COMPILATION OF INPUT–OUTPUT TABLES: CANADA*

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This paper provides a description of the annual Input–Output Tables for Canada. It describes the accounting framework and notes its close affinity to the one described in the United Nations report, *A System of National Accounts*. It demonstrates the ready derivation of GDP and Expenditure on GDP, both in current and constant prices, from the Input–Output Accounts as well as their relationship to the other subsystems of the Canadian System of National Accounts, particularly the Income and Expenditure Accounts and Real Domestic Product by Industry. Compatibility of basic accounting records of the transactors with the rectangular (commodity-industry) format of the Canadian tables is described. The need to have a consistent commodity classification and to develop a consistent valuation of all transactors in the economy is emphasized. The particular formulation of the Input–Output Impact tables is noted. The problem of deflating trade margins and the resolution of this problem is described. A strong plea is made for the economics profession to pay more attention to the problem of aggregation; all economic analysis is approached with blinkers but the aggregation problem isn't even recognized as a blind spot in most analyses.

INTRODUCTION

The Input–Output Division of Statistics Canada prepares and publishes annual Input–Output Tables for Canada in both current and constant prices. The latest tables refer to the year 1977 and with this there is a completely consistent historical series of annual tables from 1961.

The accounting framework of the Canadian tables given in chart 1 bears a close relationship to the one described in the United Nations report, *A System of National Accounts*.[1] The inputs and outputs of industries are presented in separate tables; both inputs and outputs are classified by commodity. Commodities are clearly distinguished from industries, the number of commodities exceeding the number of industries. The Canadian Input–Output Tables are rectangular rather than square. The commodity-industry format has important advantages over the traditional inter-industry square format: (i) it admits as much detail as is available in the basic economic records; (ii) the meaning of each entry is straightforward because observed transactions are not combined with fictitious transfers; and (iii) it provides a statistical audit of the consistency, integrity and comprehensiveness of economic statistics. The last advantage assumes particular importance when input–output statistics are developed in a central statistical organization like Statistics Canada.

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[1] *A System of National Accounts*, Studies in Methods, Series F, No. 2, Rev. 3, United Nations, New York, 1968.

CHART 1

THE ACCOUNTING FRAMEWORK OF CANADIAN INPUT-OUTPUT TABLES

	Commodities	Industries	Final demand categories									Total
			PE	*FCF*	*VPCW*	*VPCA*	*GGCE*	X_D	X_R	Less *M*	Less *GR*	
Commodities		*U*				*F*						*q*
Industries	*V*											*g*
Commodity indirect taxes Other indirect taxes Less subsidies Wages and salaries Supplementary labour income Net income of Unincorporated business Other operating surplus		*YI*				*YF*						*n*
Total	q'	g'				e'						

Final Demand Categories

- *PE*—Personal expenditure on goods and services
- *FCF*—Fixed capital formation, business and government
- *VPCW*—Value of physical change in inventories, withdrawals
- *VPCA*—Value of physical change in inventories, additions
- *GGCE*—Gross government current expenditure on goods and services
- X_D—Domestic exports of goods and services
- X_R—Re-exports of goods and services
- *M*—Imports of goods and services
- *GR*—Government revenue from sales of goods and services

Notation

- *V*: is a matrix of the values of outputs
- *U*: is a matrix of the values of intermediate inputs
- *F*: is a matrix of the values of commodity inputs of final demand categories
- *YI*: is a matrix of the values of primary inputs of industries
- *YF*: is a matrix of the values of primary inputs of final demand categories
- *q*: is a vector of the values of total commodity outputs
- *g*: is a vector of the values of total industry outputs
- *e*: is a vector of the values of total inputs (commodities plus primary) of final demand categories
- *n*: is a vector of the values of total primary inputs (industries plus final demand categories)

GLOSSARY

Matrices

V: is a matrix of the values of commodity outputs. In it, each row shows the distribution by commodity of the output of an industry; each column shows the distribution by industry of the output of a commodity. The data relate to domestic output only. The gross output of an industry is the aggregate value of goods and services produced and work done by the industry. It is equal to the value of industry's sales plus any increase (less any decrease) in the value of physical change in stocks of finished products and work in progress.

U: is a matrix of the values of intermediate commodity inputs. In it, each row shows the distribution by industry of the input of a commodity, each column shows the distribution by commodity of the input of an industry.

F: is a matrix of the values of commodity inputs of final demand categories: personal expenditure on consumer goods and services; fixed capital formation, business and government; value of physical change in inventories, withdrawals and additions; gross government current expenditure on goods and services; exports; imports; and government revenue from the sales of goods and services.

YI: is a matrix of the values of primary inputs of industries. Primary inputs are those inputs which are not current outputs of other industries. These are indirect taxes, subsidies, wages and salaries, supplementary labour income, net income of unincorporated business and other operating surplus. (In this report, the term "other operating surplus" and "surplus" have been used interchangeably.)

YF: is a matrix of the values of the primary inputs associated with final demand categories. These consist of indirect taxes, labour income, and depreciation which is part of surplus. The labour income includes wages and salaries and supplementary labour income paid by the government and personal sectors. The estimate of surplus (depreciation) relates to the government sector and non-profit institutions in the personal sector.

Industry—An industry is defined as a group of operating units (establishments) engaged in the same or similar kind(s) of economic activity, e.g., coal mines, clothing factories, department stores, laundries. In the Input–Output Tables only domestic industries which primarily produce goods and services for sale at a price which is normally intended to cover their cost of production are included; also included are entities, such as owner-occupants of housing and subsistence farming, which produce goods and services similar to and use processes and resources similar to typical industries even though they do not produce for market. Other entities, although listed in the Standard Industrial Classification, are not included as industries in the Input–Output Tables as they do not meet the above criteria; examples are most government departments, public hospitals, schools (except private) and universities.

