ON THE BIASES IN FOREIGN TRADE INDICES*

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Starting out from the observation that both imports and exports may be viewed as the difference between domestic consumption (use) and production, static standard theory of biases in consumption and production indices is brought to bear upon trade indices: Laspeyres tends to overrate when applied to imports and to underrate when applied to exports; for Paasche, the opposite holds true. Hence, terms of trade tend to be underrated (exaggerated) when based upon Laspeyres (Paasche) price indices. The problem of extending these conclusions to the case of changes in production frontiers and preference maps is discussed. When homotheticity is absent, correlation between price and quantity relatives may upset the simple conclusions. This is of special importance in the large-country situation. Dynamics further complicate the situation. A cobweb mechanism in exports may thus reverse the static results.

In contrast to what is the case with consumption and production indices, little systematic economic theory appears to have been developed for possible biases specifically related to export and import volume and price indices.

For all price indices, including those in foreign trade, of course, it is true that if price and quantity are negatively (positively) correlated, Laspeyres exceeds (falls short of) Paasche, and that by the identity, volume times price equals value, Laspeyres quantity and price indices will either both exceed or fall short of Paasche (Allen, 1975, pp. 62–63). Combined with the economic hypothesis that prices and quantities in foreign trade are determined by the intersection between demand and supply curves for exports and imports, respectively, these statistical and definitional generalities have been used to explain empirically observed relations between Laspeyres and Paasche in United States foreign trade (Lipsey, 1963, pp. 62–74), or to predict such relations for particular commodity groups, including exports (Allen, 1975, pp. 64–65, 115–25, etc.). When shifts in the supply curve predominate, we can expect negative correlation; when demand shifts predominate, positive correlation; with obvious consequences for the indices.

Useful as such considerations are, they fail to utilize the existing theory of consumption and production indices. Its relevance for foreign trade indices follows from the obvious observation that, ignoring inventory changes, we can always write

$$\text{import} = \text{domestic consumption } \textit{minus} \text{ domestic production}$$

and

$$\text{export} = \text{domestic production } \textit{minus} \text{ domestic consumption}$$

and apply the theory of consumption and production indices to these expressions

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for imports and exports.[1] In the following we shall do that and make an attempt to combine the results with the statistical correlation argument.

1. The Small-Country Situation

To simplify, let us first adopt the *small-country assumption* with both export and import prices being given exogenously. At given preferences and technology, a foreign price increase will lead to domestic substitution that in consumption will go against, and in production, in favor of the commodity (output) in question. Considering the signs of consumption and production in the two expressions above, we immediately have the following proposition:

Laspeyres tends to exaggerate when applied to imports and to underrate when applied to exports, whereas for Paasche the opposite is the case. An important corollary is that terms of trade tends to be underrated when calculated as the ratio between Laspeyres export and import price indices and to be exaggerated when calculated on the basis of Paasche price indices.

Let us develop this proposition more precisely for imports. As an illustration to the formulae given below, the reader may find a simple geometrical example useful. Assume that some domestic resources are completely specialized and can be used only for producing a commodity that is exported and has a given foreign price; this commodity is not consumed domestically.[2] The remaining domestic resources can be used alternatively for producing two commodities that, as a matter of fact, both compete with imports. In Figure 1, FF is the production frontier for the two import-competing commodities, $M1$ and $M2$. At given import prices, export expressed in terms of import commodity $M1$ equals X. There is a well-behaved community indifference map, I_0, etc. Assuming trade to be balanced and all prices given, the initial production point for import-competing goods is $Q_0 = (q_{1,0}, q_{2,0})$ with the consumption point $C_0 = (c_{1,0}, c_{2,0})$. By assumption, C_0 is situated to the NE of Q_0. This assumption is maintained at changes of the production and consumption points so that none of the two import goods becomes export goods. (See Figure 1.)

