ON THE EFFECTS OF DIFFERENT PATTERNS OF PUBLIC CONSUMPTION EXPENDITURES

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The multiplier effects resulting from an isolated increase in the level of public consumption within different public branches are investigated and the policy implications are discussed. The article begins with a theorethical analysis which shows why and in which ways these multipliers can be expected to differ between public branches. Thereafter, an empirical investigation is given, based on simulations with an econometric model of the Swedish economy. In this model the public activities are divided into 13 different public branches. The effects of an increase in public consumption on employment, imports and private consumption are found to differ considerably depending on which branch of the public sector is expanded. Some implications for short run stabilization policy are discussed. The article ends with a special analysis of the implications for a medium term planning problem: the trade off between private and public consumption growth. This analysis throws new light on the old topic "private or public consumption". In an economy with highly differentiated production in the public sector the trade-off is shown not to be unique. The sacrifice of private consumption growth corresponding to a given growth of public consumption expenditures will vary considerably according to the distribution of the public consumption growth within the different branches of the public sector.

1. INTRODUCTION

An isolated increase in the level of public consumption gives rise to a multiplier effect in the economy. The public sector will demand more input goods from the industrial sectors. Employment will increase both in the private and public sectors. Accordingly, there will be an increase in the demand for consumption goods. These first round effects will then work through the economy via the pattern of interindustrial deliveries and via the Keynesian consumption multiplier.

There is no reason to believe that the resulting effects on the economy are independent of the branch of government in which the increased consumption takes place. Instead, we will get different multiplier effects for different public branches depending on the mix of inputs employed in the branches. Even though these differences might be of a considerable magnitude and therefore important from a policy point of view, they are usually not considered in even large scale econometric models.

The purpose of this paper is to work out these multiplier effects on employment, imports and private consumption for the Swedish economy and to discuss the implications of the results. In particular we will analyse how the trade-off between private and public consumption varies according to different distribution patterns of public consumption growth within the various branches

*The authors express their gratitude to Sten Bergman and Mervyn King for helpful comments and suggestions.

of the public sector. Our basic tool of analysis is a medium-term model of the Swedish economy developed by the authors (the IUI-model).[1] An important feature of this model is its detailed specification of the public sector. Public consumption takes place in 13 different branches. Concerning multiplier effects the model captures interindustrial multipliers as well as Keynesian income propagation through private consumption.

Among former studies dealing with effects of public expenditure on a disaggregated level that should be mentioned are Morishima (1972) and Forsell (1975). Both of these authors' works are similar in spirit to what is attempted here. In their studies, however, different expenditure patterns vis-à-vis the private sector are not linked to different kinds of government activities.

2. A Simplified Model

We start with a theoretical analysis, carried out by the help of a simplified version of the IUI-model. In this simplified version we omit, among other things, the lag structure of the original model and suppress all classification converters.

The model has 23 producing sectors. For each one of these we have the basic accounting identity that total supply equals total demand.

$$M_i + X_i = A_i X + PC_i + LF_i + \overline{PI_i} + \overline{OI_i} + \overline{\Delta S_i} + \overline{EX_i} \qquad i = 1, \ldots, 23 \tag{1}$$

where

M_i = imports to sector i
X_i = gross production in sector i
A_i = row-vector of input coefficients
PC_i = private consumption of sector i goods
LF_i = public expenditures on sector i goods
PI_i = gross private capital formation of sector i goods
OI_i = gross public capital formation of sector i goods
ΔS_i = change in inventories of sector i goods
EX_i = exports from sector i.

Exogeneous variables are denoted by a bar.

The relation between gross production and value added (VA_i) in sector i is given by

$$VA_i = X_i\left(1 - \alpha_i - \sum_{j=1}^{23} a_{ji}\right), \qquad i = 1, \ldots, 23 \tag{2}$$

where α_i denotes the sales tax ratio on sector i goods and a_{ji} the input–output-coefficients.