Commodity—A commodity is defined as a good or service normally intended for sale on the market at a price designed to cover the cost of production.

Establishment—An establishment is defined as the smallest unit that is a separate operating entity capable of reporting all elements of basic industrial statistics—the main elements of input and output. It is typically a factory, mine, store, farm, airline, hotel or similar unit and in most cases it is a separate company.

Purchasers' prices—The cost of goods and services in the market to the point of delivery to the purchaser. The purchasers' price can vary with circumstances, e.g., where the purchaser picks up the commodity at the producer's point of shipment rather than having it hauled by a common or contract carrier at the expense of the producer.

Producers' prices—The selling price at the boundary of the producing establishment excluding sales and excise taxes levied after the final stage of processing; equals purchasers' price less transport, trade and commodity indirect tax margins where applicable.

Canada is one of the pioneering countries in both the theoretical and empirical development of rectangular input–output tables, a development pursued by practitioners in Statistics Canada and in other institutes. Preliminary Input–Output Tables for the year 1961 were published by Statistics Canada in 1969 in two volumes: *Input–Output Structure of the Canadian Economy, 1961*; *Vol. I* (catalogue 15–501), *Vol. II* (catalogue 15-502). In these volumes a comprehensive description of the detailed accounting framework, analytic uses, the published tables, classification systems and selected definitions and an extensive section on the mathematical treatment of analytic uses were given. This document gives the most complete description of the development of the accounting framework and the classification systems that underlie the current Canadian tables, although subsequent work has led to refinements in the classification systems, particularly in the articulation of the categories of final demand, in comparison to the preliminary 1961 tables. Further evidence of the contemporary Canadian work on the development of rectangular input–output tables, applied in a provincial context, can be found in *Rapport intérimaire sur le système de comptabilité économique du Québec, le système et son fonctionnement*, Bureau de la statistique du Québec et Laboratoire d'économétrie, Université Laval: juillet 1967. Another illustration of the application of the Canadian input–output accounting framework in a provincial context can be found in *Input–Output Study of the Atlantic Provinces, 1965*, by Kari Levitt; *Vol. I, Social Accounting Matrix and Models* (catalogue 15-503E). *Vol. II Structural Analysis and Data Sources* (catalogue 15-504E), published by Statistics Canada.

Current price input–output tables. Three data tables are prepared and presented for each year: (1) Make Matrix, (2) Use Matrix, and (3) Final Demand Matrix. At the most detailed level, the system is balanced with 191 industries, 595 commodities and 136 categories of final demand. In addition there are seven primary inputs: wages and salaries, supplementary labour income, net income of unincorporated business, other operating surplus, commodity indirect taxes, other indirect taxes, and subsidies. At this level of detail, some of the entries in the various matrices are confidential under the provisions of the Statistics Act. Though all the tables are prepared at the L level of aggregation, it is only at aggregation level S and M that we publish these tables. The dimensions of these aggregations are given below:

Designation of Aggregation	Number of Industries	Number of Final Demand Categories	Number of Commodities and Primary Inputs
S	16	14	49
M	43	29	100
L	191	136	602

The three data tables are published both at aggregations S and M. As well, two impact (Inverse) tables—one at each aggregation S and M (see discussion of the definition of impact tables in a later section)—and a table on commodity margins at aggregation M are published annually. Thus, in total, nine table are published for each year.[2] In addition to these public tables, we provide at request, additional detail or special aggregations, subject to confidentiality, of the input–output data at the L level of aggregation.

Constant price input–output tables. As in current prices, the balancing of the constant price tables and the complete deflation process are carried out for each year for 191 industries, 595 commodities and three primary inputs (commodity indirect taxes, other indirect taxes, and subsidies) and 136 categories of final demand. Gross Domestic Product at factor cost is calculated residually through the use of double deflation.

Three data tables are prepared and presented for each year: (1) Make Matrix, (2) Use Matrix (the matrix of the values of intermediate commodity inputs as well as primary inputs), and (3) Final Demand Matrix. Though all of the tables are prepared at the L level of aggregation, it is only at the aggregation levels S and M that tables are published.[3] The dimensions of these aggregations are identical to the ones in the current price tables except that the factor inputs are combined into Gross Domestic Product at factor cost. Impact tables are not presented in constant prices though we would do so at request.

The Input-Output Accounts and their Relationship to the System of National Accounts

Input–Output Tables form part of the broader Canadian System of National Accounts which includes (in addition to Input–Output Tables) the Income and Expenditure Accounts, Real Domestic Product by Industry, Productivity Studies, the Financial Flow Accounts, the Balance of International Payments and balance sheets showing the assets and liabilities of the economy. The System of National Accounts was originally developed to satisfy the need for consistent and comprehensive measures of economic activity. As demands for data for economic analysis have grown over the years, the conceptual framework has been extended and refined; at the same time flows of data have been established to fill in the System through the exploitation of existing sources of information, the development of new sources, and the design of new estimating techniques.

The best known Accounts of the System are the Income and Expenditure Accounts which were designed, in the main, to provide current and comprehensive, though relatively aggregative, measures of the output of the economy in two ways: (1) as the value of the "final" expenditure on goods and services by the various sectors of the economy, less total imports of goods and services, and (2) as the income accruing to (or costs of) primary factors of production engaged in the production process plus certain non-factor costs such as indirect taxes.