Assume now that the price of commodity $M2$ increases. The production point moves to $Q_1 = (q_{1,1}, q_{2,1})$ and, at given export in terms of commodity $M1$, the consumption point moves to $C_1 = (c_{1,1}, c_{1,2})$ which with our assumptions is situated on a lower indifference curve as compared with C_0. The ten points on the abcissa, A_1 to A_{10} indicate all the value sums we need in the formulae,

$$A_1 = \sum p_1 q_0$$

$$A_2 = \sum p_1 q_1$$

$$A_3 = \sum p_0 q_1$$

$$A_4 = \sum p_0 q_0$$

[1]Consumption is here taken to include both final and productive consumption. Since, however, the theory of consumer indices carries over directly to input indices (just substitute isoquants for indifference curves), we argue in the following as if only final consumption were involved.

[2]We might assume that it is consumed domestically in a given quantity but with zero income and price elasticities.

$$A_5 = \sum p_1 c_1$$
$$A_6 = \sum p_1 c_0^*$$
$$A_7 = \sum p_1 c_0$$
$$A_8 = \sum p_0 c_1^*$$
$$A_9 = \sum p_0 c_1$$
$$A_{10} = \sum p_0 c_0$$

where p is import price. Summation is over import goods $M1$ and $M2$, but for simplicity we have dropped the commodity subscript. The asterisk refers to a point of tangency, defining compensating income changes.

From Figure 1, and more generally by virtue of the concavity of the production frontier for import competing goods and the convexity of the indifference curves, we have for the import volume indices:

	Measured Index	"True" Index
Production		
Laspeyres	$\sum p_0 q_1 / \sum p_0 q_0 \leq \sum p_0 q_0 / \sum p_0 q_0 = 1$	
Paasche	$\sum p_1 q_1 / \sum p_1 q_0 \geq \sum p_1 q_1 / \sum p_1 q_1 = 1$	
Consumption		
Laspeyres	$\sum p_0 c_1 / \sum p_0 c_0 \geq \sum p_0 c_1^* / \sum p_0 c_0$	
Paasche	$\sum p_1 c_1 / \sum p_1 c_0 \leq \sum p_1 c_1 / \sum p_1 c_0^*$	
Imports		
Laspeyres	$\sum p_0 (c_1 - q_1) / \sum p_0 (c_0 - q_0) \geq \sum p_0 (c_1^* - q_0) / \sum p_0 (c_0 - q_0)$	
Paasche	$\sum p_1 (c_1 - q_1) / \sum p_1 (c_0 - q_0) \leq \sum p_1 (c_1 - q_1) / \sum p_1 (c_0^* - q_1)$	

Following standard theory, we have defined the "true" production index as a number that shows the capacity of the economy to produce a certain commodity mix (Moorsten, 1961). Since the production frontier is unchanged, the "true" production index is 1 (Fisher and Shell, 1972, p. 51). The "true" consumption volume index we have defined in line with Hicks as the compensating consumer income that would leave the consumer indifferent to the price change. The measured and the "true" import volume indices, finally, may be derived from the same considerations or, what is equivalent, simply defined as an (externally) weighted means of the corresponding consumption and production indices, the weights being the values of consumption and production respectively in base or current period. For the measured Laspeyres import volume index, for instance, we have

$$\frac{\sum p_0 c_0 \dfrac{\sum p_0 c_1}{\sum p_0 c_0} - \sum p_0 q_0 \dfrac{\sum p_0 q_1}{\sum p_0 q_0}}{\sum p_0 c_0 - \sum p_0 q_0} = \frac{\sum p_0 (c_1 - q_1)}{\sum p_0 (c_0 - q_0)}$$

