TRUE HOUSEHOLD EQUIVALENCE SCALES AND CHARACTERISTICS OF THE POOR IN THE UNITED STATES

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In this paper we deal with the question of which measures of economic well-being are adequate to identify those groups of households in the U.S. whose economic conditions justify public concern and assistance. We derive a utility based measure of economic well-being from the estimation of a complete set of consumer demand equations. The demand system is Lluch's Extended Linear Expenditure System (Lluch, 1973). Household characteristics are incorporated using the scaling method proposed by Barten (1966). Using the welfare indicator derived, we study the composition of the poorest part of the population, using data from the 1972–73 Consumer Expenditure Survey. We compare our results with those obtained using various other welfare indicators, including the official U.S. poverty line. We show that using different family composition adjustments significantly and systematically affects just who are considered to be at the bottom of the welfare distribution. We finally suggest that program designers therefore can improve their target efficiency by carefully selecting from among the acceptable indices of welfare when defining program eligibility.

1. INTRODUCTION

In a recent article in this journal, Datta and Meerman (1980) forcefully assert that *per capita* household income (PCY) is preferable to household income for study of the distribution of economic well-being.[1] They also show that the choice is empirically important: trend, and the classification of households by decile, are sensitive to the difference in income concepts.[2]

Datta and Meerman recognize that PCY is a proxy for adult equivalent income. They state that it gives results quite similar to those derived from merging a set of ten equivalence scales. Our experience, however, is that the equivalence scales in common use in the United States, and another that we developed, give quite different results than are obtained with PCY. Furthermore, PCY is conceptually quite different from most other equivalence scales, which derive from the empirical examination of data on consumption by households, not from data on

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[1]For the record and despite what follows, one author of this paper rejects out of hand the idea that adjusting income for family size is preferable to unadjusted household income in making welfare comparisons in a developed country. For formal arguments see e.g. Pollak and Wales (1979), Lebergott (1976, Chapter I).

[2]Danziger and Taussig (1979) show that inequality over time in PCY differs in level and trend from inequality in "household *per capita* income of individuals," to use Datta and Meerman's terminology.

household income. Obviously, among the set of equivalence scales PCY is an extreme choice—it counts infants as adults.[3]

In this paper we deal with the question of which measures of economic well-being are adequate to identify those groups of households in the U.S. whose economic conditions justify public concern and assistance, and, confronted with a variety of measures, how to choose among them.

The most common measure of economic well-being is current income, as measured by, say, annual earnings plus nonearned income. Many argue, however, that large transitory components in annual income make this measure unattractive. Since permanent income, the more desirable measure, is generally not observable, proxy measures are suggested. One such proxy measure is total consumption expenditures, which presumably suffers less than income from transitory fluctuations. Another measure of economic well-being frequently employed is based on Engel's first law: the food share. Households spending a large share of their income on food are assumed to be worse off than households spending a small share.

Not only must we determine the yard-stick that should be used to measure economic well-being of a household of given composition, but we must determine how to compare the economic statuses of households of different composition. Using the food share implicitly solves this problem. Watts (1977) has suggested broadening this idea to encompass the share spent on "necessities" (the iso-prop index).

Frisch's concept of the income elasticity of the marginal utility of income is based on consumer demand theory. Goldberger (1967) has shown that this measure is proportional to the proportion of income spent on all commodities at subsistence levels of consumption, where subsistence is as defined in the Linear Expenditure System. Frisch's measure is also directly related to the notion of true household equivalence scales. True—i.e. constant utility-household equivalence scales with respect to differences in family composition—are defined analogously to true cost of living indices (see Muellbauer, 1974). We will estimate this measure, using the U.S. Consumer Expenditure Survey, 1972/73, and compare it with a number of other frequently used measures.

First, in section 2, we will briefly describe how to derive a set of true household equivalence scales from the estimation of Lluch's (1973) Extended Linear Expenditure System (ELES), following Kakwani (1977, 1980). In section 3, we present the estimation results. In section 4, we will compare our new measure of economic well-being with four commonly used measures, including the official U.S. poverty-line, and we will focus on the characteristics of the poor under the alternative measures. Section 5 discusses the various approaches and draws policy implications.