The items of final expenditure are identical in both the Income and Expenditure and the Input–Output subsystems. On the Gross Domestic Product side, the Input–Output breakdown of primary inputs is less detailed. The term operating surplus in the Input–Output Accounts encompasses profits and other investment income, capital consumption allowances, inventory valuation adjustments, etc.

[2] *The Input–Output Structure of the Canadian Economy*, (catalogue 15-201).

[3] *The Input–Output Structure of the Canadian Economy in Constant Prices*, (catalogue 15-202).

The Input–Output Tables, and the estimates of Gross Domestic Product derived from them, use somewhat different statistical sources than the Income and Expenditure Accounts. Notwithstanding the different approaches used, the estimates of Gross Domestic Product for the economy as derived from the Input–Output Accounts very closely approximate those in the Income and Expenditure Accounts.

The notable difference in the two subsystems is in the estimation of operating surplus. In the Input–Output Accounts, the data are derived from establishment-based surveys, such as annual Census of Manufactures, and surplus is essentially calculated in the context of a balance between inputs and outputs. In the estimate of the components of surplus for Income and Expenditure Accounts, the data sources are surveys of companies and other administrative records. The use of different statistical reporting units yields differences in industrial allocations.

On the Expenditure side, there are some minor differences in the estimates of current government expenditure, exports and imports, inventories and personal expenditure. The differences in the estimates of current government expenditure and exports and imports are definitional. One example of a definitional difference is the treatment of research grants for defence which are treated as transfer payments in the Input–Output Accounts but as purchases of government services in the Income and Expenditure Accounts. In imports and exports, the transfers of funds by trade unions are treated as transfers in Input–Output Accounts and as sales/purchases of goods and services in Income and Expenditure Accounts.

The valuation of physical change of inventories presents difficulty in both the Input–Output and the Income and Expenditure Accounts. There is a lack of data on the valuation practices as well as on the commodity content of inventories. The two subsystems use different techniques for measuring the value of the physical change of inventories (VPC). For example, in the Income and Expenditure subsystem, the value of the change of inventories in 1977 is equal to closing inventories in 1977 less closing inventories in 1976. In the Input–Output subsystem, VPC of inventories equals closing inventories in 1977 less opening inventories in 1977. The closing inventories of 1976 may differ from the opening inventories of 1977 because of: classification changes; changes in the basis of valuation; and births and deaths of establishments.

Consumer expenditure makes up about three-fifths of the Gross Domestic Product at market prices, yet this is one sector not supported directly with regular surveys. In the process of commodity balancing (see discussion of this important tool in the next section) it is necessary to have estimates of the categories of consumer expenditure and their commodity content. Thus considerable care, using a range of data sources, is taken in estimating the various categories of personal expenditure and the commodity composition of each category.

Occasional Family Expenditure Surveys provide information on the pattern of consumer expenditure. In addition, the Retail Trade Surveys when used with information on class of customer and commodity content of sales by type of store provide estimates of consumer expenditure. Balanced tables for preceding years do provide a reference point, and retail sales by type of store can be short run indicators of the consumption of related groups of commodities, although experience suggests that this approach be used with caution. In many cases, estimates can be verified against the trend in real consumption per person. Additional information on consumer expenditure is available in the form of data on direct selling by manufacturers, a detailed breakout of the disposition of energy from the Quarterly Report on Energy Supply and Demand in Canada, data from trade associations, and of course from the operating statements of some of the private non-profit organizations which form part of the Personal Sector. The outputs of certain commodities like imputed and paid rent and domestic services are allocated directly to consumers.

The Income and Expenditure Accounts show a separate entry called "Residual error of estimate," as a measure of the statistical discrepancy between the "income side" and the "expenditure side" of the accounts. In the Input–Output Tables, where each Industry and each Commodity Account must balance, there is no "residual error of estimate." The supply side is always equal to the disposition side for any commodity and all commodities. Deducting from the total supply (or disposition) a common intermediate expenditure total gives GDP or Expenditure on GDP. Thus GDP cannot be different from Expenditure on GDP.

The relationship between the constant price Input–Output tables and its counterparts in the other subsystems of the Canadian System of National Accounts is articulated in a later secton on Constant Price Input–Output Tables.

Commodity Balancing in Input–Output Tables

The Introduction described the general accounting framework of the Canadian Input–Output Tables and noted that a feature of the rectangular framework was its compatibility with basic accounting records. However, considerable work is still required to transform data as reported by the economic transactors in the economy into the comprehensive and consistent economic accounting framework that is characteristic of Input–Output Tables, and the major elements of this process are outlined in the following paragraphs. Input–Output Tables relate the supply of each commodity to its disposition and this is done by constructing a *commodity balance* for each commodity. Producers report production of a commodity at producers' values while those acquiring the same commodity, be they industries purchasing inputs into the production process or final demand transactors, report the cost as incurred by them, at purchasers' values. A commodity balance necessitates a uniform valuation for a commodity for all transactors, and effecting such a system of uniform valuation requires the identification and measurement of the *margins* (trade, transport, tax, etc.) that make up the difference between the value that the purchaser pays and what the producer receives for a commodity.

There are also major areas—for example, the *construction* industry, *government* expenditures and *capital formation*—where the basic accounting records do not furnish the commodity information necessary to construct the I–O tables. The approaches taken in the face of these problems are described in subsequent sections.

The fundamental element in statistical compilation is classification. Input–Output Tables classify transactions both by industry and by commodity. The industrial classification of transactors (the Standard Industrial Classification) has been in place for some time and presents no major problems. However, an accounting of economic activity requires a measure of what is done as well as who does it. A commodity balance is such a complete accounting, and as commodity balances are compiled only in the context of the Input–Output Tables, it is not surprising that serious problems are encountered in imposing this additional demand on the existing data base.

