to section 2 of an earlier paper (Khamis 1971) with j referring to a division of the country and $T=2$.

purchasers prices and quantities produced or consumed for many items for various purposes in addition to those required for estimating GDP or other national accounting aggregates. For example, indices of producer, retail and consumer prices and quantity indices of different kinds are usually calculated and appropriate data on item prices and quantities, in addition to expenditure data, are in many cases collected and utilized. Technical co-operation statistics programmes of international and regional agencies and similar programmes provided on a bilateral basis to developing countries could be integrated in such a way as to better meet national requirements for data. The ICP provides a useful framework for such an integrated approach to technical co-operation, especially in relation to the National Household Survey Capability Programme. There is, therefore, a satisfactory basis to justify intensifying efforts to collect item price and quantity data to enable the application of aggregation methods at a much more detailed level of disaggregation of detailed expenditure categories. It is frequently argued that such a detailed data collection programme is not feasible. Such arguments may turn out to be ultimately justifiable but they can only be judged after serious efforts are first undertaken to collect the item price and quantity data. The author is of the opinion that there are sufficient pragmatic grounds to exert all possible efforts for the compilation of the detailed information. If multilateral measurement of GDP and PPPs is to be undertaken by expenditure on final product as well as by industry of origin or by other approaches, the need to calculate a single set of global PPPs and international prices for as many representative items as possible becomes of utmost importance.

Finally, since interest is growing in the multilateral measurement of GDP and PPPs by more than one approach (including value added by industry of origin), it is essential to ensure the applicability of the GK method to approaches other than expenditure on final product. The conditions for the GK method to lead to unique and positive PPPs and average prices require that the quantity data entering the equations (1) and (2) are non-negative. This does not, however, preclude the application of the GK method to the value added approach or other approaches involving subtracting one aggregate (e.g. intermediate inputs) from another aggregate. A suggestion for overcoming the difficulty in the case of national temporal comparisons was given by the author (Khamis 1972, p. 106). It is also possible to prove the applicability of the GK method to the value added by industry approach subject to generally satisfied necessary and sufficient conditions (i.e. when some quantities are negative as in the case of input items). Also for multilateral comparison purposes, another solution is to derive a satisfactorily accurate set of global PPPs and average prices using, e.g. the ICP expenditure approach with appropriate modifications. Such a set can benefit by more disaggregation of the ICP 153 detailed categories to enable the utilization of more item price and quantity data. The resulting global PPPs and average prices thus derived may then be used also for other approaches to international comparisons of GDP and its components to prepare tables according to the alternative valuations discussed above. Such a solution, if agreed to, requires national and international support for the continued progress of the existing programme of work of the ICP, especially along lines that enable more direct use of more item price and quantity data.

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