THE MEASUREMENT OF REAL INCOME

BY D. USHER

Queen's University, Kingston, Canada
Nuffield College, Oxford

This paper is about the theory of the measurement of real income. By "theory of measurement" I mean the characterization of statistical terms as variables in a model, just as real consumption is characterized as an indicator of utility and the consumer price index is characterized as the cost of attaining a given level of utility in the economic theory of index numbers developed by Konus, Frisch and others half a century ago. I identify five logically distinct and internally-consistent concepts of real income: maximum sustainable consumption, consumption plus the output of new capital goods, consumption plus the increase in the capital stock where capital can be measured in two quite separate ways, and the sum of actual consumption and consumption forgone in the investment process. The last of these concepts is the most appropriate as a guide to producing long time series of real income for measuring a country's rate of economic growth.

Many of the difficulties statisticians encounter in the attempt to construct time series of real income and related aggregates such as real capital, the rate of economic growth, and the rate of technical change can be traced back to a conceptual problem in the way real income is defined. The problem has to do with the inevitable discrepancy between the concepts we employ—formally as terms in models of the economy or informally in economic argument—and the numbers we attach to those concepts. Real income is defined in the first instance as a term in the familiar one-sector model described below. Statistics of real income must be constructed with data from a world that is more complex than any model we might devise. How then do we decide whether a particular number obtained by a given computational rule is an appropriate measure of real income or not? We would like to redefine real income in terms of the ultimate constituents of the economy, but it is impossible, even in principle, to do so. What we can do is to redefine income in terms of a second and larger model incorporating some of the properties of the economy that the one-sector model assumes away. There is no pretence that the second model is the real or true model, but the attempt to redefine real income does bring out aspects of the concept of income that would otherwise be overlooked.

What comes to light in the course of this paper is that there is no single, ideal, all-purpose definition of real income in the larger model. When transferred from the one-sector model to a more detailed model of the economy, real income spreads out and subdivides into a family of concepts. Properties that originally hung together now go their separate ways, each attaching itself to a distinct concept of income. The implication for the development of statistics is that each usage of real income—usages such as short term forecasting, budgeting, or the measurement of economic growth—draws upon a distinct set of attributes and may therefore require a time-series specially designed to emphasize those attributes at the expense of others which are less important for the purpose at hand.

Real income is represented by the term Y in a model of the economy consisting of (i) a production function

(1) $$Y = f(K, L, t)$$

where Y is measured in pounds of potatoes per year, K is measured as a number of machines, L is the number of men employed, and t, representing time, is included to allow for the possibility of technical change; (ii) an identity

(2) $$Y = C + S$$

where C and S are consumption and saving, both measured as pounds of potatoes per year; (iii) a relation converting investment to new capital goods

(3) $$S = pM$$

where M is the number of machines that can be created by the sacrifice of potatoes that might otherwise have been consumed, and p is a relative price which in this model is assumed to be constant [This equation could have been avoided by setting $p = 1$; we do not do so because we will want the equation later on.]; (iv) a relation between capital formation, depreciation and the growth of the capital stock

(4) $$M = \Delta K + d(K)$$

where depreciation, $d(K)$, takes the form of evaporation each year of a part of the capital stock, ΔK is the increase in the capital stock from one year to the next, and all machines, other than those which have evaporated, are assumed equally productive. We might also suppose that income each year is allocated between consumption and saving to maximize intertemporal utility as a function of consumption in every future year, but that is not necessary for the definition of real income. The statistician's task is to construct a time series of Y from time series of prices and quantities.

Suppose for convenience that there is no technical charge (so the term t drops out of the production function), no depreciation, and no change from year to year in the size of the labour force. On these assumptions, the term Y can be equally well thought of as:

(i) Maximum sustainable consumption: real income is the amount that can be consumed this year consistent with the community being as well off at the end of the year as it was at the beginning.

(ii) A measure of total output: real income is the value at constant prices of the output of consumption goods and capital goods together.

(iii) The value at constant prices of current consumption plus the increase in capacity of the economy next year embodied in currently-produced capital goods.

(iv) The value at constant prices of current consumption plus the increase in capacity of the economy in the long run embodied in currently-produced capital goods.

(v) The value at constant prices of current consumption plus current saving, the latter measured as an amount of consumption goods forgone.

Each of these interpretations of real income may be thought of as a variant of equation (2) above $- Y = C + S$. They all have the term C in common and they differ, one from another, in what they contain in place of S. The first replaces S with a measure of net change in wealth as defined below. The second replaces S with pM from equation (3). The third and fourth replace S with variants of the term $p\Delta K$ from equation (4). And the fifth retains S itself. It is convenient to reserve the term 'investment' to refer generically to any or all of these five attributes of the difference between income and consumption.[1]

These five attributes attach themselves to a single concept of investment as long as we are prepared to adhere to the one-sector model or to measure income in current dollars. All this changes when we try to express real income as a property of a larger model of the economy. Then, as we shall show, the five attributes separate, attaching themselves to distinct concepts of income to be discussed one by one below.

The argument of the paper proceeds as follows. We begin with a discussion of real consumption covering familiar ground the whole way but setting up a basis for the discussion of investment and income to follow. Next we present a more general model than that in equations (1), (2), (3) and (4), in terms of which the separate families of real income can be defined. Then we define the five concepts of income one by one, considering in each case whether and to what extent it can be represented with some degree of accuracy by time series of prices and quantities. Finally at the end of the paper we consider briefly the usefulness of some of these concepts of income in the light of purposes for which time series of real income might be employed. My own interest in the matter is to find the definition of real income most suited for developing a time series of real income with which to measure a country's rate of economic growth. The measure of real income as the sum of real consumption and real saving is singled out as being especially appropriate for that purpose, but it does appear that other concepts might be more appropriate for budgeting and forecasting.

A. Real Consumption

There is a well developed theory of how to measure real consumption. It is called the economic theory of index numbers. The theory can with equal justification be said to be about the measurement of the price level, but that is of no consequence, for propositions about price indices can be translated into propositions about real consumption with only a slight change of terminology and emphasis. Following the standard practice in the theory of index numbers we employ a model of a community which acts like a representative consumer, who maximizes welfare subject to the constraint of a production possibility curve that shifts outwards in the course of economic growth.

Let the utility function of the representative consumer be

$$U(q_1, \ldots, q_n) \tag{5}$$

[1]These aspects of investment are discussed by T. K. Rymes in *On Concepts of Capital and Technical Change*, Cambridge University Press, 1971, ch. 8, "Professor Hicks and the Concepts of Capital and the Production Function".

where q_1 is the amount of the good i consumed, and let the production possibility curve of the year t be

(6) $$F^t(q_i, \ldots, q_n) = 0$$

A superscript t may be added to q_i to signify that q_i^t is the amount of the good i bought in the year t. Similarly p_i^t is the price of the good i in the year t. Quite arbitrarily we assign the Year 0 to be the base year.

Real consumption can be defined as a measure of welfare or as a measure of productive capacity, the former being a reflection of the height of the indifference curve attained and the latter being a reflection of production possibilities in the current year. As a measure of welfare, real consumption in the year t is defined as the amount of money required at prices that obtained in the year 0 to enable the representative consumer to be as well off as he is in the year t. Formally, the measure of real consumption as an indication of welfare in the year t for a base year 0 is

(7) $$C_0^{wt} = \sum_{i=1}^{n} p_i^0 q_i^{wt}$$

where w refers to welfare, 0 is the base year, t is the current year, i refers to a commodity of which there are n, the p_i^0 are base year prices and the q_i^{wt} are quantities that minimize C_0^{wt} subject to the constraint

(8) $$U(q_1^{wt}, \ldots, q_n^{wt}) \geq U(q_1^t, \ldots, q_n^t)$$

where the q_i^t are quantities actually consumed in the year t.