There are three logically separable phases to constructing commodity balances. The obvious first requirement is a system of classification for commodities (goods and services) that can be consistently applied to all branches of economic activity. This requires that a commodity be coded consistently whether produced by a manufacturing industry, imported or exported, transported by boat, train or aeroplane, or purchased by some industry or in some final demand category. The classification system actually encountered in each of these spheres of activity tends to be peculiar to it, reflecting special interests and historical development. It is not surprising to find that export commodity classes reflect the importance of goods exported, and the same is true of import commodity classes, etc. But commodity balances require consistent commodity classes applied to all transactions, and in sufficient detail to be analytically meaningful. Thus the development and continued application of a consistent commodity classification system is a major undertaking by the input–output analyst.

The second requirement is to fill major holes in the basic data bases. In spite of the vast array of statistics collected, there are significant and important areas affording very little direct knowledge, and, even that, often at very infrequent intervals. The data base for manufacturing is more highly developed than for most other industries; yet the annual census provides no data on purchased services, and the commodity composition of inventories is unknown. Some industries are surveyed only at decennial census intervals, and then little detail on operating structures is obtained. In such cases, the required series are estimated using related indicators, *ad hoc* studies and occasional surveys, etc. For example, taxation statistics for corporate and unincorporated businesses provide evidence on the gross output of some industries, and implicit input structures in other cases, even though the classification of multi-establishment companies is not consistent with the establishment-based concept of the Input–Output Tables. The point to be noted here is that various sources, some having direct reference but others providing only inferences and indicators, must be used in compiling the Input–Output Tables.

The diverse classification systems peculiar to the different spheres of economic activity are confronted and moulded into a common system of adequate consistency and the resulting data are then analysed. Varied and unpredictable differences immediately become apparent. This brings us to the final phase in commodity balancing. Supply must equal disposition for each of the 595 commodities. The under- or over-allocation of commodities must be examined and eliminated. The discrepancy may be due to several reasons—production may be unreported or misclassified; imports and exports may be improperly valued, as well as misclassified; there may be timing inconsistencies, etc. There are no ready-made statistical approaches to solving such imbalances. The only approach is laborious investigation; one has to go back to the basic records to locate the sources of such imbalances. It is our experience that the commodity balance approach, with its detailed accounting of output by industries and of use by industries and final demand transactors, uncovers major problems in both statistics reported and classification.

VALUATION

The Input–Output Tables are initially constructed in current prices with the entries valued at purchasers' prices, with subsequent calculation of the tables at producers' prices. Purchasers' prices represent what the purchaser pays while producers' prices are what the producer receives; the difference is the trade, tax and transportation margins. In producers' prices a commodity has the same valuation base throughout the system.

Note that what we call "Producers' Prices" are, in the UN SNA terminology, "approximate basic values". Our valuation in producer prices is readily understood by the industry experts and the data are easily comparable with other published series on industry statistics. We believe that it is quite important to keep our concepts as close as possible with those used by industry analysts and other users who are not familiar with the nuances of SNA terminology.

Our published data tables are always in producers' prices. The impact tables are manipulated from the producer price files and so are the constant price Input–Output Tables. But purchaser price tables are available to any user on request.

MARGINS

There are seven margins distinguished in the Input–Output Tables: retail margins, wholesale margins, tax margins, transport margins, gas margins, storage margins and pipeline margins. In the majority of cases, particularly trade and transport margins, data on the appropriate margin to be attached to each commodity are not directly available and must be estimated based on partial evidence and professional judgement.

Trade margins. Data on margins by commodities are generally not available. For wholesale and retail trade margins, gross trading margins by type of store are first calculated. Occasional commodity surveys provide a basis for allocating the type of store margins to groups of commodities. These estimates by type of store and by commodity group are fitted into a statistical framework and balanced. To complete the first stage the margins by commodity groups are expanded into the full detailed input–output commodity classes using proportionality assumptions. This exercise provides the first approximation of the margins on each commodity. In the next stage, i.e. at commodity balancing, margins may be modified in the light of the commodity balances and price spread studies. The industrial distribution of these margins is based on the Census data on class of customer and other relevant information and estimates.

Transport margins. Transport margins are developed by mode of transportation (such as air, rail, water, services incidental to water, truck and other services incidental to transportation) giving the cost of transportation by commodity going from producer to purchaser. Most of the transport margins are generated by railways and trucks. Railways can distinguish freight revenue by about 300 commodity groups. This statistical and commodity coverage is not quite complete but useful to get a first approximation of the values of rail transport margins. The annual Truck Commodity Origin and Destination Survey gives revenue and tonnage by approximately 450 commodity groups. Certain *ad hoc* studies and surveys, with a big dose of professional judgement by industry experts, complete our first estimates for transport margins.

Gas margins are straightforward as all the gas margins apply to commodity Natural Gas. Pipeline distribution margins are directly allocatable to natural gas and crude oil. Storage margins apply to a very few primary outputs such as wheat and barley; most of the storage charges are not on commodity margins but charges for storing household effects.

Tax margins. Examples of tax margins are federal excise taxes, import duties, gasoline taxes, liquor gallonage taxes, profits of provincial government run liquor commissions. Provincial sales taxes are allocated directly to categories of final demand as well as such intermediate industries as construction. All other tax margins are distributed by commodity. It is a painstaking exercise to go through the myriad tax levies and tax exemptions. This tax margin allocation is further subjected to an informal consultation with tax experts and other knowledgeable persons.