The original model contains import functions for the 23 production sectors. The specification of these contains in many cases a lag structure. A basic element in the functions is that imports in sector i depends on total demand for sector i

[1] A full account of this model is given in Jakobsson (1977) and Dahlberg (1977).

goods.[2] Here, however, we make the simplification that imports are a constant fraction of gross production in that sector:

$$M_i = h_i \cdot X_i, \qquad i = 1, \ldots, 23 \tag{3}$$

Labour productivity is assumed to be constant. Therefore we get employment (L_i) in the production sectors as a constant fraction of value added in each sector.

$$L_i = VA_i \cdot \frac{1}{\lambda_i}, \qquad i = 1, \ldots, 23 \tag{4}$$

where λ_i = labour productivity in sector i.
Total wage bill in the production sectors is given by

$$BILL = \sum_{i=1}^{23} w_{pi} L_i, \tag{5}$$

where w_{pi} is average wage rate in sector i.

The different activities in the public sector are determined by the level of public consumption OC_l ($l = 1, \ldots, 13$) in 13 different branches of central and local government. Government expenditure in the different sectors is determined by the following input–output relationship:

$$LF_i = \sum_{j=1}^{13} \gamma_{ij} OC_j, \tag{6}$$

where γ_{ij} is an input coefficient for public consumption.

Employment in the public sector (OL) is given by

$$OL = \sum d_j \cdot OC_j, \tag{7}$$

where d_j denotes labour requirements for a unit of public consumption in branch j. The public wage-bill is given by

$$OBILL = w_{0j} \cdot d_j \, OC_j, \tag{8}$$

where w_{0j} denotes average wage level in branch j.

While the original model contains a detailed specification of income formation in the household sector, for our purposes it is sufficient to consider only two sources of income, namely, wage income and transfers from the public sector. We then have disposable household income as

$$DISP = BILL + OBILL - T + S, \tag{9}$$

where T = tax payments of the household sector (including wage taxes and social security contributions that are assumed to be born by the wage earners).

S = transfers to households.

[2]For a similar treatment of imports within the framework of a large scale econometric model, see Barker (1970).

Also in the formulation of the tax function we here simplify the very detailed specification[3] of the original model. In particular we suppose that tax payments are a linear function of the total wage-bill in the household sector:

$$T = p\,(BILL + OBILL) + T_0. \tag{10}$$

Concerning household consumption we suppose that total household expenditure (y) is a constant fraction (c) of disposable income ($DISP$). The distribution of expenditure on commodities is determined by a linear expenditure system with habit formation.[4] For the purpose of simplification we here use the following formula for describing the relation between household income and consumption of different commodities:

$$PC_i = \beta_i y + q_i; \qquad \sum \beta_i = 1. \tag{11}$$

Expression (11) completes our simplified model.

3. Reduced Form Analysis

We are now interested in the effects of different patterns of government spending on consumption, employment and imports. In order to analyse these effects we must rewrite our model in a reduced form. It is natural first to find a solution in terms of X. Thus we first derive:

$$\begin{aligned}PC_i = {} & q_i + \beta_i \cdot c(S - T_0) + \sum OC_l \cdot \overbrace{\frac{D_{il}}{w_{0l}\, d_l(1-p) \cdot \beta_i c}} + \\ & + \sum \frac{X_k}{\lambda_k} \underbrace{\left[\beta_1(1-p) w_{pk} \left(1 - \alpha_k - \sum_{k=1}^{23} a_{jk}\right) c\right]}_{E_{ik}}.\end{aligned} \tag{12}$$

By substituting (3), (6) and (12) in (1), we get

$$\begin{aligned}X_i = {} & \left[\sum_j \left(a_{ij} + \frac{E_{ij}}{\lambda}\right) X_j + \sum_l (D_{il} + \gamma_{il})\, OC_l + \beta_i c(S - T_0) + \right. \\ & \left. + q_i + PI_i + OI_i + \Delta S_i + EX_i\right] / \,[1 + h_i].\end{aligned} \tag{13}$$

Rewriting (13) in matrix form, we get

$$X = BX + \Gamma OC - \beta c T_0 + Q, \tag{13)'$$

where

$X = (X_1, \ldots, X_{23})$
$B = 23 \times 23$ matrix with typical element $b_{ij} = (a_{ij} + Eij/\lambda_i)/(1 + h_i)$
$\Gamma = 23 \times 13$ matrix with typical element $t_{il} = (\Delta_{il} + \lambda_{il})/(1 + h_i)$
β = row vector with typical element $\beta_i/(1 + h_i)$
Q = column vector with typical element $(\beta_i cS + q_i + PI_i + OI_i +$
$+ \Delta S_i + EX_i)/(1 + h_i)$.

