NOTES AND MEMORANDA

INDEX NUMBERS AND THE COMPUTATION OF FACTOR PRODUCTIVITY: A FURTHER APPRAISAL

(A Note on Abram Bergson's, "Index Numbers and the Computation of Factor Productivity," *The Review of Income and Wealth*, September 1975, pp. 259–78)

STEVEN ROSEFIELDE

University of North Carolina, Chapel Hill

In his recent article on index numbers and factor productivity [1], Abram Bergson explored the conceptual foundations of productivity indices by considering the biases that might arise if the production functions implicit in the rules of aggregation adopted for productivity indices were misspecified. Although his analysis was cast in general terms, without explicit mention of the Soviet Union, the significance of his paper is best appreciated as an amplification of the meaning of his past research on the sources of Soviet economic growth and the measurement of Soviet factor productivity. To this end Bergson identified a number of predictable biases which depend on the concavity of the transformation locus, the elasticity of factor substitution and interdependence in the production function, and argues that a proper understanding of these phenomena requires the recognition of a fundamental asymmetry between input and output indices. More specifically, where the underlying production function is CES including Cobb-Douglas as a special case, indices of inputs are "conventionally" aggregated "geometrically" (the weights are powers of the terms, $K^{\alpha}L^{1-\alpha}$) and outputs "arithmetically," $(p_iq_i+p_jq_j)$. All productivity indices therefore are ratios of arithmetic and geometric indices; that is, linear and convex relationships which Bergson concludes may be especially misleading if the functional form of the production function is misspecified.

Insofar as the distinction between "arithmetic" and "geometric" indices describes Bergson's own work [2, 3, 4, 5] in which productivity is estimated *without* the aid of econometric techniques, his strictures are unexceptional and illuminate subtleties that were brushed over in the past. The preoccupation with arithmetic output indices and input indices using *hypothetical* geometric power weights however obfuscates the general theoretical problem.

As a rule, the true form of the production function cannot be known *a priori*. The form of the input relationship must either be determined empirically, or be inferred from an unproven body of theory. When as in Bergson's case, a hypothetical input form is applied in specifying the rules of input index formation, and the values of the parameters are chosen according to some expedient rule such that α and $1-\alpha$ are set equal to relative imputed national income factor shares, it is plausible to argue that an appreciation of the asymmetry between "arithmetic" and "geometric" indices can assist in identifying the probable direction of bias, *ceteris paribus*.

TABLE 1

ALTERNATIVE ECONOMETRIC ESTIMATES OF THE RELATIONSHIP BETWEEN SOVIET INPUTS AND AGGREGATE INDUSTRIAL OUTPUT 1950–73

Source	Input Specification	Disembodied Technical Progress	γ	Parameters δ_1	δ_2	λ	β	ρ	σ	R^2	Comments
1. Weitzman [12]	$[\delta K^{-\rho}+(1-\delta)L^{-\rho}]^{-1/\rho}$	$e^{\lambda t}$	0.80	0.64		0.02		1.50	0.40	0.9995	Official Soviet data; sector of origin value added weights, adjusted for changes in man-hours.
2. Kumar and Asher [8]	$[\delta K^{-\rho}+(1-\delta)L^{-\rho}]^{-1/\rho}$	$e^{\lambda t}$	1.00	0.99		0		0.11	0.90	0.9980	Same as Weitzman; different starting points for non-linear estimation procedure.
3. Desai [6]	$[\delta_1 K^{-\rho}+\delta_2 L^{-\rho}+1-\delta_1-\delta_2 R^{-\rho}]$	$e^{\lambda t}$	Not Reported	0.02	0.68	0.04		2.70	0.27	0.9994	Same as Weitzman; three factor model including natural resources, R.
4. Gomulka [7]	$[\delta(BK)^{-\rho}+(1-\delta)(AL^{-\rho})]^{-1/\rho}$	—	1.00	0.29 −1.11		0.06		0	1.0	0.8076	The terms B and A represent a bundle of parameters prefitted to take account of exogenous shocks. First differences are used instead of time series in the nonlinear estimation procedure.
5. Rosefielde and Lovell [9]	$[\delta K^{-\rho}+(1-\delta)L^{-\rho}]^{-1/\rho}$	$e^{\lambda t+\beta t^2}$	0.89	0.42		−0.006	0.001	4.55	0.18	0.9992	Same as Weitzman; value added by sector of delivery weights using adjusted factor costs.