Use of Imports versus Domestic Production

The value of inputs into the Use Matrix as well as the Final Demand Matrix includes (where applicable) imports as well as commodities produced by the domestic economy. The basic economic records such as the Census of Manufactures make no distinction between imports and domestic production for raw materials purchased for further processing. Similarly the Final Demand transactors such as the Household sector, Government sector, Business sector (for capital formation) do not record the purchase of commodities in terms of their origin—imports versus domestic production.

We note that some countries produce an Intermediate Transactions Matrix broken down into Domestic Production versus Imports. They must have formidable basic records to produce such tables. Or, maybe, they are far more imaginative than we are. Recently OECD asked us to fill a standardized questionnaire wherein the origin of inputs between imports versus domestic production is maintained. We could not fill this part of the questionnaire.

In our tables, we show Imports as a negative column in the Final Demand Matrix. One could though assume that imports are a fixed share of total supply for each of the individual users, and prorate imports over all users. This way one could produce a matrix of Inputs for imports and domestic production separately. But this is not a statistical table, only a display of assumptions.

There are a certain very limited number of commodities that are not produced at all in Canada; these include tropical fruits, coffee, tea, rubber, etc. Use of these commodities is shown in the Canadian I–O tables as rows just like use of other commodities.

We have assumed that the valuation of imports which is equivalent to the "producer price" of domestic production is imports c.i.f. to the border plus import duties.

Transfer of Secondary Outputs

The Canadian I–O tables are not inter-industry but have commodity-industry dimensions. The number of commodities is larger than the number of industries. We do not purify industries by transferring their secondary or subsidiary output except for own account construction (see below for construction). Industries by definition will produce their own principal products, but if there is a subsidiary output, it is left in the industry. We think that this reflects the transactions better and then we can easily relate our data to other statistics published in an industry framework. It is here that the role of Input–Output Tables as a statistical audit for other economic statistics becomes relevant and important.

Construction

Construction is defined in the Canadian I–O tables as work put-in-place both by establishments classified to the construction industry and by the own-account construction activities of other industries. Thus the total activity of construction is brought together in the I–O tables and transfers are accordingly made from the various industries to the Construction activity vectors of the I–O tables. We have broad "information" on the labour component and the material component of construction activity broken by own-account and contract but we have no regular surveys giving us input detail by commodity. It is somewhat easier to imagine a particular commodity going into a typical construction structure but it will be most difficult to estimate its allocation by own-account versus contract. The sheer lack of input detail forced us into putting the entire construction activity together. As well, there are some details available from *ad hoc* studies giving input detail of structures. These two considerations forced us to put the entire activity together.

Government Sector

The accounts published by governments usually follow a functional classification, with only minor consideration given to an economic object classification. The first major problem is to convert the functional detail into an economic object classification. The second problem is that governments are not asked to fill a structured questionnaire as are the industrial respondents. Governments keep their records in a way which is satisfactory for legislation needs, but not necessarily for statistical needs. Records of the various levels of government, federal, provincial and municipal, are unstructured and then the commodity detail is lacking. The available data are for very broad categories. The input–output analyst must estimate the detail by reference to detail buried behind the published records, and diligently code myriad information spread over tabular and textual forms. The commodity detail information is rather weak in the basic public accounts of the Government sector.

Capital Formation

Statistics Canada collects information by industry on total capital formation in construction and in machinery and equipment. Hardly any information is collected on the types of commodity that an industry purchases on capital account. One has to study each commodity which could possibly be capitalized and then decide its allocation to a particular industry. Surprisingly a large number of commodities can quite easily be allocated. Some types of machinery are industry-specific hence easily codable to a given industry. Once one has exhausted the commodity supply side, then one checks the allocation totals to see if they are different from the survey ones. Going back and forth a couple of times will in most cases fix rather well the allocation by detail and total.

Impact Table

The definition of the Impact tables as published in the Input–Output Structure of the Canadian Economy involves two sets of assumptions. The first has the function of allocating the production of commodities among industries. The second establishes the production functions of industries which in turn determine the requirements of industries for commodity inputs.

The simplest assumption concerning the allocation of commodity production among industries is that industries will preserve their observed share of the market for each domestically produced commodity irrespective of the levels of commodity production. The mathematical expression of this assumption is the following matrix equation:

$$g = Dq \tag{1}$$

In this equation vector g represents the values of industry outputs and vector q the values of domestically produced commodity outputs. Matrix D is a matrix of coefficients which is calculated by dividing each element in a column of the output matrix V of Chart I by the corresponding total commodity output. It will be referred to as the Domestic Market Share Matrix.

The simplest way of defining the production functions of industries is to suppose that the values of the inputs of each industry are fixed proportions of the value of the total output of the industry and are thus independent of the composition of this output. This assumption, which has always been a basic assumption of input–output analysis, is now being referred to in the literature as the "industry technology" assumption. The mathematical expression of this assumption is the following matrix equation:

$$Ui = Bg \tag{2}$$

In this equation matrix U is the matrix of commodity inputs of Chart I. Vector i is a column vector, equal in dimension to the number of industries, whose elements are all equal to 1. The matrix product Ui, therefore represents a vector containing the sum of the intermediate inputs of all industries classified by commodity. Matrix B is a matrix of coefficients which is obtained by dividing each element in a column of matrix U by the corresponding total industry output. Matrix B will be referred to as the Industry Technology Matrix. Vector g represents, as before, industry outputs. The equation as a whole states that total intermediate inputs classified by commodity can be calculated as the product of the Industry Technology Matrix and the vector of industry outputs.