[3]PCY has nowhere been theoretically justified either. Its use is attributable primarily to habit and custom. When Kuznets was establishing the field, micro data sets were rare, computers even more scarce, and utility was a textbook idea disdained by all practical men. That having been said, it must be clear that income rather than consumption could be justified by appeal to utility maximization (where income would be the sum of earnings plus the value of leisure plus nonearned income). We cannot foresee, however, a time when infants and adults will be shown to be formally equivalent.

2. True-Constant Utility-Household Equivalence Scales

As is well known, in the absence of price differences, the estimation of true household equivalence scales from a full system of demand equations is complicated by an identification problem (Muellbauer, 1974). As Kakwani (1977) has shown, this problem can be overcome if we apply Barten's approach to incorporate household characteristics in a demand system (Barten, 1966) to Lluch's Extended Linear Expenditure System.

This demand system can be derived from the assumption that households maximize a lifetime utility function under a lifetime wealth constraint. The demand equations derived from this system for the current period look as follows (see Appendix):

$$q_i = a_{i0} + b_i z + a_i' h, \qquad i = 1, K; \tag{1}$$

i.e. the demand for good i, q_i, is a linear function of income, z, and a vector of household characteristics, h.[4]

This demand system differs from the more familiar Linear Expenditure System in that income, instead of total consumption expenditures, appears on the right-hand side; that is savings or dissavings are endogenous in any given time period.

Lluch developed his model in a continuous time framework, with infinite time horizon. He argues that consumers replan their consumption at the beginning of each period, given new information on future price and income developments. Having accepted that interpretation, we will base our welfare comparisons on utility derived in the current period, that is the year during which the data were collected.

This utility level can be represented by the familiar Stone–Geary function:

$$U = \sum_{i=1}^{K} \beta_i \log\left(\frac{q_i}{m_i} - \gamma_i\right), \tag{2}$$

where β_i and γ_i are parameters, and m_i is a commodity specific weighting factor that is a function of household characteristics. For instance, if m_i equals family size for all commodities, welfare comparisons can be made on a *per capita* income basis. If m_i equals 1.0 for all commodities, welfare comparisons can be made using total household income. Generally the m_i's will differ among households in a more complex way, and they will vary among commodities. We specify

$$m_i = 1 + d_i' h, \qquad i = 1, K \tag{3}$$

where h is a vector of household characteristics to be discussed below.

The parameters β_i and γ_i in equation (2) and the parameter vector d_i in equation (3) can be obtained, once the demand system (1) is estimated. These parameters will enable us to construct constant utility equivalence scales by employing the cost-function dual to the utility function (Muellbauer, 1974). The Appendix gives details. In the next section we present our estimation results.

[4]Prices do not enter these equations because, since we are dealing with cross-section data, we assume that all households face the same prices.

3. Estimation Results

The demand equations given in (1) are estimated for food, housing, clothing, transportation and a remainder category, other, using the 1972–73 Consumer Expenditure Survey. The vector of household characteristics, h, includes the age and sex of the head of the household, and whether or not there are children in various age classes. The log of total family size is included, to represent possible overall scale effects. The O.L.S. estimation results are presented in Table 1.

Not surprisingly, log family size has a large and significant impact on all consumption expenditures. The age distribution of the children and the age of the head of the household also appear to be an important determinant of consumption.

Female headed households spend, *ceteris paribus*, less on food, transportation and other, but there is no significant impact on housing and clothing.