[3] In the original model household taxation is covered by an extended version of the tax model presented in Jakobsson–Normann (1973).

[4] See Parks (1969). For estimation of this model on Swedish data, see Dahlman–Klevmarken (1971).

Provided the matrix $(I-B)$ has full rank the system (13)′ can be solved for X in the following way:

$$X=(1-B)^{-1}(\Gamma OC-\beta cT_0+Q). \tag{14}$$

We are interested in how the solution X is affected by changes in the vector OC. Obviously the properties of the B matrix are essential in this connection.

By recalling the definition of the typical element in the B matrix it is easy to show that B is a positive matrix (i.e. all the elements of B are positive in value) with the characteristic that all column-sums are less than one. It is then well-known[5] that this implies that $I+B+B^2+\ldots+B^n+\ldots=(I-B)^{-1}$. Since B is positive then $I+B+\ldots B^n+\ldots>0$ and hence $(I-B)^{-1}>0$.

It is also clear that Γ is a positive matrix. Consequently,

$$\frac{\delta X}{\delta OC}=(I-B)^{-1}\cdot\Gamma>0 \tag{15}$$

Therefore an increase in public consumption in any branch will always, ceteris paribus, give rise to an increase in production in the private sectors.

The structure of Γ reflects the fact that the effects on the private sector of an increase in public consumption takes place

(i) via increased demand for consumption goods from publicly employed people (D_{il} element in formula (12)).

(ii) via increased direct public expenditure in the private sectors (γ_{il} element in formula (12)).

Let us now turn to our main task, namely the effects of changes in OC on total employment, total private consumption and imports. Concerning total employment (TL) we get by (2), (4) and (6)

$$TL=\sum_i\frac{X_i}{\lambda_i}(1-\alpha_i-\sum a_{ji})+\sum_j d_j\,OC_j, \tag{16}$$

whereby

$$\frac{\delta TL}{\delta OC_j}=\sum\frac{\delta X_i}{\delta OC_j}\frac{1}{\lambda_i}\cdot(1-\alpha_i-\sum a_{ji})+d_j \tag{17}$$

where $\delta X_i/\delta OC_j$ is given by the matrix $(I-B)^{-1}\cdot\Gamma$.

So a change in OC_j results in a direct effect on public employment (d_j) and an indirect multiplier effect on private employment. The latter effect appears as the sum of partial effects on each specific branch. Obviously we could expect that

$$\frac{\delta TL}{\delta OC_j}\neq\frac{\delta TL}{\delta OC_k};\qquad \gamma\neq k$$

The effect on employment from public spending will vary according to where the spending takes place.

[5] See e.g. Dorfman, Samuelson and Solow (1958), pp. 254-257.

Turning to private consumption, we have by (12)

$$(18)\qquad PC = \sum PC_i = \sum_i q_i + c(S - T_0) + \sum_l OC_l \cdot w_{ol}\, d_l \cdot (1-p)c +$$

$$+\sum_k X_k(1-p) \cdot w_{pk}(1-\alpha_k - \sum a_{jk})c \cdot \frac{1}{\lambda_k},$$

whereby

$$(19)\qquad \frac{\delta PC}{\delta OC_j} = \sum_k \frac{\delta X_k}{\delta OC_j} \frac{(1-p)c}{\lambda_k} \cdot w_{pk}(1-\alpha_k - \sum a_{jk}) + w_{oj}\, d_j \cdot (1-p)c.$$

The above expression is very similar to that which holds for the employment derivative.