The mathematical expression of the accounting balance between total supply and total disposition is as follows:

(3) $$q + m + a + v = Bg + \overset{*}{e} + X_D + X_R$$

where

m is a vector of the values of imports

a is a vector of the values of government production

v is a vector of the values of withdrawals from VPC

X_D is a vector of the values of domestic exports

X_R is a vector of the values of re-exports

and $\overset{*}{e}$ is a vector of the values of the following final demand categories:

$$PE + FCF + VPCA + GGCE$$

(3a) $$q = Bg + \overset{*}{e} + X_D + X_R - m - a - v$$

Equations (1) and (3a) yield the following open determination model.

(4) $$g = (1 - DB)^{-1} D(\overset{*}{e} + X_D + X_R - m - a - v)$$

Equation (4) defines linear transformation of final demand categories into industry outputs.

Industry output can easily be transformed into commodity output by using equation (3a).

$(1 - DB)^{-1}D$ post-multiplied by exogeneously-specified final demand calculates industry outputs. This model does not account for any leakages from domestic industries. To the extent that imports and/or withdrawals from inventories and/or government production of goods and services share with the domestic industries in the supply of a commodity, the impact of an increase in final demand on domestic industries will be reduced. These leakages are specified below.

(5) $$m = \hat{\mu}(Bg + \overset{*}{e} + X_R)$$

where vector m represents imports and $\hat{\mu}$ is a diagonal matrix of coefficients whose elements are calculated as the ratios of imports to use, use defined as $Bg + \overset{*}{e} + X_R$. It is to be noted that this import share assumption implies that domestic exports of a commodity are supplied from domestic industries that produce the commodity. Of course, domestic exports may have imports indirectly embodied in them to the extent that producing industries import their intermediate inputs.

$$(6) \qquad a = \hat{\alpha}(Bg + \overset{*}{e} + X_D)$$

where vector a represents government production of goods and services and $\hat{\alpha}$ is a diagonal matrix of coefficients whose elements are calculated as the ratio of government production to use, use defined as $Bg + \overset{*}{e} + X_D$

$$(7) \qquad v = \hat{\beta}(Bg + \overset{*}{e} + X_D)$$

where vector v represents withdrawals from inventories (*VPC*) and $\hat{\beta}$ is a diagonal matrix of coefficients whose elements are calculated as the ratio of withdrawals to use, use defined as $Bg + \overset{*}{e} + X_D$.

Substitution of equations (5), (6) and (7) into (3a) yields the following:

$$(8) \quad q = Bg + \overset{*}{e} + X_D + X_R - \hat{\mu}(Bg + \overset{*}{e} + X_R) - \hat{\alpha}(Bg + \overset{*}{e} + X_D) - \hat{\beta}(Bg + \overset{*}{e} + X_D)$$

$$(8a) \quad q = (I - \hat{\mu} - \hat{\alpha} - \hat{\beta})Bg + (I - \hat{\mu} - \hat{\alpha} - \hat{\beta})\overset{*}{e} + (I - \hat{\alpha} - \hat{\beta})X_D + (I - \hat{\mu})X_R$$

Equations (8a) and (1) yield the following open output determination model which makes an allowance for leakages out of intermediate demand as well as final demand.

$$(9) \qquad g = [I - D(I - \hat{\mu} - \hat{\alpha} - \hat{\beta})B]^{-1} D[(I - \hat{\mu} - \hat{\alpha} - \hat{\beta})\overset{*}{e} + (I - \hat{\alpha} - \hat{\beta})X_D + (I - \hat{\mu})X_R]$$

The precise mathematical expression defining the inverse used in the Impact (Inverse) Tables is

$$[I - D(I - \hat{\mu} - \hat{\alpha} - \hat{\beta})B]^{-1} D$$

AGGREGATION

Another problem in economic accounting which is brought into sharp focus in the construction of Input–Output Tables is aggregation, a topic which has not received much attention in the literature.

As noted above, the Canadian Input–Output Tables are completely balanced at the L level of aggregation but are published at aggregation level S and aggregation level M. The levels of aggregation are hierarchical. Level S can be derived from level M and level M can be derived from level L. The hierarchical basis of aggregation is true for industries, commodities and categories of final demand.

Gross output of industries and gross intermediate inputs of industries do not change with any aggregation. We do not eliminate intra-industry inputs at any level of aggregation.

But one must realize that the same data base could give different results depending upon the level of aggregation the model builder has used. Aggregation is quite a serious problem but unfortunately one does not hear much of it in most economics textbooks. Aggregation forces a particular point of view, a particular analysis. Indeed, aggregation itself is a model.

Let me elaborate the above point using the Canadian input–output tables. Input–output impact tables are published at two levels of aggregation for all the seventeen years 1961–1977. Still another level of detail (which we do not publish so as to avoid disclosure of information on individual firms) is used internally for model building. We looked at a particular commodity and a particular industry whose definitions remained the same at the two published aggregations. The impact of this particular commodity on the particular industry was different depending upon which (level of aggregation) impact tables one looked at. This is a seemingly anomalous situation. However, this apparent anomaly can be explained by the fact that other commodities (other than the one under study) were aggregated differently in the two aggregations. But the fact remains that different aggregations give different results.

One might draw a lesson. One must avoid the temptation, as much as possible, to aggregate data in the early stages of economic analysis. Aggregation is easier to do than disaggregation. The implications of this simple statement must be well understood. We strongly recommend to pay attention to keeping micro data sets. We say "keeping" micro data sets because data sets from the respondents are made available in this form. With them one would be able to arrange custom-made aggregations for economic analysis. It goes without saying that the data retrieval system should be efficient. Without an efficient data retrieval system, one will remain a prisoner of dictated (by others) aggregations and one's horizon of economic analysis will remain narrow indeed.