As a measure of productive capacity, real consumption in the year t for a base year 0 is

(9) $$C_0^{pt} = \sum_{i=1}^{n} p_i^0 q_i^{pt}$$

where the superscript p refers to productivity and the quantities q_i^{pt} are chosen to minimize C_0^{pt} subject to the constraint

(10) $$F^t(q_1^{pt}, \ldots, q_n^{pt}) = 0$$

The rate of economic growth when real consumption is an indicator of welfare is the average annual rate of appreciation of C_0^{wt} between the initial and the final year of the series. The rate is not independent of the choice of base year unless indifference curves are homothetic. Similarly, we can define a rate of economic growth when real consumption is an indicator of productivity.

Since statistics of q_i^{wt} and q_i^{pt} are not normally available and can at best be estimated in roundabout ways, it is often convenient to approximate C_0^{wt} and C_0^{pt} by a Laspeyres type measure

(11) $$C_0^t = \sum_{i=1}^{n} p_i^0 q_i^t$$

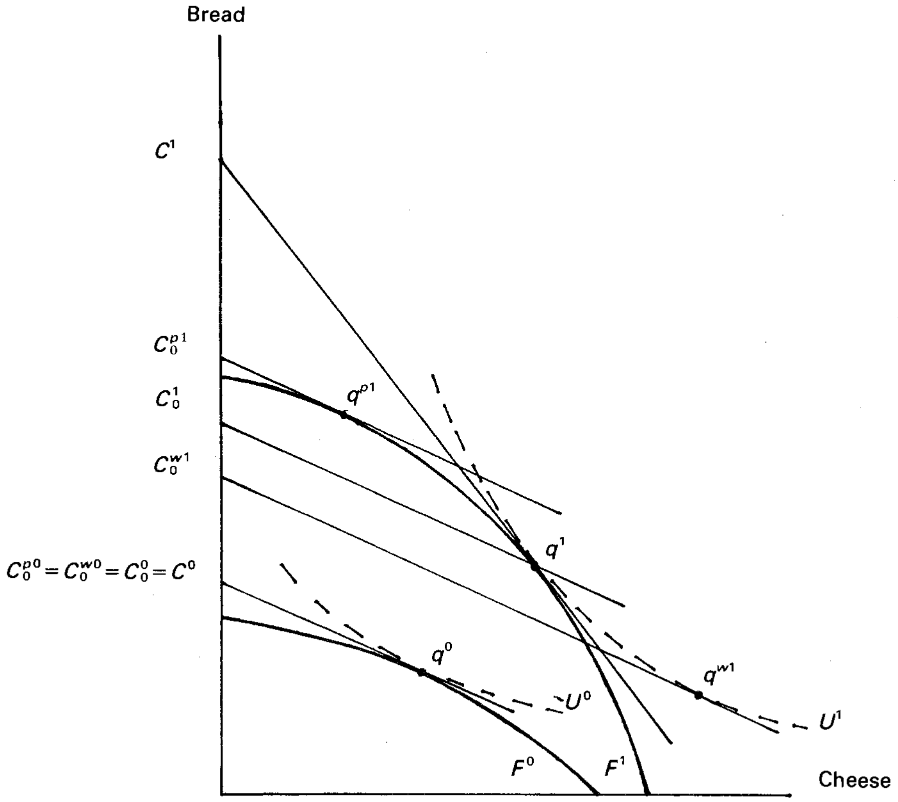


Figure 1

which can be shown to approach both C_0^{wt} and C_0^{pt} to a first order approximation.[2]

All these measures are illustrated in Figure 1 for two time periods $t = 0$ and $t = 1$ and for a two commodity world where i can be either bread or cheese. Between the year 0 and the year 1, the production possibility curve shifts outward from F^0 to F^1. There is a stable set of indifference curves of which we show the curves that happen to be tangent to F^0 and F^1. Money incomes in the years 0 and 1, with bread as the numeraire, are indicated by C^0 and C^1 respectively, and the rest is self-explanatory.

All this is well known to readers of this journal who are acquainted with the economic theory of index numbers. But it is important for what follows to emphasize certain aspects of the procedure by which real consumption is defined. As we move from a model with one consumption good to a model with n consumption goods, the simple notion of consumption represented unambiguously as C in equation (2) breaks up into two distinct families, welfare measures

[2]The theory of index numbers is very largely concerned with the question of how time series of C_0^{wt} or C_0^{pt} might in practice be constructed from time series of prices and quantities. See in particular Professor Sir John Hicks' article "The Valuation of Social Income" (*Economica*, 1940, pp. 109–24), where the concepts C_0^{wt} and C_0^{pt} are first clearly distinguished, and R. G. D. Allen's new book *Index Numbers in Theory and Practice* (Macmillan, 1975) where the theory is set out in detail.

and productivity measures, each with as many members as there are base years. In our discussion of real income to follow we are concerned to distinguish among the families of measures where again each family will have a separate member for each choice of the base year.

Though the model developed in this section is considerably more realistic than the one-sector model, it remains abstract in some important respects. In particular, it contains the implicit assumption that output consists of amounts of exactly n commodities that persist unchanged from the beginning of time to the end of time. This abstracts from what is probably the most difficult problem the national accountant has to deal with, namely, that of comparing different types of consumption goods—like Victrolas and hi-fi sets—and different types of capital goods—like clipper ships and jumbo jets—upon a common scale. The national accountant must somehow develop a rule for apportioning the infinitely-diverse and ever-changing flow of income into n supposedly uniform and invariant categories. Procedures have been devised, but no one—least of all the national accountants—supposes them to be satisfactory.

It is worth noting in passing that there is a way around this problem when real consumption is measured as an indicator of welfare. In that case, statistics of real consumption can in principle be obtained by a simple conceptual experiment performed with the aid of the representative consumer. Suppose we choose 1961 as the base year. To discover the real consumption in some other year, say 1950, all we need to do is to send a representative consumer from the year 1961 back to 1950, give him the average income in 1950, and allow him to choose the best bundle of goods available at 1950 prices. Then we bring him back to 1961 and ask him how much income he needs in 1961 to be as well off as we allowed him to be in 1950. His answer to that question is what we mean by real consumption in 1950, assessed at 1961 prices; it is the value of C_0^{wt} where 0 is 1961 and t is 1950.This is quite a useful little test because it enables us to compare real consumption over different years in a wide variety of circumstances, even if the nature of goods consumed is changing in the course of time. We can in principle compare real consumption in 1950 and 1961, even if our consumers bought ordinary radios in one year and hi-fi sets in the other. I cannot think of any similar test for measuring real consumption as productive capacity. If hi-fi sets could not have been made at all in 1950, then productive capacity in that category is simply incomparable, and considerations of taste, welfare or usefulness would have to be introduced for real consumption to be measured at all.