Notes

γ = a scale parameter

δ_2 = the labor intensity parameter

β = variable component of technical progress

δ = the elastic of substitution ($\sigma = 1/1+\rho$)

L = labor

δ_1 = the capital intensity parameter (In Gomulka's case δ_1 was estimated separately for two subperiods.)

λ = constant Hicks neutral technical progress

ρ = a parameter which determines the elasticity of factor substitution

K = capital

R = natural resources

For a thorough appreciation of the strengths and weaknesses of each specification, the reader should consult the papers in which they were initially reported. The list above does not include nonlinear estimates using CIA data. In general the growth rates and factor substitution elasticities computed from this alternative data series are lower than counterpart estimates using official gross value of output data.

However, one may easily fall into the trap of confusing *ceteris paribus* inferences from hypothetical geometric input specifications properly adjusted for bias with inferences on the true state of productivity [10, 12]. In order for the distinction between arithmetic and geometric indices to be really useful, the correct specification of the relationship between outputs and inputs must be estimated econometrically with considerable statistical certainty; otherwise the ambiguities introduced by the specification problem may distort our perception of productivity more than the biases caused by the asymmetry between arithmetic and geometric indices.

Table 1 which draws on evidence taken from a variety of production function studies of postwar Soviet economic growth brings out the empirical problem associated with the use of geometric input indices for the measurement of productivity [6, 7, 8, 9, 12]. As is easily seen, the best statistical form of the input function varies widely. Estimates of the elasticity of factor substitution σ range from 0.18 to 1, $(0 \leq \rho \leq 4.55)$. The capital intensity parameter, δ_1, varies from 0.02 to 0.99. Moreover, although the low elasticity of capital-labor substitution indicates that the CES specification better explains postwar Soviet production relations, in at least one case, Gomulka's, the Cobb-Douglas form is found to be superior.

The marked differences among these estimates are attributable to a variety of factors including:

1. the data series (CIA or Soviet gross value of output data)
2. use of time series or first differences
3. the number of primary factors specified
4. the weights used to aggregate industrial outputs
5. other adjustments to the data
6. the specification of technical progress
7. the exogenous estimation of special parameters
8. the starting points chosen to initiate the nonlinear regression routine.

Had other functional forms been estimated both this list and Table 1 itself could be easily expanded. Enough evidence has been provided however to illustrate that our empirical perception of productivity may be much more sensitive to the form in which inputs are geometrically aggregated than to the bias imputable to the asymmetry between arithmetic output and geometric input series.

If the relative merit of the alternative specifications reported in Table 1 could be readily determined on scientific grounds, statistical or otherwise, the empirical ambiguity surrounding the measurement of productivity could be dispelled. Although some sorting out is surely possible, as is widely understood, discriminating preferred from dispreferred specifications is a controversial matter, a fact that poses serious difficulties for scholars concerned with accurately measuring productivity. Hope perhaps can be found in the possibility that the residuals computed from these diverse specifications may not be as dissimilar as might be supposed from a casual glance at the parameters. Some evidence sustaining this view is available. If it should be borne out by further research, Bergson's analysis would remain valid and provide important insight into the systematic biases associated with the computation of productivity using weighted CES input indices. However, if the form of the input specification proves to be indeterminant, the practical value of Bergson's bias assessment would be placed in considerable doubt. Productivity will have to be calculated according to some bounded, mean representation of all viable geometric input specifications, making allowance for a range of error imputable to specificational indeterminism and index bias. If the range of error attributable to ambiguities of specification is large, the relative importance of the biases associated with the asymmetry between arithmetic output and geometric input indices will necessarily be diminished.

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