Constant Price Input–Output Tables

With the decision to establish a program of annual input–output tables in current prices, tables at constant prices were a logical corollary. In Canada one of the roles of input–output tables is to serve a supportive and integrating role for other parts of the system of national accounts and input–output tables at constant prices are necessary to completely fulfill this function. While, on the face of it, the construction of input–output tables in constant prices simply involves restating the commodity values from the current price tables in base year values, in fact a number of problems were encountered, and this section describes some of these problems, how they were approached and how they were resolved. It is important to note that there are a number of problems to which we do not yet have definitive solutions and these areas are continuing to be explored and developed as resources permit.

At the time the project was initially started the literature available to guide us was relatively sparse. More recent references, particularly the United Nations *Manual on National Accounts at Constant Prices*[4] have reinforced most elements of the approach we adopted.

Before discussing some of the problems and some of the features of the Canadian I–O tables a brief general description of the approach to deflation is

[4]ST/ESA/STAT/SER.M/64; United Nations, New York, 1979.

necessary. The Canadian I–O tables in current prices are published at producers' values, but are in fact compiled as noted above at both producers' values and at purchasers' values. It might be possible to carry out deflation on either valuation basis, but deflation of producers' values offers a number of advantages, and this is in fact done. The Canadian Manufacturing Industry Selling Price Indexes are measured at the factory gate before inclusion of commodity indirect taxes or outward transportation, and these price indexes serve as the basis for calculation of the deflators for a majority of the commodities.

In valuation at producers' prices all commodity transactions are on a uniform basis of valuation thus permitting the use of a single price deflator across a commodity vector. In tables at purchasers' values, each transaction is potentially a unique amalgam of producers' value and various margins, and thus presents almost impossible demands on the construction of price deflators. In tables at any basis of valuation the problem of deflating the tax, trade and transport margins on commodities must be faced.

In the following paragraphs the development of the constant price input–output tables is examined from two points of view. The paragraphs immediately below note some of the aspects of relating the constant price input–output tables to other accounts in the Canadian System of National Accounts. Subsequent paragraphs discuss some of the problems peculiar to input–output tables, problems not encountered in constructing the tables at current prices. Of course all aspects of the tables are interrelated and these distinctions are made for the purpose of exposition.

The traditional measures of production published in Statistics Canada to which the input–output tables can be related are Gross National Product, Gross National Expenditure (the National Income and Expenditure Accounts) and Real Domestic Product by Industry. Only Gross National Expenditure (GNE) and Real Domestic Product by Industry (RDP) are published in constant prices.

The input–output tables encompass both GDP by industry and Expenditures on GDP and thus provide the only mechanism whereby the RDP measures can be reconciled with constant price GNE. While the theoretical concept of the RDP measures is similar to constant price GDP from the input–output tables there are of course differences in methodology.[5] The input–output tables employ double deflation for all industries but the RDP indexes are constructed from single indicators or double deflation taking no express account of some service inputs. Where double deflation is employed in RDP, inputs are recorded at purchasers' values and the outputs at producers' values. Thus complete reconciliation of the RDP indexes with the input–output tables cannot be definitive but problem areas can be identified.

We have also observed that there can be a significant difference between the growth in production at factor cost and at market prices—such as when imports (which are taxed, but do not form part of domestic production) account

[5]While the preparation of both annual constant price input–output tables and annual RDP indexes appears to have some elements of duplication, the RDP indexes do present a long historical series (from 1935 for total RDP, from 1919 for the Index of Industrial Production) of real output by industry measured on a consistent basis. The annual RDP indexes also serve as the basis for monthly estimates of real output by industry in the current period.

for a changed share of domestic demand, or when the commodity composition of domestic demand shifts to more (or less) highly taxed commodities.

As was noted above, the estimates of Expenditure on GDP from the input–output tables can corroborate or refine the final demand categories in the GNE accounts. In constant prices, there is another dimension to the relationship between the final demand estimates from the input–output tables and GNE from the Income and Expenditure Accounts. Taking consumer expenditure as a particular example, final demand at constant prices from the input–output tables is calculated as the sum of commodities at producers' values, deflated with appropriate price indexes, and the various margins at constant prices.

The GNE account is estimated at purchasers' values, and deflated with purchasers' price indexes, principally elements of the Consumer Price Index (CPI). Thus the input–output tables present a powerful tool for reconciling different price indexes at different levels of valuation. Our experience with this exercise has been surprisingly good. The first add-ups revealed a few problems but analysis of these cases provided straightforward solutions in most instances, leaving only a few areas such as clothing, which is recognized as notoriously difficult to price, for further development.

There are also some problems in deflation which are largely peculiar to the internal characteristics of the input–output tables. The principal ones are noted below.

It was perhaps not surprising to discover that the commodity classification system developed for the current price input–output tables did not always result in commodity classes that were sufficiently homogeneous for deflation purposes—either from the point of view of the diversity of varieties within classes or of market differentiation. In the next review of the commodity classes attention will be directed to the needs for deflation, but considerable progress has been made. In the current tables imports of a commodity are routed with the domestic production of the same commodity. To the extent that import price indexes are available corresponding to the input–output commodity classes, imports in the input–output tables are deflated with import prices. Domestic output and exports are deflated with domestic prices, and all categories of domestic use, intermediate and final, are deflated with a weighted import and domestic price index. Similarly where exports attract a different price from the domestic market, exports are deflated with a special price index. Within the domestic market some commodities exhibit price discrimination, and the flow of these commodities to each market is deflated by the price appropriate to that market. For the commodity pipeline transportation it was observed that prices of pipelines carrying natural gas behaved differently from those of pipelines carrying petroleum products. This became two commodities for deflation. At present the price deflator for transportation margins is a weighted average for all modes of transport and for all commodities, but work is under way on developing price indexes for transport margins classified both by commodity and by mode of transport.