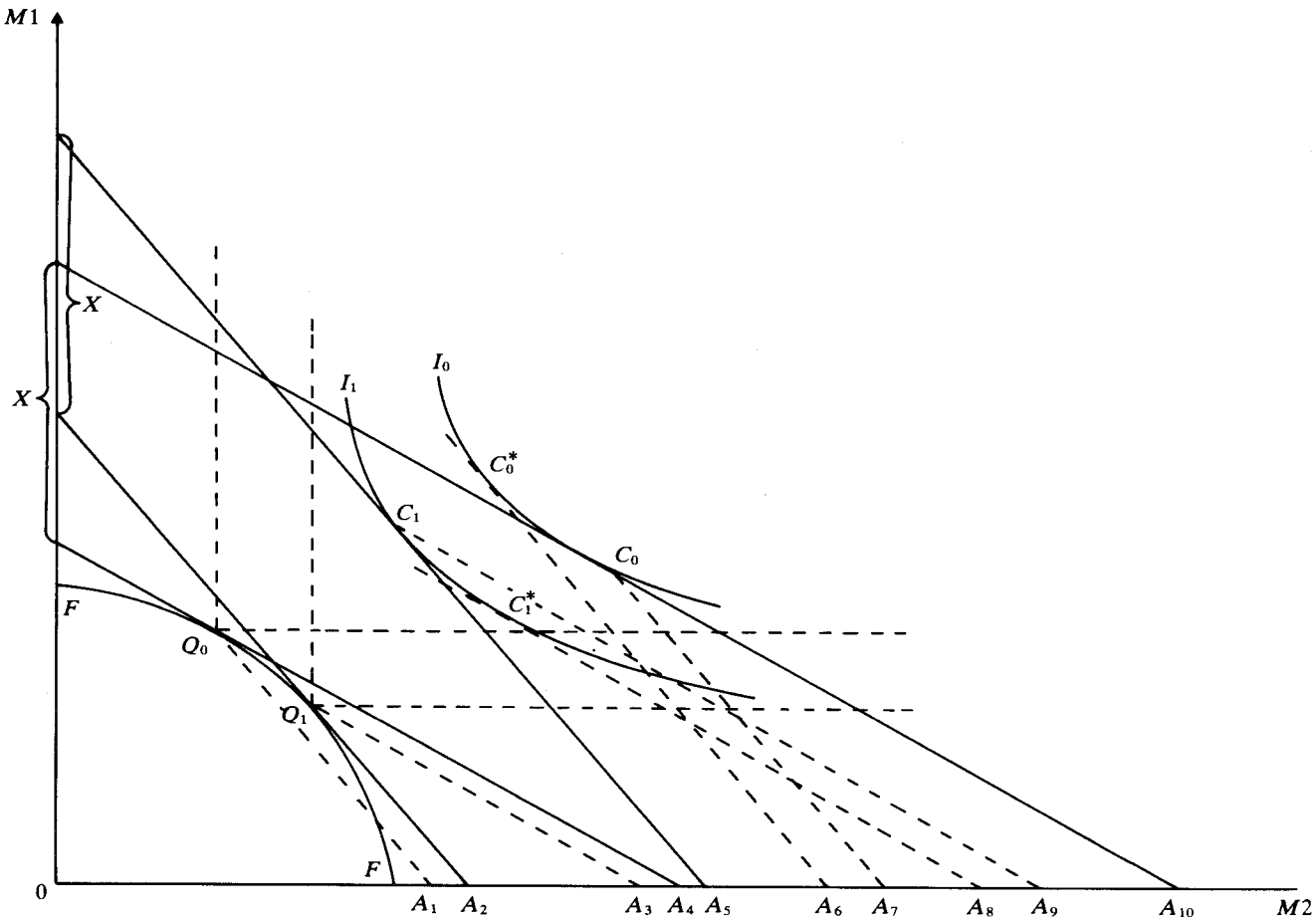

Figure 1

and likewise for the measured Paasche and the "true" Laspeyres and Paasche import volume indices. Notice that the "true" import indices refer to two different preference levels, and we cannot know *a priori* whether Laspeyres exceeds Paasche or vice versa. We know only that the biases are in relation to two different "true" indices. This problem disappears if we assume homotheticity in the preference function (Samuelson and Swamy, 1974), but generally such an assumption is not warranted.[3]

The biases of the import price indices follow directly from the identity, volume index × price index ≡ value index of imports.