For imports we simply have

$$(20)\qquad M = \sum M_i = \sum k_i X_i$$

and consequently

$$(21)\qquad \frac{\delta M}{\delta OC_1} = \sum k_i \frac{\delta X_i}{\delta OC_1}.$$

The next section will be devoted to a presentation of the empirical estimates of $(\delta M/\delta OC_i)\,(\delta TL/\delta OC_i)$ and $(\delta PC/\delta OC_i)$ for the 13 different branches of government that appear in our econometric model.

4. Multiplier Simulations

In the *IUI* model the public sector is first split into two subheadings: 1) those services produced under the direct control of central government and 2) those produced under the control of local governments. These in turn are divided respectively into the seven and six branches listed in Table 1.

TABLE 1

PUBLIC BRANCHES IN THE IUI ECONOMETRIC MODEL

Authority	Branch Number	Kind of Activity
Central government	1	Defence
	2	Public order and safety
	3	Universities and other higher education
	4	Research hospitals
	5	Social security
	6	State roads
	7	Other services produced by the central government
Local governments	8	Fire protection
	9	Lower education
	10	Health
	11	Welfare services
	12	Local roads and streets
	13	Other services produced by local governments

For each of the 13 branches the kind of multipliers theoretically derived in the preceding section have been estimated by simulations in the original non-simplified model. The results can be found in Table 2. A general observation from the table is that each kind of multiplier varies considerably in size between different public branches. Observing the effects on employment within the public sector itself ($\partial OL/\partial OC_i$) we find that the highest multiplier (branch 4) is almost four times greater than the lowest one (branch 12). Going one line further down observing the effects on private employment ($\partial L/\partial OC_i$), the highest multiplier (branch 12) is about six times the lowest one (branch 4). Concerning the aggregated employment effects ($\partial TL/\partial OC_i$) the highest multiplier (branch 4) is about four times greater than the lowest one (branch 3). Going further down in the table we find that the effects on imports and private consumption created by a unit increase in public consumption expenditure varies even more than the employment effects. For example, the rise in private consumption ($\partial PC/\partial OC_i$) connected with an increase of public consumption expenditures in branch 12 is about seven times greater than that induced by an equivalent expansion in branch 9.

5. Stabilization Policy: Some Implications

Some of the policy implications given by Table 2 are quite obvious. For example, assume that we want to reduce unemployment by raising public expenditures. We then know that the additional employment created will vary in magnitude and in placement within the private and public sectors, depending on where the consumption expenditures were increased. A rise of the public consumption within branch 4 will yield the highest increase in aggregate employment, with most of the new employment in the public sector itself. On the other hand, expanding the consumption in branch 12 will give us a considerable employment effect within the private sector. Branch 4 (research hospitals) is labour intensive and requires very small inputs from the private sector. In contrast, branch 12 (local roads and streets) has a very small production of its own: most of its services (road work) are brought from private firms.

A classical problem in short term stabilization policy is how to increase domestic demand without deteriorating the balance of payments. A simple policy-guide to that problem can be obtained by constructing the ratio ($\partial L/\partial OC_i$) / ($\partial M/\partial OC_i$) from Table 2. This ratio expresses, for a unit increase of public consumption expenditures, how much employment is connected with a unit increase in imports. From Table 2 it is clear that the highest ratio is found in branch 4 (research hospitals). Also branch 2 (public order and safety) and branch 9 (lower education) have very high ratios. The lowest ratio is found in branch 1 (defense). Consequently, public consumption expenditures within this branch should not be expanded for employment purposes only.

6. Medium Term Planning: the Trade-Off Between Private and Public Consumption

Leaving short term policy and facing the problems of medium term planning, the implications of Table 2 are no longer obvious. We shall here use the

TABLE 2

THE EFFECTS ON PUBLIC EMPLOYMENT (*OL*), PRIVATE EMPLOYMENT (L), TOTAL EMPLOYMENT (TL = OL + L), IMPORTS (M) AND PRIVATE CONSUMPTION (PC) IN DIFFERENT PUBLIC BRANCHES CREATED BY RISE OF 1 MILL. SW CR (1968 PRICES) IN YEARLY PUBLIC CONSUMPTION EXPENDITURES (OC)