Before we proceed, it might be helpful as a way of explaining what this essay is about to distinguish among five types of theoretical problems associated with the construction of economic statistics. These might be called the problems of meaning, existence, aggregation, index numbers and purpose. The problem of *meaning* with which this essay is primarily concerned is how to express terms like consumption, investment and income unambiguously as elements in a model of the economy, a simple model such as that at the beginning of this paper or a complex and detailed model such as we are about to construct. The problem of *existence* is in some respects a misnomer. It is really about whether we are justified in using a single statistic to represent a concept in a wide variety of circumstances, whether for instance we are justified in employing the same time series of real consumption regardless of the choice of the base year. It is easily shown that real consumption exists in this sense if and only if indifference curves are homothetic. The term existence is not quite apposite in this context because real consumption is well-defined even if it does not exist in this special sense. I use the term nonetheless because it has been used in this way in discussing the concept of real capital. The problem of *aggregation*, with which we are not concerned at all in this essay, is how to connect statements about individual actors in the economy with comparable statements about the economy as a whole, how, for instance, to build a community demand curve for a commodity out of the demand curves of all the people in the community. The problem of *index numbers*, which we try to avoid whenever we can but which does crop up from time to time, is about how to represent a function by means of time series of prices and quantities which are, after all, the only data we are likely to possess about the economy. For instance, once we have defined real consumption by deciding whether it is a welfare measure or productivity measure we are after and by choosing a base year for the time series we are confronted with an index number problem in attempting to measure real consumption with the available data. The problem of the *purpose* of statistics is not as easily stated as the others but it is equally if not more important. Ultimately we are going to use statistical time series to represent terms in arguments, either formally as when real consumption is used in a savings function within a large econometric model or informally as when statistics of real consumption are used to show whether people are becoming better off in the course of time. The purpose of the statistic must stand as the final criterion in decisions about how the statistic is to be constructed—what the primary data are to be and how they are to be processed. But it is not self-evident what statistical procedures are warranted by the purpose at hand. The tailoring of statistics to a purpose will typically involve the national accountant in an unwelcome but probably inevitable element of subjectivity and personal judgement.

We now return to the measurement of real income. We shall consider the five meanings discussed above, and we shall show how each gives rise to a unique definition of real income in terms of a larger model of the economy to be set out immediately below.

B. An Intertemporal Model for Measuring Real Income and Real Investment

Since, by definition, consumption takes place in the current year, it proved possible to analyse the concept of real consumption by means of an atemporal model of the economy. That model will not do for the analysis of real investment because investment is a relation between events in different years. For comparing definitions of real income and real investment, it is necessary to have before us an intertemporal model of the economy. Think of the economy as a representative consumer who maximizes intertemporal utility subject to the constraints of a technology that changes, and presumably improves, over time and of an initial endowment of capital goods. The representative consumer chooses a stream of consumption commencing immediately and continuing into the indefinite future.

In the year t, he maximizes

$$U(u(q^t), u(q^{t+1}), \ldots, u(q^{t+\tau}), \ldots) \tag{12}$$

where U is intertemporal utility and u is annual utility.[3] The utility function U is maximized subject to the constraint of a sequence of annual technologies.[4]

$$f^\tau(q^\tau, M^\tau) = g^\tau(K^\tau, L^\tau) \tag{13}$$

$$\tau = t, t+1, t+2, \ldots$$

where the function f may be thought of as the production possibility curve and g may be thought of as the production function, and depreciation relations

$$K^{\tau+1} - K^\tau = M^\tau - \zeta K^\tau \tag{14}$$

where q, M, K and L are vectors;

$$q^\tau \equiv \{q_1^\tau, \ldots, q_n^\tau\} \tag{15}$$

is a vector of outputs of the n types of consumption goods consumed in the year τ;

$$M^\tau \equiv \{M_1^\tau, \ldots, M_m^\tau\} \tag{16}$$

is a vector of outputs of the m types of capital goods in the year τ;

$$K^\tau \equiv \{K_1^\tau, \ldots, K_m^\tau\} \tag{17}$$

is a vector of stocks of the m types of capital goods available during the year τ;

$$L^\tau \equiv \{L_1^\tau, \ldots, L_r^\tau\} \tag{18}$$

is a vector of the amounts of r types of labour available during the year τ (it is assumed that the labour force remains constant over time so that $L^i = L^j$ for all values of i and j); and ζ is a diagonal matrix of rates of depreciation on the m types of capital goods. The technologies f^τ and g^τ are made dependent on τ to allow for the possibility of technical change.

Though this model is more complex and more general than the one-sector model above, it remains very abstract in ways that can be of importance for the measurement of real income. First, the form of the utility function of equation (12) implies that the economy can be looked upon as though it were a single consumer. This assumption is common to the whole economic theory of index numbers and there is nothing we can do about it. Second, as mentioned in connection with the discussion of real consumption, it is assumed that commodities remain unchanged in their form and nature from the beginning to the end of time. This assumption is never strictly true but it is probably closer to the truth, in the sense that changes in the nature of commodities do less violence to our time series, for consumption goods than for capital goods and for short time series than

[3]It would be less restrictive and in some respects simpler to let U be a function of $q^t, q^{t+1}, \ldots$ directly without the intermediary of the annual utility function u. The reasons for introducing u are to connect this model with the definition of real consumption in the preceding section, and because two of the definitions of real income depend on a notion of annual utility.

[4]A less restrictive form of the annual technology would be $f(q, M, K, L) = 0$ but it is convenient for our purposes to use the form in equation (13). The loss of generality does not impede the argument we are making.

for long ones. Thirdly, the maximization process in the model is described as though people have perfect foresight. Obviously statistics that would be quite unambiguous if people really had perfect foresight, statistics constructed out of data on events that will not take place for many years to come, can be of no practical use unless the data can be estimated with a fair degree of accuracy.

We now discuss the concepts of income one by one, defining them formally and considering whether and under what assumptions they can serve as the bases for time series constructed from the data at hand.

C. Income as Maximum Sustainable Consumption

$$Y = C + \Delta W \tag{19}$$

This is the concept of income that Professor Hicks[5] described as 'the maximum value which he (the income recipient) can consume during the week and still be expected to be as well off at the end of the week as he was at the beginning.' The term ΔW in equation (19) is the increase in wealth in the course of the year. It is typically measured as the sum at base year prices of tangible investment and capital gains. This concept of income has been vigorously advocated as the appropriate basis for the assessment of income tax[6], but its merits in that context are quite separate from its advantages or disadvantages as a foundation for the national accounts. We are of course concerned with the latter context exclusively.

Real income as maximum sustainable consumption can be defined analogously to real consumption as an indicator of welfare. The value of real income for the year t when the year 0 is the chosen base year is

$$Y_0^t = \sum_{i=1}^{n} p_i^0 \hat{q}_i^t \tag{20}$$

where Y_0^t is the costs of the cheapest bundle of goods $\hat{q}_i^t$ that can be purchased at base year prices p_i^0 and that would provide at least as much utility as our economy can provide each year when the intertemporal utility function U in equation (12) is maximized subject to the technology constraints and to the additional constraint that annual utility is maintained constant forever

$$u(\hat{q}^t) \geq u(q^t) = u(q^{t+1}) = u(q^{t+2}) = \ldots \tag{21}$$

The corresponding definition of real consumption is

$$C_0^{wt} = \sum_{i=1}^{n} p_i^0 \hat{q}_i^t \tag{22}$$

where $\hat{q}_i^t$ is the cheapest bundle of goods that could be purchased at prices p_i^0 and for which

$$u(\hat{q}^t) \geq u(q^t) \tag{23}$$

[5] J. R. Hicks, *Value and Capital*, Second edition, Oxford, 1946, p. 172. There is some question in my mind as to whether equation (19) reflects what Professor Hicks intended, when he defined income as he did. A case might be made that he meant something similar to the definition at which I arrive at the end of this paper.