TABLE 1

Demand Equations of ELES, Household Characteristics Included

	Food	Housing	Clothing	Transportation	Other
1. Child <6 years[a]	−0.35	0.38	−0.11	−0.55	−0.26
2. Children 6–11; <6	−0.03	0.19	−0.13	−0.64	−0.29
3. Child 6–11	0.15	0.04*	−0.04*	−0.36	−0.08*
4. Child 12–17	0.24	−0.13*	0.09	−0.15*	−0.03*
5. Children 12–17; <6	0.20	−0.25*	−0.02*	−0.86	−0.36
6. Children 12–17; 6–11	0.32	−0.03*	0.06*	−0.36	−0.08
7. Children 18; <6	0.30	−0.48*	0.13	0.04*	−0.17*
8. Children 18; 6–17	0.51	−0.13	0.16	0.48	0.45
9. Child 18+	0.16	−0.10*	0.10	0.78	0.40
10. Age of head <35	−0.03*	0.07*	0.02*	0.18	−0.08
11. Age of head 55–64	−0.03*	−0.41*	−0.13	−0.38	−0.10
12. Age of head 65+	−0.22	−0.62	−0.26	−0.83	−0.28
13. Female head	−0.22	−0.09*	0.01*	−0.66	−0.34
14. Log family size	0.70	0.38	0.21	0.46	0.32
15. Income after tax	0.02	0.07	0.02	0.03	0.03
16. Constant	1.03	1.77	0.32	1.43	0.91
R^2	0.40	0.24	0.27	0.19	0.25

*Not significant at 1 percent level.

[a]The first 9 variables are dummy variables equal to one if a household has one or more children in the indicated age classes, and zero otherwise. Variables 10, 11 and 12 are dummy variables equal to one if the head of the household has the indicated age, and equal to zero otherwise.

These results are transformed into a set of true household equivalence scales and presented in Table 2, for a selection of household types (see Appendix for details).

As expected from the estimation results of the demand equations, the age and sex of the household head are important variables and the scale is also very sensitive to the age of children—much more so than to family size. In the equivalence scale implicit in the official U.S. poverty line, in contrast, the age of children plays no role, but family size is very important (Table 3). This is perhaps not surprising since the U.S. poverty line equivalence scale (see Orshansky, 1965) is obtained by specifying food "needs" for households of

TABLE 2

TRUE (CONSTANT UTILITY) HOUSEHOLD EQUIVALENCE SCALES[a]

	Age of Head			
	35	35–54	55–64	65+
One person				
Male	64	66	54	40
Female	48	50	38	24
Two persons				
Husband and wife	82	84	71	57
Female head, child 6–11	62	64	52	38
Three persons				
Couple, child <6	81	83	71	57
Couple, child 6–11	88	90	78	64
Couple, child 12–17	91	93	80	66
Couple, child 18+	108	110	98	84
Four persons				
Couple, 2 children <6	88	90	78	64
Couple, 2 children 6–11, <6	88	90	78	64
Couple, 2 children 6–11	96	98	85	71
Couple, 2 children 12–17, 6–11	98	100	87	73
Couple, 2 children 12–17	99	101	89	75
Couple, 2 children 18+, 6–17	117	119	106	92
Couple, 2 children 18+	115	117	105	91
Five persons[b]				
Couple, 3 children 6–11, <6	94	96	83	—
Couple, 3 children 12–17, 6–11	104	106	93	—
Couple, 3 children 18+, 6–17	123	124	112	—

[a]A family consisting of a husband and wife with two children, ages 12–17 and 6–11, the age of the husband being between 35 and 54, is 100.

[b]Adding more children to the household adds 4 or 5 percentage points to the scale up to family size 8. After that only 2 to 3 percentage points should be added.

TABLE 3

POVERTY LINE EQUIVALENCE SCALE COMPARED WITH THE CONSTANT UTILITY SCALE

	Poverty Scale			Constant Utility Scale			
	Age of Head			Age of Head			
Family Size	<65	65+		<35	35–54	55–64	65+
1	58	55		64	66	54	40
1, Female				48	50	38	24
2, Husband, wife	73	68		82	84	71	57
3, Husband, wife, 1 child	80		(child 6–11)	81	83	71	57
4, Husband, wife, 2 children	100		(6–11; 12–17)	98	100	87	73
5, Husband, wife, 3 children	120		(6–17; 18+)	123	124	112	—
6, Husband, wife, 4 children	114		(6–17; 18+)	127	129	116	—

different composition. Consequently it is more sensitive to family size than an equivalence scale based on expenditures on all consumption goods. Economies of scale for housing and transportation, for instance, are much larger than for food.