	Public Branch in which the Yearly Consumption Expenditures are Raised												
	Central Government							Local Governments					
Kind of Multiplier	1	2	3	4	5	6	7	8	9	10	11	12	13
$\partial OL/\partial OC_i$ (thousands of working hours)	27.8	38.1	23.3	102.1	27.3	24.1	32.8	24.7	28.8	50.2	65.3	16.8	22.7
$\partial PL/\partial OC_i$ (thousands of working hours)	12.9	6.7	7.8	16.8	10.9	28.0	9.4	6.9	6.0	10.4	10.9	35.6	17.8
$\partial L/\partial OC_i$ (thousands of working hours)	40.8	44.8	31.1	118.9	38.2	52.1	42.1	31.6	34.8	60.7	76.2	52.4	40.5
$\partial M/\partial OC_i$ (mill. Sw Cr, 1968 prices)	0.26	0.09	0.08	0.22	0.13	0.33	0.12	0.07	0.07	0.16	0.15	0.32	0.20
$\partial PC/\partial OC_i$ (mill. Sw Cr, 1968 prices)	0.21	0.11	0.13	0.34	0.19	0.44	0.17	0.09	0.08	0.19	0.32	0.54	0.26

information given in the table to investigate a typical medium term problem, the trade-off between public and private consumption under given resource constraints and with given economic targets.

With given production possibilities and with full capacity utilization every increase in public consumption will force us to sacrifice a certain amount of private consumption. The latter amount will vary in size depending on the pattern of public consumption with respect to the branches. Our primary question concerns the size of this variation and its relation to a given growth in public consumption *when* our freedom in choosing a pattern is limited by given production possibilities and resource constraints. We formulate the question more precisely as follows. Consider a given increase in the amount of public consumption. Depending on the branch pattern of the increased spending there will be a certain amount of private consumption forgone. How large will the range of variation in this pattern be with respect to different spending patterns?

To answer the question posed we have taken as a bench-mark one of the main alternatives for the development of the Swedish economy between 1974 and 1980, which was worked out with the IUI econometric model within the framework of a medium term study of the Swedish economy.[6] This means that we have adopted the values of the exogenous variables that go with this alternative and that we have restricted ourselves to the same resource constraints: a certain level of employment and a certain level on imports (the level that gives balance in foreign payments with regard to the exogenously determined exports).

Let us denote the change in public expenditure in this alternative by the exogenously determined vector $\overline{\Delta OC} = (\overline{\Delta OC_1}, \ldots, \overline{\Delta OC_{13}})$. According to our multiplier analysis this change gives rise to certain increases in imports and labour requirements and to a certain increase in the value of public consumption. These changes are given by:

$$\overline{\Delta TL} = \sum_{i=1}^{13} \frac{\partial TL}{\partial OC_i} \overline{\Delta OC_i} \tag{22}$$

$$\overline{\Delta M} = \sum_{i=1}^{13} \frac{\partial M}{\partial OC_i} \overline{\Delta OC_i} \tag{23}$$

$$\overline{\Delta OC} = \sum_{i=1}^{13} \overline{\Delta OC_i} \tag{24}$$

Our task now, is to investigate how private consumption can be varied by choosing different values of the public consumption in different branches, while holding total public consumption constant. The resource requirements of the new policy should equal those implied by $\overline{\Delta TL}$ and $\overline{\Delta M}$.

Generally, it is clear that a choice of public branches with small multipliers makes it possible to transfer resources towards production and imports of private consumption goods. This change in potential supply has to be met by a corresponding change in household demand of private consumption goods.

[6]IUI (1976).

We therefore need an instrument for demand management. The instrument we shall use is the tax parameter p (see eq. (10)), which in the original solution had the specific value $\hat{p}$. Shifting the value on $p(\Delta p = p - \hat{p})$ yields a uniform percentage shift of the tax-schedule for all income classes. Within the framework of our multiplier analysis the partial effects on employment, private consumption and imports of a change in p have been investigated. In what follows these partial effects will be denoted by $\partial TL/\partial p$, $\partial PC/\partial p$ and $\partial M/\partial p$.