[6] Henrey Simons, who advocated this definition as a basis for the assessment of income tax, was careful to distinguish between the definition appropriate for that purpose and the definition appropriate for the national accounts; he explicitly denied that capital gains should be a component of income in the latter sense. See Simons, *Personal Income Taxation*, 1938, p. 47.

where q_i^t are the amounts actually consumed in the year t. The difference between Y_0^t and C_0^{wt} may be thought of as the measure of real investment.

Equation (19) above is the Laspeyres approximation to this concept of real income. The corresponding measure of real investment is the change over the year in real wealth, the difference $W_0^{t+1} - W_0^t$, where real wealth, W_0^t, is the value at base year prices of all future consumption commencing at the year t,

$$W_0^t = \sum_{\tau=0}^{\infty} \sum_{i=1}^{n} \bar{p}_i^\tau q_i^{t+\tau} \qquad (24)$$

and where the $\bar{p}_i^\tau$ are forward prices τ years ahead as assessed at the base year 0.[7]

The definition of real income as maximum sustainable consumption is subject to major defects which are worth considering in some detail because there is developing a substantial body of opinion that this definition is appropriate, or even ideal, not only for tax policy but for the national accounts as well.[8] We discuss three problems: i) Real income is defined as a function of data about events which occur after the end of the accounting period. ii) Real investment might be positive and substantial without any sacrifice of potential consumption in the current year. iii) This definition of real income is fundamentally incompatible with the meaning of real income in most of the contexts in which economists now use the term.

(i) One need only glance at equations (20) and (21) to appreciate that this concept of income is defined as a function of the whole time stream of consumption from now to the end of the world. To know current income on this definition we must know future consumption forever. This automatically bars the definition of real income as the sum of consumption and the change in wealth from serving as a basis for measurement in the national accounts because real income in the national accounts must be based on current data alone.

It is sometimes supposed that this difficulty can be circumvented and that national accountants can estimate real income in accordance with equation (19) by means of currently-available data if the range of data is extended to include stock market quotations and other evidence of the market's evaluation of future income streams accruing to capital goods. In my opinion this expectation is illusory for several reasons. First we have no quotations on the present value of human capital. Second, and more important, the market does not supply us with estimates of the real value of wealth in accordance with equation (24) above. All that can be discovered from financial data is the change from one year to the next

[7]The forward price τ years ahead assessed in the year 0 is related to the expected spot price in the year τ, p_i^τ, by the formula

$$\bar{p}_i^\tau = p_i^\tau \exp\left(-\int_0^\tau r(x)\,dx\right)$$

where $r(x)$ is the instantaneous rate of interest at time x.

[8]An attempt to measure real income in accordance with equation (2) has been made as part of an undertaking by Robert Eisner and others at Northwestern University to improve the coverage of the national accounts. See R. Eisner and A. B. Treadway, 'Non-Income Income: A foreword', a brief paper presented at the 1970 meetings of the American Economic Association and M. McElroy, 'Capital Gains and the Theory and Measurement of Income', A Ph.D. dissertation at Northwestern University, 1970.

in the money value of wealth—not the change in the real value, W_0^t of equation (24), that is required in the definition of real income. A rise in the value of shares on the stock market may be due to an increase in expected real earnings or to a fall in expected real rates of interest. We would wish to recognize an increase in expected real earnings as constituting an increase in real wealth, but we would not wish to recognize a fall in the real rate of interest, for the real rate of interest is the relative price of present and future consumption and it is precisely to abstract from price changes that we distinguish between real and money income. If we recognized a fall in the real rate of interest as constituting an increase in real wealth, we might find ourselves having to say that real income has increased from one year to the next despite the fact that no change has occurred in current consumption or in expected future consumption. Third, stock market data provide us with evidence of expected wealth rather than actual wealth, and expected wealth may rise or fall for a variety of reasons that we would not want reflected in the national accounts. Scientific discovery may create false hopes of future prosperity. Businessmen and bankers may misjudge the duration of the current phase of the business cycle. Wars may come unexpectedly, or may be expected and fail to come. All this is to say no more than that men cannot predict the future accurately, and that the market value of wealth increases or decreases periodically as expectations are formed and revised. Surely, whatever we mean by income, we do not want our measure of real income to rise or fall with every change in expectations of future consumption.

(ii) The definition of real income as the sum of real consumption and the change in real wealth violates a condition that most economists would think of as central and indispensable to the notion of real income, namely, that income reflect what is happening today rather than what will happen tomorrow. In particular, it is a requirement of the definition of income that income and consumption be one and the same if there is no provision in the current year to augment consumption later on. The measure of income in equation (19) does not possess this property.

Imagine an economy in which labour is the only factor of production, in which there is no capital, no saving, no investment, and no growth of population, but technical change occurs at a rate λ. The production function in each year t is

$$(25) \qquad C_0^t = L \exp(\lambda t)$$

where C_0^t and L are consumption and the labour force. Since L is constant, consumption grows at a rate λ, wealth in the year t is as indicated by the equation

$$(26) \quad W_0^t = \int_t^\infty \exp(-r(\tau - t))C_0^\tau d\tau = L\int_t^\infty \exp(-r(\tau-t)-\lambda\tau)\, d\tau = \frac{C_0^t}{r-\lambda}$$

and the real value of real income would have to be

$$(27) \qquad Y_0^t = C_0^t + W_0^t = C_0^t + \frac{d}{dt}\left[\frac{C_0^t}{r-\lambda}\right] = C_0^t\left[1+\frac{\lambda}{r-\lambda}\right]$$

where the rate of interest r is assumed constant over time. The term $r-\lambda$ must be positive, for otherwise wealth would be infinite and a man could sell a share, no matter how small, of his earning power in perpetuity for an infinitely large sum.

(iii) Acceptance of the definition of real income as the sum of real consumption and the change in real wealth would require the abandonment of other important usages of the term. Obviously, we could no longer identify real income as the sum of real consumption and real saving because real saving is zero in the example above, but real income exceeds real consumption nonetheless. Similarly, and for exactly the same reason we could no longer identify real income as the sum of real consumption and real capital formation. Nor even could we identify real income as the national output in the aggregate production function $Y=f(K, L)$. Forced to choose between abandoning the definition of real income as the sum of the consumption and the change in wealth and abandoning all these other usages of the term, I, and I think most economists, would have no hesitation in abandoning the definition of income as the sum of consumption and the change in wealth because these other usages of the concept of income are deeply imbedded in economic analysis while the definition of income as the sum of consumption and the change in wealth is a definition and no more.

D. Real Income as the Sum of the Outputs of Consumption Goods and Capital Goods

$$Y=C+p^0M \tag{28}$$

This second interpretation of real income is an extension of the definition of real consumption as an indicator of productive capacity in equation (9) above. The corresponding definition of real income would be designed to represent the location of the production possibility curve for capital goods and consumption goods combined. Formally, real income in the year t with respect to year 0 as the base year is

$$Y_0^{pt} \equiv \sum_{i=1}^{n} p_i^0 q_i^{pt} + \sum_{j=1}^{m} p_{j+n}^0 M_j^{pt} \tag{29}$$

and where q_i^{pt} and M_j^{pt} are quantities in the most valuable bundle of goods (at prices in the base year) that can be produced with the technology of the year t as represented by equation (13) above $f^t(q, M)=g^t(K^t, L^t)$ where K^t and L^t are amounts available in the year t. This definition of real income may be approximated by a Laspeyres measure

$$Y_0^t \equiv \sum_{i=1}^{n} p_i^0 q_i^t + \sum_{i=1}^{m} p_{j+n}^0 M_j^t \tag{30}$$

which is completely analogous to the Laspeyres measure of real consumption in equation (11) above.