Our scale does exhibit certain oddities, however. The difference between single males and females seems too large. It is also unlikely that the addition of a young child to a childless couple leaves their economic "needs" unaltered. However, the results are straightforward translations of the estimation results of the simple demand equations with household characteristics incorporated. As such, this scale has a stronger theoretical and empirical foundation than the scales commonly employed. This, of course, does not imply that our equivalence scale is the "correct" one in any absolute sense. The base of our scale is the Extended Linear Expenditure System (ELES) in which we incorporated household characteristics by scaling the commodities in a way first suggested by Barten. Consequently, our results are subject to all objections against the assumptions for ELES and the scaling procedure. But, at least, these assumptions are more explicit than in the *ad hoc* approaches, and our results can be judged accordingly. We will use the results in the next section to make welfare comparisons across households.

4. Alternative Welfare Indicators and the Poor in the U.S., 1972/73

The choice of a particular welfare indicator by which to measure the relative economic well-being of a population has large effects on the characteristics of those who are considered poor.

Using the official U.S. poverty line, we find that 16.2 percent of BLS households were poor in the years 1972/73; i.e. 16.2 percent have an after tax income,[5] adjusted for family size and age of head, below the poverty line. Accepting that 16.2 percent of the population are to be considered poor, we ranked the 16.2 percent of households in our sample who were worst off according to five alternative welfare measures. Table 4 reports the definitions of the welfare indicators employed; Table 5 the composition of the poor.

TABLE 4

Welfare Indicators

Name	Definition
Consumption	Total household consumption expenditures
Income	Total household after tax income
"Real" household income	After tax income adjusted with the constant utility equivalence scale
Poverty ratio	After tax income divided by the poverty line
Per capita income	After tax income divided by family size

[5]The U.S. poverty line is a before tax income concept. However, since we used after tax income in all our estimations, we compare the poverty line with after tax income, to isolate the effects of employing different equivalence scales.

TABLE 5

SELECTED HOUSEHOLD CHARACTERISTICS OF THE POOREST 16.2 PERCENT OF THE POPULATION, UNDER VARIOUS WELFARE INDICATORS

	Consumption	Income	Poverty Ratio	Real Income	*Per Capita* Income
Average age of household head	61.2	56.2	51.6	49.0	47.7
Average family size	1.6	2.1	2.9	2.8	3.8
Average number persons <18, per household	0.26	0.52	1.08	0.93	1.69
Before-tax income ($)	3,406	1,214	1,530	1,702	2,493
Percentage from wages	37.6	22.9	32.1	43.7	54.6
Percentage public assistance and social security	38.1	62.6	49.7	36.9	27.6
Taxes paid ($)	195	63	285	257	295

The choice of one welfare indicator rather than another significantly affects the demographic characteristics of the poor. For example, in Table 5, the average age of head varies between 61.2 and 47.7. Average family size increases across the table from 1.6 to 3.8 persons. Holding the number of poor *households* constant, therefore, the number of poor *persons* differs by more than 100 percent between one definition of economic well-being and another. Moreover, the measures capture different segments of the population. The per capita income measure emphasizes the "working poor": 54.6 percent of their income consists of wages. By the current income measure, this percentage is only 22.9 percent.

These patterns are also illustrated in Table 6 where we present the probability of being poor under the various welfare indicators. Of households with at least one wage earner, 8.9 percent are poor under the *per capita* measure; but the after tax income measure yields only 4.8 percent.