The biases for the export indices follow by analogy. To set up a geometric example corresponding to that in Figure 1, we would have to assume that consumers use up a given quantity of an imported good with the available resources producing two export goods, both consumed domestically. In the figure, the consumption line would now lie below the production line and that feature reverses all conclusions from the import case.[4]

So far we have only been discussing price changes at given production frontier and preference map. Changes in foreign trade may, of course, also stem from shifts in the production frontier or the preference map (quite apart from utilization of resources and income distribution). In the small-country case such changes do not in themselves give rise to index problems, however. By assumption prices are unaffected by any kind of domestic change; Paasche and Laspeyres as well as the "true" price indices are unchanged (Fisher and Shell, 1972, pp. 14, corollary 3.1., and 80, corollary 8.1.). It has been argued (*ibid.*, pp. 58-9), erroneously in my opinion, that when shifts in frontier and preferences and prices occur simultaneously—more often than not they do just that, of course—the "interesting" price indices are Paasche for consumption and Laspeyres for production and, hence, Laspeyres for consumption and Paasche for production in so far as quantity is concerned. Be that as it may, working with trade indices we apply by implication the same index form, Laspeyres or Paasche, to both consumption and production price and quantity, respectively. It can be shown (*ibid.*, pp. 18-9 and 70 f.f.) that in case of quantity augmenting taste-changes and Hicks-neutral technical progress with some other changes, Paasche price (hence,

[3]If constant returns prevail in production and if productive inputs predominate in both exports and imports, then homotheticity may be a reasonable simplification.

[4]One country's import is the Rest of the World's export. Hence, one country's Laspeyres import price index is the Rest of the World's Laspeyres export price index (ignoring c.i.f.–f.o.b. differences). The very same measured price index tends thus to exaggerate in relation to the importing country's "true", base-level import price index and to underrate in relation to the Rest of the World's "true", base-level export price index. But this requires the former to be lower than (or equal to) the latter "true" price index.

Considering in the same way a country's Paasche import price index, it follows that a country's "true", current-level import price index exceeds (or is equal to) the Rest of the World's "true", current-level export price index.

Both of these equilibrium conditions—the theory assumes that all consumers and producers on both sides optimize successfully—may be fulfilled simultaneously. If, moreover, we have homotheticity everywhere, the importing country's two "true" import price indices coincide; and so do the Rest of the World's two "true" export price indices. But then all four "true" price indices must coincide, with Laspeyres equal to Paasche and to the "true" indices. Our assumptions, including that of homotheticity, are consistent therefore only with a steady state in which either all prices or all quantities change in the same proportion (Samuelson and Swamy, 1974, p. 589, col. 2).

Laspeyres quantity) indices are in need of no weight adjustment while that is the case for Laspeyres price (Paasche quantity) and the related "true" indices. This is an argument for concentrating upon Paasche price and Laspeyres quantity index for both imports and exports; but that, as we know, implies an upward bias in the terms of trade.

Whereas foreign prices are unaffected by domestic changes in frontier and preferences, the opposite is not generally true. To enter a new market, foreign sellers may let lower prices be accompanied by increased advertising although in the long term we would expect increased advertising and higher prices to go together. On the production side we should expect a price increase for exports or imports to be accompanied by a shift of existing resources as well as accumulation to export and import-competing industries and, hence, a shift outwards in the export and import-competing production frontiers. But such interdependencies do not change the arguments of the preceding paragraph.

Finally, a word about the statistical correlation which may be added as an auxilliary argument to the economic-theoretical arguments.[5] In case of spontaneous domestic changes no bias arises for this reason; prices are unaffected and there is zero correlation between quantity and price. When foreign price changes, and preference and frontier changes occur simultaneously but independently, we should also expect zero correlation. The weight adjustments needed in certain indices (see above) do not involve any particular bias. Things are more complicated when foreign price changes induce changes in preferences and frontiers. On the consumption side we argued that in the short term advertising might be accompanied by lower prices, in the long term by higher prices. Whatever effect advertising has on preferences, the correlation at any given moment of time for the totality of imports is difficult to predict and we may perhaps be justified in expecting zero correlation. The income effect through terms of trade gains or losses, on the other hand, operates to create negative correlation in case of both exports and imports. On the production side we should, of course, expect positive correlation as a result of reallocation of resources and accumulation. Generally we can therefore assume that, apart from the possibility of strong income effects on consumption of export goods from changes in export prices, the net effects from foreign prices to preferences and frontiers strengthen our arguments via the correlation between prices and quantities. On the import side the tendency is definitely to create negative correlation and, hence, for Laspeyres to exceed Paasche; on the export side possibly to create positive correlation with the opposite effect.