This definition of real income is favoured by a number of economists and national accountants who feel that real income ought to be an indicator of what the economy can do this year, rather than of how well off we are by comparison with some other year.[9] A case can be made that the Laspeyres approximation of equation (30) is at least close to the formula used to measure real income in the national accounts. It would certainly be convenient to discover that standard practice in the national accounts provides us the information we require with as much accuracy as can reasonably be extracted from the available data.

It turns out, however, that there are some very serious problems with this definition of real income: (i) Unlike the measure of real consumption as an indicator of productive capacity in equation (9), the measure of real income in equation (30) carries no necessary implications about representative consumer's capacity to make himself better off now or at any future time because quantities of investment goods, M_j, are not arguments in the utility function. (ii) The definition of real income as the sum of real consumption and the output of new capital goods is frivolous in its response to technical change, in the sense that economically-equivalent technical changes may or may not lead to increases in real income depending on irrelevant characteristics of technical change. (iii) This measure of real income shares the property of the measure of real consumption as an indicator of productive capacity that it cannot take account of changes over time in the nature of goods and that a rule must be introduced extraneously to compare new and old types of goods on a common scale.

(i) The analogy between real income in equation (30) and real consumption in equation (9) is incomplete because there is no consistent set of indifference curves, tangent to each and every production possibility curve $f^t(q^t, M^t)$ at the chosen values of q^t and M^t, comparable to the set of indifference curves in Figure 1.

Suppose that bread is the only consumption good and tractors the only capital good. One can construct a sequence of production possibility curves for bread and tractors such as is illustrated in Figure 2, but there is no corresponding set of indifference curves because the number of tractors is not an argument in the utility function. Admittedly, the relative price of bread and tractors is a reflection of the rate of substitution in use between them and in each year there may be constructed a set of what might be called pseudo-indifference curves, each of which is the locus of combinations of bread and tractors for which the intertemporal utility function of equation (5) is constant. What differentiates the intertemporal case illustrated in Figure 2 from the atemporal case in Figure 1 is that the pseudo-indifference curves in Figure 2 are not invariant over time even though taste itself is invariant in the sense that the intertemporal utility function of equation (12) preserves its form from year to year. Pseudo-indifference curves are incomparable from year to year because the rate of substitution in use between bread and tractors today depends upon the state of technology tomorrow and because technology is changing over time. Pseudo-indifference curves may cross as illustrated by the broken curves of Figure 2. One cannot infer from the positions of pseudo-indifference curves such as U^0 and U^1 in Figure 2 whether the representative consumer is better off in year 1 than he is in year 0, not even if one curve lies entirely outside the other. In the atemporal model depicted in Figure 1, a representative consumer moving from a

[9]See R. Moorsteen, "On Measuring Productive Potential and Relative Efficiency", *Quarterly Journal of Economics*, 1961, pp. 451–67, and F. Fisher and K. Shell, *The Economic Theory of Price Indices*, 1972.

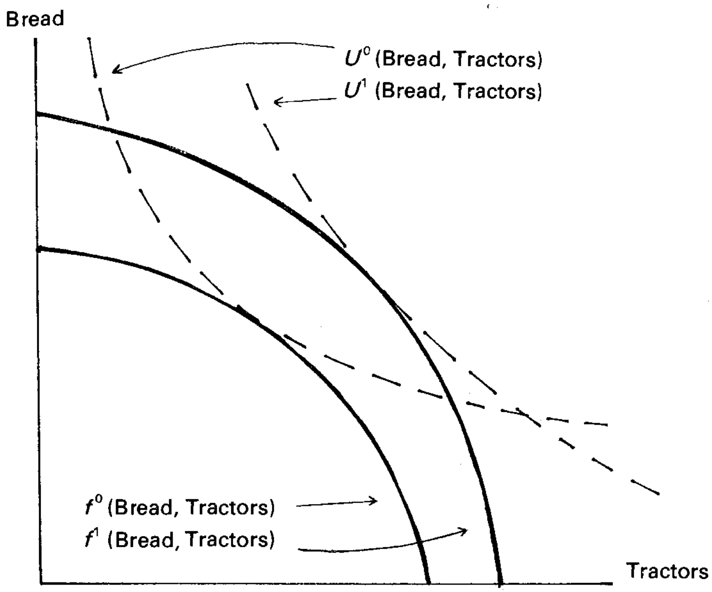


Figure 2

lower to a higher indifference curve is better off by definition, a uniform outward shift of the corresponding production possibility curve gives the representative consumer the opportunity to become better off, and he will become better off unless the shift of the production possibility curve is accompanied by distortions in relative prices. By contrast, a uniform outward shift of the production possibility curve for bread and tractors in Figure 2 may not signify that the representative consumer has the opportunity to become better off, even in a perfectly competitive economy without taxes or other distortions in the price mechanism.

(ii) The second problem with the definition of real income as the sum of real consumption and the real output of capital goods is really an elaboration of the first. The source of the problem is that capital goods share many of the qualities of intermediate products. They are produced at the cost of consumption goods that might have been produced instead, and they yield extra consumption goods in due course. The difference between capital goods and intermediate goods is one of timing. Intermediate goods are created and used up in a single accounting period; a statistician could transform goods from capital goods to intermediate goods, or vice versa, by lengthening or shortening the accounting period. The sequence of creation and use of capital goods may be represented as $C \rightarrow K \rightarrow C$. Capital has two links to consumption. The first is between the consumption that must be given up to create capital goods and the amount of capital goods created; the second is between capital and the amount of consumption goods it can produce. Technical change may occur in either link of the chain. If it occurs in the second link, we are made better off without there being any increase in the quantity of capital goods produced; the capital we produce today becomes more productive, but we do not necessarily produce more capital than we would otherwise have done, and the measure of real investment need not show an increase. If technical change occurs in the first link, a given sacrifice of consumption goods today yields a larger supply of new capital goods, and there are corresponding increases in our measures of real investment and real income. The two types of technical change may be equivalent in their impact on the intertemporal welfare of the representative consumer, but one type is reflected as an increase in real investment while the other is not.

Specifically, if real income is the sum of the values at constant prices of C and M according to equation (28), then the rate of change of Y has to be an average of the rate of change of C and the rate of change of M. But for an economy with a given initial value and rate of change of C, and hence a given value of the intertemporal utility function U, the rate of change of M can be greater or smaller depending on the extent to which technical progress is embodied in new capital goods. Consequently the rate of increase of Y can be larger or smaller without there being any impact at all on the welfare of the participants in the economy. This may not matter very much when statistics of real income are used as a tool in economic forecasting, but it can make complete nonsense of statistics intended to reflect the rate of growth of a country in the long run.[10]

(iii) The concept of real income as the indicator of the location of the production possibility curve tends to break down when we try to allow for changes over time in the nature of goods themselves. It has already been pointed out in the section on real consumption that there is a major difference between welfare and productivity in this respect. For welfare measures, there is, at least in principle, a simple procedure for comparing real incomes in circumstances where different kinds of goods are consumed. Productivity measures, on the other hand, cannot be defined at all unless either the same goods are consumed at all times or considerations of taste are introduced surreptitiously to compare goods consumed at different times on a common scale.