TABLE 6

PROBABILITY OF BEING POOR FOR SELECTED HOUSEHOLD CHARACTERISTICS, USING VARIOUS WELFARE INDICATORS, OVERALL PROBABILITY KEPT CONSTANT AT 16.2 PERCENT

	Consumption	Income	Poverty Ratio	Real Income	*Per Capita* Income
At least one earner	8.7	4.8	6.3	8.0	8.9
Black	29.7	27.3	33.9	30.7	32.1
Female	40.8	33.7	30.3	22.4	22.9
Unmarried	38.4	30.7	26.7	22.6	19.2
South	20.4	35.4	20.2	20.4	20.5

Black households are most likely to be counted as poor under the *per capita* measure and the official U.S. poverty measure. The consumption measure emphasizes female headship as we would expect given that female headed households show consistently low consumption levels at every income level. Finally, we note that the probability of being counted as poor if one lives in the South varies from 20.2 to 35.4 percent.

The variation in these probabilities suggests that there is only a small group of households which would be counted as poor no matter which measure of economic well-being was applied. Indeed, only 2.7 percent of households are poor under all of the five measures employed.

Almost 30 percent of these approximately 5.3 million "5 times poor" households are black, 55 percent have a female head, 40 percent are over 65 years of age and more than 50 percent consist of single person households. There is no wage earner in 62 percent of these households. Average before tax income is $1,170, of which 62 percent comes from social security and public assistance programs. Forty-eight percent live in the South.

Apart from these households, which are poor by every measure, there seems to be quite a bit of discretion in the choice of who is poor and who is not: a discretion that is embodied in the choice of the welfare indicator. As Tables 5 and 6 show, this choice implies a choice for or against the aged, for large or small households, for whites or blacks, workers or nonworkers, etc.

Of the measures employed, the real income measure, i.e. the income measure adjusted by the constant utility equivalence scale, appears to be the most defensible on theoretical economic grounds. The utility indicator used to derive this ratio is based on all consumption goods, takes into account that households of different composition may have different needs, and makes use of the fact that different scale effects exist for the consumption of different goods. Moreover, it acknowledges the fact that households respond to "price" (in this case: household composition) changes, by buying more or less of the goods that became relatively less or more expensive as household composition changes.

The total consumption and total income measures ignore the impact of household composition entirely. The *per capita* measure very roughly adjusts income measures for household composition, while the poverty-line equivalence scale is based on nutritional requirements only. The real income measure avoids these problems and finds a solid base in economic theory. Nevertheless, we do not claim it to be an ideal welfare indicator. We will discuss its limitations in the next section.[6]

5. Discussion and Conclusion

Implicitly, we have specified the key determinants of systematic variations in utility across households to be: the level and composition of expenditures, age and sex of the household head, and the number of children and their ages. Obviously, many other factors systematically bear on household satisfactions. Some, like inherited personality traits, are, of course, beyond our purview. However, the relationship between objective and subjective measures of satisfaction is not as direct as we imply—quite clearly personality intervenes. There are other problems. In choosing among consumption categories at any given level

[6]See also Nicholson (1976) for a critical appraisal of different methods of estimating equivalence scales.

of income there are many trade-offs, and many variables are ignored. Other neglected variables are intimately bound up with the determinants of total income and hence with the components of total consumption. In the first category consider such publicly provided goods as library services and police services. Two households may have the same after-tax income, but one may put a higher value on the services of the public sector that are supported by its taxes. In the latter category, consider labor force participation rates, nonpecuniary job attributes, and perhaps region and city size. Households with similar consumption but different in available leisure time and amenities on and off the job are likely to differ in their levels of satisfaction. Furthermore the measure omits elements affecting household productive efficiency; the stock of durables, education levels, and health status are examples. Finally, we have taken only cursory account of the families' past and future by adding to the demand analysis such "life cycle" variables as age of head and age of children. In particular we have ignored the household's past income history, its expected future income and its savings behavior.

Ignoring this large array of variables cannot be justified on the grounds that they are not relevant for public policy. It is sobering to realize that every one of them is used in some U.S. tax or transfer program to define equals, to establish program eligibility, or to otherwise explicitly affect net benefit levels in some other fashion. Further, the list of attributes upon which governments draw in their efforts to promote horizontal and vertical equity goes beyond those enumerated here (race, ethnic origin, or source of income, if examples are required).

Not only does our specification suffer because a large number of relevant variables are missing, but the included variables are themselves controversial.