Probably the major problem in constructing constant price input–output tables is the deflation of trade margins; the measurement of the output (constant price margins) of trade industries is always difficult whether within or outside the input–output framework. While a number of approaches were considered, two have been tried, with results that have been acceptable but not fully satisfactory. The suggestion of calculating margins in constant prices as the product of the margin rate in the base year and the volume of the commodity being traded was rejected on conceptual grounds. Constant margin rates imply a constant proportionality between the volume of a commodity being sold and the quantity of trade margin or service attached to that good. Observation of the market place in action suggests this is not so. Marketing techniques are constantly changing and the quantity of distribution service is generally becoming less. Filling stations require drivers to pump their own gasoline; free delivery becomes charged for; packaging and display are "economized" to reduce costs. The first approach adopted was to use the margin rates for each year from the current price tables. Thus the margin rates at current and at constant prices were the same, with the changing rates reflecting changes in the quantum of services. This approach worked reasonably well and is incorporated into our published tables for the years 1961–1971. It may also be noted that this was a period of relatively stable prices. This technique was originally carried into the 1970's with less agreeable results.

The use of current year trade margin rates exacerbated the effects of rapidly changing prices combined with a great diversity in mark-up practices. The next step was to emulate Goldilocks and the Three Bears in search of some formula that seemed "just right." The closest to Baby Bear's porridge proved to be using a margin rate calculated as the average of the given year's rates and the base year's rates from the current price tables. The two principal criteria by which different approaches were evaluated were reasonable productivity measures for the trade industries and an implicit price at purchasers' values that most closely approximate observed purchasers' prices. As mentioned above, this is one of the areas we plan to continue to investigate.

Indirect taxes and subsidies present few difficulties in constructing constant price input–output tables at both factor cost and market prices. Commodity indirect taxes at base year prices are calculated as the product of the base year tax rate and the quantum of the commodity taxed. Some slight imprecision can arise because it is not known just what margins should be added to the producers' value to calculate the tax. For highly taxed commodities such as alcoholic beverages, where in Canada at least, the tax is a large multiple of the producers' value of the commodity, any (say) timing error in the producer's value of the commodity as recorded is greatly inflated in the tax calculation. In the case of subsidies, in the majority of cases for the significant subsidies it has been possible to identify the commodity being subsidized; the procedure then becomes analogous to commodity indirect taxes. In the more trivial cases the subsidy is calculated as a fixed (base year) proportion of the constant price output of the industry receiving the subsidy.

One other problem, mentioned last because it arises less frequently, is the problem of rebasing constant price input–output tables. Periodic rebasing is necessary in constructing longer time series and the general Canadian practice has been to rebase at about ten-year intervals. These time segments are linked in a backward chained Laspeyres measure, with the chaining periods being the ten year intervals. Chained indexes are unfortunately not additive, and this non-additivity gives rise to adjusting entries which are implicit when the constant price series is expressed in index number form (the indexes of RDP) or explicit when given in actual currency units (as the constant dollar GNE). The matrix presentation of the input–output tables does not lend itself to the inclusion of adjusting entries, and thus we cannot present chained rebased input–output tables in the same mode as the other constant price production accounts in the Canadian System of National Accounts. At the present time Canada has annual tables for seventeen years 1961–77, with the whole period based on 1971 prices; in addition the tables for years 1961–71 are available at constant 1961 prices. But the use of one base year becomes less defensible as the time period lengthens.

Conclusions

The Input–Output Accounting framework, when set in commodity–industry dimension, provides a powerful statistical audit mechanism for improving consistency, integrity and comprehensiveness of economic statistics. This aspect of the compilation of input–output tables has, in our mind, not been delineated properly in the literature. Compilation of input–output tables forces a re-examination of various and unintegrated commodity classification schemes prevalent in any central statistical organization; it brings to the forefront, as a matter of routine, any statistical holes in economic statistics; it brings to light the various and unintegrated valuation practices in economic series; it provides a meaningful, comprehensive and integrated data base to reconcile the expenditure side of the national accounts with the income side in both current and constant prices.

The compilation of input–output tables serves several roles but the one which has so far dominated, at the cost of others, is the role in providing the framework for impact analysis with given shocks in the final demand. In Canada, our program, as an illustration, serves at least the following roles:

(i) Analysis of the structure and studies of the impact (direct and indirect effects) of exogenous factors on the economy in contrast to the macro economic analysis possible from the data bases of the other sub-systems of the Canadian System of National Accounts (CSNA).
(ii) Market analysis in the context of a detailed, comprehensive and consistent data base on the supply and demand of commodities.
(iii) Basis for new frontiers in productivity analysis.
(iv) A statistical audit of the consistency, integrity and comprehensiveness of economic statistics.
(v) Benchmarks for the production accounts in the CSNA.
(vi) Source of algorithms in the development of weighting patterns for price indexes, say, for capital formation.

When compilation is viewed in its broader role, the periodicity of input–output tables becomes an important question. Input–output tables can serve better their broad role and be of high quality if they are produced annually. Time series improve quality of output and the experience of the staff remains relevant and useful.

We made a reference to aggregation problems in this paper. It is absolutely essential to develop an efficient data retrieval system. Furthermore, basic input–output tables must be balanced at as micro a level as possible and this micro data set must be preserved. It is worth emphasizing that the economics profession should pay more attention to the problem of aggregation of economic series and the errors which ensue with various levels of aggregation.