As this point is of some importance, it is worth illustrating a simple example. In the year 1950, barley is grown by means of a technology employing wooden ploughs, and output consists of barley as the consumption good and ploughs as the investment good. Subsequently in 1961, wheat is grown by means of a technology employing tractors, and output consists of wheat as the consumption good and tractors as the investment good. Suppose that wheat could not be grown in 1950 because tractors had not yet been invented, and that the growth of population has rendered it impossible for people in 1961 to revert to the earlier technology.

We can easily measure real consumption as an indicator of welfare. Suppose we want to measure real consumption in 1950 with respect to 1961 as the base year. All we need to do is to send the representative consumer from 1961 back to 1950, to have him consume the average amount of barley that was consumed in 1950, and ask him how much wheat he would need to consume in 1961 to be as well off as he had been in 1950. We can compare real consumption in practice because we can compare in our minds what it would be like to consume wheat or barley. The statistician may decide that wheat and barley are equivalent as food or that one has a certain premium on the other. There is, on the other hand, no test

[10]For a full development of this point, see D. Usher, *The Measurement of Economic Growth*, Queen's Discussion paper #131. See also D. W. Jorgenson, "The Embodiment Hypothesis", *Journal of Political Economy*, February, 1960, pp. 1–17.

that will enable us to compare real consumption in 1950 and 1961 as measures of productive capacity alone. Barley and wheat are technically distinct because there is no common production possibility curve encompassing the two. What we do in practice is to identify wheat and barley in accordance with their usefulness or, to put the matter in a more general context, we use welfare criteria for classifying diverse goods into *n* arbitrary categories of consumption and we then look upon the *n* categories of consumption as though they were themselves consumption goods that remain invariant as to their form in the course of time. There may be a pure measure of real consumption as an indicator of welfare but there cannot be a measure of real consumption as an indicator of productive capacity alone. All the problems of comparing wheat and barley are of course carried over into the comparison of ploughs and tractors except that the implicit utility comparison is not so direct or so evident.

New types of capital goods do not come off the assembly line complete with labels certifying which of our *m* categories of capital goods they belong to and how many units of the old types of capital goods they are. Confronted with the great diversity of capital goods available at any moment of time, and with changes in composition of capital goods in the course of time, the statistician must one way or another identify all capital goods as amounts of *m* standard types if he is to construct a time series of investment at all. He must decide how many transatlantic sailing ships are equivalent to one jumbo jet, or how many abaci are equivalent to one modern electronic computer. The statistician is understandably reluctant to make judgements of this kind and he appears to circumvent them by means of the deflation procedure. Instead of identifying capital goods, one by one, as amounts of the *m* capital goods in the model, he measures real capital by dividing the money value of investment by a price index of capital goods. Of course, the problem of comparability of capital goods is not really avoided. It is merely tucked away in a thousand little statistical decisions about such apparently innocuous matters as when to incorporate a new product into the price index or when to change the base of the price index. Looked at as a whole, the statistical procedures are frequently so complex that the statistician himself may not quite realize what he is doing. Nonetheless, a decision is made as to how many abaci equal one electronic computer, for without such an equivalence the time series of real income could not be compiled at all.

Space does not permit us to discuss these matters in detail, but two points are of special importance. First, since there is no purely and exclusively technical comparison between capital goods available in different years, the methods employed in the national accounts for constructing time series of real investment and real income cannot legitimately be identified with the concept of real income as an indicator of productive capacity alone. The support which this concept appears to gain from its association with the statistical procedures in the national accounts is at least in part based on a mistaken impression of what information the national accounts contain. Second, the statistical problems we are discussing interact with and compound the theoretical difficulties discussed above concerning the fact that equivalent technical changes can have different effects upon the measure of real income depending upon whether they take the form of increasing the productivity of capital goods or lowering the price of new capital goods. If the statistician identifies the newly developed computer with a relatively small number of abaci, then the rate of investment is relatively small too, and it is as though technical change increased the productivity of capital goods as a whole. On the other hand, if the statistician identifies the newly developed computer with a large number of abacuses, it is as though there were a substantial fall in the relative price of new capital goods and real investment would be said to grow more rapidly than otherwise. The essential arbitrariness in identifying new types of capital goods with old types of capital goods gets hidden in the unavoidable complexities in the construction of the national accounts, but it is there, nonetheless, and real investment may grow rapidly or slowly depending upon what the statisticians decide to do.

E. Income as the Sum of Real Consumption and the Increment to Real Capital

$$Y = C + p_0 \Delta K \tag{31}$$

Investment, like consumption, can be measured from the point of view of productivity or from the point of view of welfare. The productivity measure, discussed in the preceding section, is an indicator of the location of the production possibility curve of capital goods. The welfare measure we are now about to consider is an indicator of what the new capital goods can do. In the simple economy depicted in Figure 2, the corresponding measures of real income are respectively indicators of the location of the production possibility curve of bread and tractors and of the location of the pseudo-indifference curve attained. In this section, we shall see whether we can construct a pure use measure of real investment, a measure that increases if capital goods produced this year can do more than capital goods produced last year, irrespective of the locations of the production possibility curves from which they themselves were created.

An aggregate of quantities of different types of machines weighted according to their usefulnes is commonly referred to as a measure of "real capital stock" and is designated by the letter K. The question is whether a reasonable measure of real capital can be devised. At the outset of this discussion, it is important to recognize that two separate and distinct concepts of real capital are employed in economic analysis.[11] We shall refer to these concepts as "instantaneous productive capacity" and "long-run productive capacity", and will illustrate them as properties of

[11]The contrast between the two concepts of real capital is discussed by Zvi Griliches in 'Capital Stock in Investment Functions: Some Problems of Concept and Measurement', an essay in C. Christ *et al.*, *Measurement in Economics*, Stamford University Press, 1963. Griliches argues, with some justification, that measures of both concepts of the capital stock might be approximated by weighted averages of amounts of capital goods available, with service prices as weights in the measure of instantaneous productive capacity and stock prices as weights in the measure of long-run productive capacity. The concept of aggregate capital stock, as instantaneous productive capacity, has been analysed by Franklin Fisher in 'Embodied Technical Change and the Existence of an Aggregate Capital Stock', *Review of Economic Studies*, 1965, pp. 263–88. Fisher shows that only under very restrictive assumptions is it possible to define an aggregate capital stock which is at once a stable function of the numbers of the different kinds of machines in the economy and a perfect surrogate for their joint effect within the production function. Fisher's results do not rule out the possibility that existing measures of the capital stock may be adequate for purposes such as use within big econometric models designed to predict the course of the economy. Fisher does not discuss the concept of aggregate capital stock as long-run productive capacity.

the simple intertemporal model in equations (12), (13) and (14). Real capital stock as an indicator of instantaneous productive capacity will be designated as $\bar{K}$ and real capital stock as an indicator of long-run productive capacity will be designated as $\bar{\bar{K}}$.