The ideal utility function would take account of many variables beyond the ones used in this paper. Extending the framework to take the labor-leisure choice into the demand system (i.e. adding leisure as one of the "goods" in the utility function) would be quite consistent with much ongoing research. The addition of savings to the utility function seems also a natural extension and extending the model even further to embrace the fertility decision is probably feasible. But reaching out to the determinants of living arrangements (e.g. to include the circumstances under which children and grandparents live apart from their nuclear families) would probably exceed our grasp. It is to some degree feasible to redefine income, or to expand the set of commodities to include publicly provided goods and the flow of services from durables to take account of habit persistence, differences in needs and tastes at different stages of the life cycle, and expected future income. We intend to add some of these components to this work in the future.

In the choice of a welfare indicator for policy purposes yet another factor enters: to which group of households is the policy measure directed? If one aims to assist the economically deprived aged, total consumption or income are clearly the welfare indicators that maximize the number of eligible aged. If one wants to focus specifically on young children, the working poor, or minorities one should choose an indicator like the poverty ratio or per capita income. Different welfare indicators are indeed used to define eligibility for different public programs. The poverty line frequently defines "need" in establishing eligibility for such programs as Aid to Families with Dependent Children. For tax purposes, household income is the main indicator of economic status.

It has been traditional to redefine income inadequacy by appealing to consumption data, and to use the resulting measures for statistical counts of the poor for establishing eligibility for public programs. We have pursued the same route but in a quite formal way. In particular, we merged Barten's approach to introducing demographic variables in a formal demand system with Lluch's Extended Linear Expenditure System. In this way we derived a set of true-constant utility-equivalent scales with attractive properties. Our equivalence scales are flatter than other scales in wide use, as we should expect, for our scales make use of all categories of consumption expenditures. In addition, they reflect the importance of the age and sex of the household head, and the ages of the children.

Finally we have shown that using different family composition adjustments significantly and systematically affects just who are considered to be at the bottom. The more radically a welfare measure departs from simple total household consumption, the younger and larger the family, the more likely the head is to be male, and the more likely it is that the head works. Program designers can therefore improve their target efficiency by carefully selecting from among the acceptable indices of welfare when defining eligibility.

Our calculations in section 4 were based on the assumption that 16.2 percent were poor, a number we obtained by employing the official U.S. poverty line to the BLS 1972/73 Consumer Expenditure Survey Data. This measure of poverty has recently been under attack since it does not take into account in-kind transfers like Medicaid and Food Stamps. Levels are not all that is at issue, however. Clearly the choice of the measure used, income or consumption, in-kind transfers included or excluded, will have an important impact on the number of households that should be considered poor. But, as this paper suggests, it is equally important in identifying who is considered poor to pay special attention to adjusting the measure chosen to allow for differences in household composition.

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Appendix

Lluch derives the Extended Linear Expenditure System in a continuous time framework, with infinite time horizon. The same results can be obtained from a simpler two-period model. In addition, we include the effect of household characteristics. This yields the following maximization problem:

$$\max U = \sum_{i=1}^{K} \beta_i \log\left(\frac{q_{1i}}{m_i} - \gamma_i\right) + \frac{1}{1+\delta} \sum_{i=1}^{K} \beta_i \log\left(\frac{q_{2i}}{m_i} - \gamma_i\right)$$

$$\text{subject to} \quad \sum q_{1i} + \frac{1}{1+\pi} \sum q_{2i} = Z \equiv \text{wealth}$$

$$\sum_{i=1}^{K} \beta_i = 1; \qquad \frac{q_{1i}}{m_i} > \gamma_i, \qquad \frac{q_{2i}}{m_i} > \gamma_i, \qquad i = 1, K;$$

all prices are set equal to 1.0 in both periods. Thus, q_{1i} is expenditure on item i in period 1; q_{2i} is expenditure on item i in period 2. The parameter δ is a subjective utility discount factor and π is an interest rate. The household specific weighting factors m_i are specified as $m_i = 1 + d_i'h$, with h a vector of household characteristics of length N.