2. The Large-Country Situation

When the country is no longer small in the above sense, it will have to import along rising foreign supply curves or export along downward sloping

[5]Homotheticity is a sufficient condition for Laspeyres to exceed (fall short of) Paasche consumer (output) price index, granted of course the usual convexity conditions. Negative (positive) correlation between price and quantity relatives is a both necessary and sufficient condition for Laspeyres to exceed (fall short of) Paasche price index (Allen, 1975, pp. 62-3). Hence, homotheticity is a sufficient but not a necessary condition for negative (positive) correlation between consumer (output) price and quantity relatives. With homotheticity there is no additional problem of correlation; but without homotheticity information about correlation adds to our understanding of Laspeyres and Paasche.

foreign demand curves. We shall only discuss the case of exports along a sloping demand curve. A country needs to be very big not to be a small-country on the import side; even a small country may encounter a sloping demand curve in its exports. But our considerations carry over to the import case.

The economic-theoretical considerations from the small-country case remain unaffected. While it is true that there is now a more complex interdependence between prices and quantities, the consequence in regard to index biases of a certain shift in foreign prices, on the one hand, and preferences and productive possibilities, on the other, are the same no matter how these shifts are brought about. But the correlation between prices and quantities is affected by the new interdependencies.

A shift to the right in the foreign demand curve for exports has in principle the same consequences in regard to actual correlation as a rise in the export price in the small-country case. Hence, the country tends to experience positive correlation between price and export quantity (with a theoretical possibility of negative correlation if income effects are strong). But when the disturbances are domestic—*viz.*, shifts in productivity or preferences—export quantity and price will tend to be negatively correlated; Laspeyres exceeds Paasche. It follows that when foreign demand, domestic productivity, and domestic preference change simultaneously, as they tend to do, we cannot know *a priori* whether Laspeyres exceeds or falls short of Paasche.

When we give up the static equilibrium assumptions upon which the arguments so far have been based and move to dynamics, the basic proposition of Section 1 no longer holds. The possibilities of specific correlation between quantity and price changes depend now upon the lag structures in the models generating these changes. The cobweb model offers an interesting and probably even a realistic example. The export of a less developed country, dominated by an agricultural commodity that reacts in line with the simple, one-lag cobweb model, will tend to experience *negative* (weighted) correlation between simultaneously occurring quantity and price changes and *positive* (weighted) correlation between quantity changes and one-period lagged price changes. The negative, simultaneous correlation will cause Laspeyres to exceed Paasche in exports (contrary to the static proposition); the positive lagged correlation will further strengthen this bias when chain-indices of the Laspeyres or Paasche type are used.[6] With more complex lag structures, these effects could conceivably, however, tend to cancel out rather than reinforce each other.[7]

3. Conclusions

For LDC's and small DC's we should not only expect Laspeyres to exaggerate (Paasche to underrate) but also find that measured Laspeyres exceeds measured Paasche on the import side. On the export side Paasche tends to exaggerate (Laspeyres to underrate), but we are left in uncertainty about the

[6]Allen (1955, pp. 163–67).

[7]The cobweb case is relevant only in relation to annual indices. For monthly indices, Zarnowitz (1961) has pointed to agricultural seasonalities as a possibility for negative simultaneous and positive lagged correlation.

relations between measured Paasche and Laspeyres unless it is known whether foreign or domestic disturbances and income effects dominate. In any case, however, there is a presumption that Laspeyres tends to underrate, Paasche to exaggerate terms of trade changes. For a big country like the United States we are left in uncertainty about the biases on both the export and import sides unless it is possible to evaluate the importance of foreign and domestic disturbances.

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