When real capital stock is measured as an indicator of instantaneous productive capacity, we say that a mix of capital goods constitutes more real capital than some other mix of capital goods if this second mix of capital goods is insufficient to produce the mix of outputs that is in fact produced with the first mix of capital goods. Specifically, suppose that in some year 0 which we arbitrarily assign to be the base year, a mix of capital goods, $K_1^0, \ldots, K_m^0$ is producing a mix of outputs $q_1^0, \ldots, q_n^0, M_1^0, \ldots, M_m^0$ with the aid of labour inputs $L_1^0, \ldots, L_r^0$, so that

(32) $$f^0(q^0, M^0) = g^0(K^0, L^0)$$

where q^0, M^0, K^0 amd L^0 are the vectors defined in connection with the intertemporal model of section B. We arbitrarily set the index of real capital stock in the year 0 at $\bar{K}^0$. The index of real capital corresponding to any other mix of capital goods $K_1, \ldots, K_m$ is set at $\bar{K}$ where

(33) $$\bar{K} \equiv \lambda \bar{K}^0$$

and where λ *is chosen such that*

(34) $$f^0(q^0, M^0) = g^0(K/\lambda, L^0)$$

Notice that according to this definition, the amount of real capital embodied in a mix of capital goods depends not only on the amounts of these goods available but on the technology of the year 0, the amounts of labour available, and the choice of outputs q_i^0 and M_j^0. It may well turn out that the mix of capital goods $K_1, \ldots, K_m$ constitutes more capital than the mix $K_1^0, \ldots, K_m^0$ with regard to the technology of the year t even though $K_1^0, \ldots, K_m^0$ constitutes more capital with regard to the technology of the year 0.

The second concept of real capital makes allowance for the durability as well as for the instantaneous productive capacity of the mix of capital goods. It would seem that the only way to account for durability in the definition of real capital—other than by appeal to wholly arbitrary statistical conventions—is to say that one mix of capital goods constitutes more real capital than another mix of capital goods if the representative consumer is better off with the first mix than he is with the second. From the system of equations our simple intertemporal model, we can see that, for given supplies of labour, utility depends on the stocks of capital goods available. As utility is a function of present and future consumption and as consumption each year depends on the amounts of capital goods available and on amounts of capital goods produced, the system of equations (12), (13) and (14) can be condensed into a type of indirect utility function

(35) $$U = U(K_1, \ldots, K_m)$$

the exact form of which depends on the technology, the f-function and the g-function, in the current year and in every subsequent year. If we designate the value in the year 0 of the intertemporal utility function of equation (12) as $\bar{U}$, then

(36) $$\bar{U} = U(K_1^0, \ldots, K_m^0)$$

because, by definition, $\bar{U}$ is the utility attained with the mix of capital goods K^0. We may arbitrarily set the index of real capital stock in the year 0 at $\bar{\bar{K}}^0$ and designate the amount of real capital corresponding to any other mix of capital goods $K_1, \ldots, K_m$ as

$$\bar{\bar{K}} = \gamma \bar{\bar{K}}^0 \tag{37}$$

where γ is determined implicitly in the equation

$$\bar{U} = U(K_1^0, \ldots, K_m^0) = U\left(\frac{K_1}{\gamma}, \ldots, \frac{K_m}{\gamma}\right) \tag{38}$$

The two concepts of capital, 'instantaneous productive capacity' and 'long-run productive capacity' are contrasted above to emphasize that it is the second, more complex concept that is appropriate in the context of measuring real income. Investment results in the appearance of new capital goods the value of which depends in an essential way on their durability as well as on what they can be used to make today. In principle, an index of real capital as long-run productive capacity can be constructed in the manner discussed above, but such an index would be of little or no value to the national accountant for four reasons: First, the index of real capital in equation (38) is defined with regard to a given utility function and a given current and anticipated technology. Suppose we decide to assess the magnitude of real capital with regard to the situation in year 0. Then, of two alternative bundles of capital goods K^* and K^{**} available in the year t it may easily happen that K^* is counted by our rule as constituting more real capital despite the fact that K^{**} is the more useful bundle in conditions in the year t. This is what authors usually mean when they say that real capital does not exist. Second, it is an essential characteristic of any measure of real investment that identical bundles of new capital goods be counted as the same amount of real investment. If output of new capital goods consists of one unit of K_1 and one unit of K_2 in year 0 and of one unit of K_1 and one unit of K_2 in year 1, then we would want to say that real investment is the same in both years. Equation (38) does not work that way. An addition to capital of one unit of K_1 and one unit of K_2 would constitute different amounts of capital in circumstances where the capital stock is evenly divided between K_1 and K_2 than in circumstances where the capital stock consists mostly of K_1 or mostly of K_2. Similarly, one unit of K_1 and two units of K_2 might be more or less additional capital than one unit of K_2 and two units of K_1 depending on the proportions of K_1 and K_2 in the existing capital stock. Third, and this is a variant of the first objection, the concept of investment as the increase over the year in the real capital $\bar{\bar{K}}$ is all but useless in an economy where technical change is altering the nature of capital goods. Recall that $\bar{\bar{K}}$ is defined with regard to base year technology. In measuring real capital in the year t, the new types of capital goods, the semi-conductors and the nuclear powered generators, could be thought of as being available without the technology necessary to work them (in which case they would be worthless) or as carrying the new technology on their backs in which case we should be counting technical change itself as part of capital formation, and the change in real capital would be indistinguishable from the change in wealth which we have already rejected as a definition of real investment. Fourth, even a perfect measure of real capital would not circumvent the problem of the interchangeability of embodied and disembodied technical change discussed in section D above.

It would perhaps be useful at this point to say a few words about depreciation and about the perpetual inventory method of measuring the size of the capital stock, for our analysis has some bearing on both of these matters. Depreciation creates special complications for the measure of real income as the output of consumption goods and new capital goods discussed in section D, for that measure is necessarily gross, and it is hard to see how any corresponding net measure might be devised. Real income in this sense is an indicator of the location of the production possibility curve of consumption goods and capital goods together. We can measure real income if we know the form of the function f in equation (13), and the rest of the model is irrelevant. Depreciation, on the other hand, is a concept closely connected to the notion of real capital, $\bar{\bar{K}}$, as discussed in this section, and, like $\bar{\bar{K}}$, it cannot be measured unless we know every function and every equation in the model. Real income as a measure of productive capacity is, therefore, incompatible with depreciation because they refer to different aspects of the economy, to attributes of income that hang together in the one-sector model but cease to do so in the more complex model. Perhaps one could obtain a rough and ready measure of real net income by imposing the one-sector model upon the data and ignoring the bits that do not quite fit together. But one can certainly understand and sympathise with the reluctance of most national accountants to deflate net income at all—their willingness to produce figures of real gross national product but not real net national product or national income.

The perpetual inventory method of measuring the capital stock in real terms is subject to and compounds almost all of the difficulties discussed in this essay, for it has to draw upon several of the equations in the one-sector model, notably equations (2), (3) and (4), and cannot tell us what we want to know about the economy unless all of the terms in all of these equations can be measured at the same time. The perpetual inventory method would of course be entirely satisfactory if the one-sector model were an accurate representation of the world. As matters stand, the perpetual inventory method requires that a long time series of the value of investment be converted to real terms, and that real investment each year be corrected for depreciation so that the annual bits of real net investment can be added together to obtain a time series of real capital stock. We have already discussed the difficulties at each stage in this process, and need not repeat that discussion in this context. Finally, since the perpetual inventory method uses bits and pieces of data from each year in the time series, it is not at all clear which member of the families $\bar{K}$ and $\bar{\bar{K}}$ is obtained, or even which family, though gross stock, as Goldsmith defines it, would seem to correspond more or less to the family $\bar{K}$ while net stock would seem to correspond more or less to the family $\bar{\bar{K}}$.