Under the additional assumption that income is the same in both periods, the constrained maximization yields the following demand functions for the first period (ignoring time subscripts):

$$q_i = \alpha_{i0} + \beta_i^0 z + \alpha_i' h, \qquad i = 1, \text{K}.$$

with

$$\alpha_{i0} = \gamma_i - \beta_i^0 \sum \gamma_j, \qquad i = 1, K$$

$$\beta_i^0 = \beta_i \mu, \qquad i = 1, K$$

$$\mu = \frac{1+\delta}{2+\delta} \frac{2+\pi}{1+\pi},$$

and the elements in the vector α_i equal

$$\alpha_{in} = \gamma_i d_{in} - \beta_i^0 \sum \gamma_j d_{jn}, \qquad n = 1, N, \qquad i = 1, K.$$

This set of demand equations is estimated in section 3. Once the estimates of α_{i0}, β_i^0 and α_{in}, $i = 1, K$, $n = 1, N$ are available, we can obtain the β_i's using:

$$\sum \beta_i^0 = \sum \beta_i \mu = \mu \sum \beta_i = \mu,$$

so

$$\beta_i = \beta_i^0/\mu = \beta_i^0/\sum \beta_i^0, \qquad i=1, K.$$

The γ's are then calculated as

$$\gamma_i = \alpha_{i0} + \beta_i^0 \sum \gamma_j = \alpha_{i0} + \frac{\beta_i^0}{1-\mu} \sum \alpha_{j0}.$$

In the same way one can solve for the d_{in}'s. Thus all parameters of the utility function can be obtained. The results are presented in Table A.1. Note

TABLE A-1

PARAMETERS OF THE UTILITY FUNCTION, DERIVED FROM THE DEMAND EQUATIONS IN TABLE 1

	Food	Housing	Clothing	Transportation	Other
β_i	0.02	0.07	0.02	0.03	0.03
γ_i	\$1,179	\$2,132	\$424	\$1,640	\$1,103
d_1	−0.31	0.15	−0.30	−0.35	−0.26
d_2	−0.04	0.06	−0.35	−0.41	−0.29
d_3	0.12	0.01	−0.10	−0.22	−0.08
d_4	0.20	−0.06	0.21	−0.09	−0.02
d_5	0.14	−0.16	−0.10	−0.56	−0.37
d_6	0.27	−0.02	0.15	−0.22	−0.08
d_7	0.25	−0.23	0.30	0.02	−0.16
d_8	0.47	−0.01	0.44	0.32	0.45
d_9	0.17	−0.01	0.30	0.51	0.41
d_{10}	−0.30	0.03	0.03	0.11	−0.08
d_{11}	−0.05	−0.22	−0.36	−0.25	−0.12
d_{12}	−0.23	−0.36	−0.71	−0.56	−0.32
d_{13}	−0.22	−0.08	−0.05	−0.04	−0.35
d_{14}	0.64	0.24	0.60	0.33	0.36

that the γ_i parameters are usually referred to as "subsistence expenditures" for or "committed consumption" of good i. Thus $\sum (\gamma_j(1+d_j'h))$ is the total subsistence expenditure for a household with characteristics h.

The cost function dual to the Stone-Geary utility function reads (e.g. Deaton and Muellbauer, 1979)

$$c(U|h) = \sum \gamma_i(1+d_i'h) + \exp\left[U - \sum \beta_i \log \beta_i + \sum \beta_i \log(1+d_i'h)\right].$$

The constant utility equivalence scales, E, are calculated as the ratio

$$E = c(U|h)/c(U|h_0),$$

with h_0 the characteristics of the reference household. Though this equivalence measure is in general a function of the utility level chosen, our estimates show E to be virtually constant and equal to the ratio of total subsistence expenditures, $\sum \gamma(1+d_i'h)/\sum \gamma_i(1+d_i'h_0)$ over a wide income interval. This is due to the relatively large estimates of the subsistence levels γ_i. Consequently we presented one set of equivalence scales only.