Now the stage is finally set for a full formal treatment of the problem: Choose the vector $(\Delta OC_1, \ldots, \Delta OC_{13}, \Delta p)$ that maximizes (minimizes)

$$\Delta PC = \sum_{i=1}^{13} \frac{\partial PC}{\partial OC_i} \Delta OC_i + \frac{\partial PC}{\partial p} \Delta p \tag{25}$$

subject to the constraints:

$$\sum_{i=1}^{13} \frac{\partial TL}{\partial OC_i} \Delta OC_i + \frac{\partial TL}{\partial p} \Delta p = \overline{\Delta TL}$$

$$\sum_{i=1}^{13} \frac{\partial M}{\partial OC_i} \Delta OC_i + \frac{\partial M}{\partial p} \Delta p = \overline{\Delta M}$$

$$\sum_{i=1}^{13} \Delta OC = \overline{\Delta OC}$$

$$\Delta OC_i \geq 0 \; \forall i.$$

Giving the results in terms of shares of total changes it is found that ΔPC_{max} is reached by the help of the following set of policy parameters $(\Delta OC_1, \ldots, \Delta OC_{13}, \Delta p)/\overline{\Delta OC} = (0, 0, 0.22, 0, 0, 0, 0, 0.17, 0, 0, 0.17, 0, 0)$. In the same way ΔPC_{min} will be reached by the following set of policy parameters: $(0, 0, 0, 0.16, 0, 0.06, 0, 0, 0, 0, 0.78, 0, 0, 0.002)$.

Our basic question was how much ΔPC could differ for a given value of ΔOC, or in other words, how large the difference is between ΔPC_{max} an ΔPC_{min} for each given $\overline{\Delta OC}$ in our LP problem. As an answer to our question the values of the objective function corresponding to the policies just presented give us the following measure:

$$\frac{\Delta PC_{max} - \Delta \dot{P}C_{min}}{\overline{\Delta OC}} = 0.28 \tag{26}$$

This is an interesting result. It tells us that the "price" in terms of sacrificed private consumption, which we have to pay for a unit increase in public consumption, can vary 28 percent for various patterns of public consumption growth.

7. Conclusions

The analysis has shown that the pattern of public consumption growth has a large influence on the development of other central economic variables. The special analysis of the trade-off between public and private consumption throws a new light on the old topic "private or public consumption". In an economy with a highly differentiated public sector the trade-off is not uniquely determined. The sacrifice of private consumption corresponding to the growth in public consumption will vary considerably according to the distribution of the public consumption growth upon different branches within the public sector.

References

Barker, T. S., 1970, "The Determinants of Britain's Visible Imports 1949–1966", in R. Stone (ed), *A Programme for Growth.* Vol. 10. London.

Dahlberg, L., 1977, Offentlig sektor, *Appendix No 3 of IUI:s långtidsbedömning 1976.* Stockholm.

Dahlman, C. J. and Klevmarken, A., 1971, *Den privata konsumtionen 1931–1975.* Stockholm.

Dorfman, R., Samuelson, P. A., and Solow, R. M., 1958, *Linear Programming and Economic Analysis.* New York.

Forsell, O., 1975, "Effects of Public Expenditures on Production, Incomes and Employment in Finland". *Review of Income and Wealth,* No 1 1977.

IUI, 1976, IUI:s långtidsbedömning 1976. Stockholm.

Jakobsson, U., 1977, "En beskrivning av IUI-modellen", *Appendix No 1 of IUI:s långtidsbedömning 1976.* Stockholm.

Jakobsson, U. and Normann, G., 1973, "A Model of the Swedish System for Personal Income Taxation". *European Economic Review.* Vol 3, No 4 1972.

Morishima, M. and Nosse, T., 1972, "Input–Output Analysis of the Effectiveness of Fiscal Policies for the United Kingdom, 1954", in M. Morishima *et al.* (eds), *The Working of Econometric Models.* Cambridge.

Parks, R., 1969, "Systems of Demand Equations. An Empirical Comparison of Alternative Functional Forms". *Econometrica,* Vol. 37, Oct. 1969.