F. Income as the Sum of Real Consumption and Real Saving

(2) $$Y = C + S$$

In an economy in equilibrium, the money value of investment is at once the money value of the sacrifice of consumption and the money value of the new capital goods produced. Our attempt to convert money investment into real investment has so far emphasized the latter part of that identity. We have tried to construct a measure of the aggregate output of capital goods or of the increment to the real capital stock. We shall now consider whether it is feasible to measure real income as the sum of real consumption and real saving and whether the resulting statistics of real income correspond to any of the meanings of the term in economic discourse. As to the first of these issues, it turns out that the measure of real saving is the simplest, most easily computed, and least subject to index number conundrums of all the measures of investment we are examining. Real saving may be measured as the money value of saving or investment deflated by the price index of consumption goods. The rule for measuring real income as the sum of real consumption and real saving is, therefore, to scale up the measure of real consumption by the proportion between the money value of income and the money value of consumption. When we measure real saving, we are not concerned with the nature and amount of capital goods produced or with the ultimate benefits in future consumption. Only sacrifice is recorded in the current year, and future benefits get recorded as part of income when they accrue. If a sacrifice of 100 tons of bread resulted in 10 new tractors last year, and if the same sacrifice results in 20 new tractors this year, we say that real saving is the same in the two years and that benefits accruing from the 10 extra tractors will be caught when income increases next year.

Defined in this way, income can be measured according to the formula

$$Y = C + S = \frac{C}{1-s} \tag{39}$$

where s is the rate of saving, the ratio of the money values of consumption and income. For this measure of real income, we need a time series of income in current dollars, a time-series of consumption in current dollars, and a price index of consumption goods. There is no need for a price index of capital goods because we do not try to compare quantities of capital goods produced at different times.

This measure of real income has the additional advantage that it can easily encompass human capital formation (the cost of education is simply transferred from the category of consumption to the category of investment), capital formation in research, or foreign investment. In fact, it is implicitly to this concept of income that authors appeal when they treat these items as part of investment, for none of the other concepts we discuss can be extended in a comparable way. By the same token, we can easily construct a time series of real net saving, though it is consistent with the spirit of this definition of real income that the measure of the dollar value of depreciation used to convert gross saving to net saving should encompass the physical deterioration of capital goods but not obsolescence due to current or expected future technical change. Ideally, we would measure depreciation by choosing a reference date, say June 1, and summing up, for each piece of equipment available on January 1, the difference in the notional values the equipment would have at the reference date in its states of repair as of January 1 and December 31 of that year.

In a sense, we obtain the measure of real income in equation (2) by treating time series of money income, the money value of consumption, and the price index of consumption goods as if they pertained to a simple one-sector model of the economy—the model described at the outset of this paper with the additional restriction that capital goods and consumption goods are assumed to be fully substitutable, one for one, at all times. We look upon these time series as though they were generated by the model set out as equations (1), (2), (3) and (4), with the actual capital stock K replaced by a notional capital stock J defined by the property that the increment to J, ignoring depreciation, is numerically and dimensionally equivalent to the quantity of saving expressed as a quantity of consumption goods. We can therefore contract equations (3) and (4) to

$$S = \Delta J \tag{40}$$

which together with equations (1) and (2) provide a complete description of the supply side of the economy.

In the light of this model, we may now define income as "the amount of consumption that could be obtained in the stationary state that would arise if technical change, net investment and population growth ceased today". I think that this definition corresponds quite closely to what we mean by the term in its usage in economic analysis, especially when real income serves as a basis for the measure of economic growth. In terms of the intertemporal model of section B, the definition of real income is formally similar to the definition of real income as maximum sustainable consumption in section C. Once again real income is the smallest value of Y_0^t such that

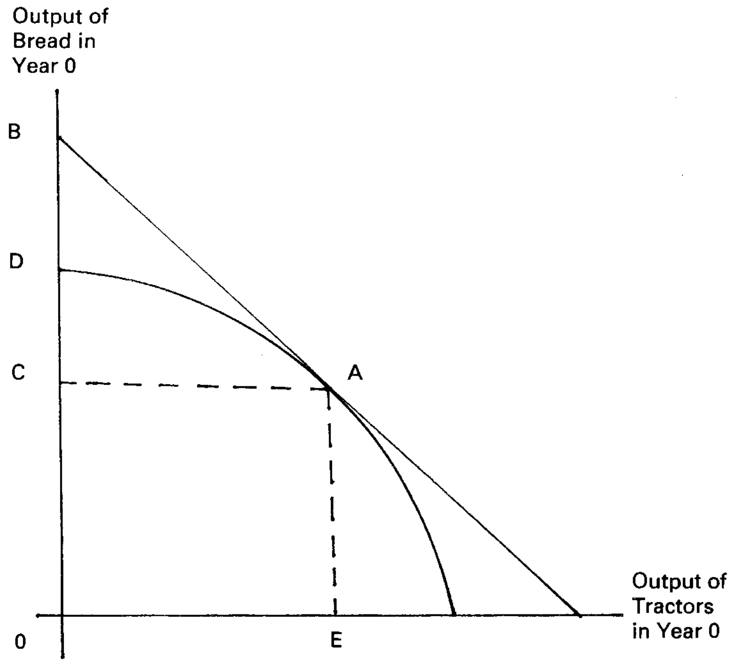
$$Y_0^t = \sum_{i=1}^{n} p_i^0 \hat{q}_i^t \tag{20}$$

where the $\hat{q}_i^t$ are quantities on the indifference curve representing the highest level of utility the system can maintain indefinitely

$$u(q_1^t, q_2^t, \ldots, q_n^t) \geq u(q_1^t, \ldots, q_n^t) = \ldots = u(q_1^{t+2}, \ldots, q_n^{t+2}) \ldots \tag{21}$$

So far, this definition of real income and the definition in section C are exactly the same. The difference between these definitions is that whereas the value of u in section C was constrained by the actual sequence of production functions in equation (13), the value of u is constrained here by the sequence as it would be if technical change stopped in the current year, if this year's technology remained in force forever. The difference is important because real income can now be measured by means of information that is, at least in principle, available to statisticians in the current year.

This definition is not without its problems. We would want to compile statistics of real income in accordance with equation (39) above, but the verbal definition differs from the definition in the equation in an important respect. The situation is illustrated in Figure 3 which reproduces part of Figure 2. As bread is the only consumption good, real income is measured according to equation (39) as a quantity of bread; real income is OB—the sum of quantity of bread produced, OC, and the quantity of tractors produced, OE, weighted by the relative price, BC/AC, of tractors and bread. On the other hand, the maximum potential consumption in the year 0 is OD, which is the intersection of the production possibility curve with the vertical axis and is necessarily less than OB. Real income



0B = 0C/(1 − *s*)

0D = maximum potential consumption in Year 0

Figure 3

as measured by equation (39) exceeds the maximum potential consumption in a stationary state because a rising supply curve for new capital goods causes each additional unit of tractors produced to represent a different amount of consumption foregone. The verbal definition and the equation would only be identical if the atemporal production possibility curve were a straight line, that is, if the supply curve for new capital goods were infinitely elastic everywhere.

The empirical significance of this point may be rather small for two reasons. First, the elasticity of supply of new capital goods depends on the length of the run. The longer the run, the more elastic the supply curve and the flatter the production possibility curve will be. The difference between the verbal definition and the equation is minimized because the very longest run is the relevant one in the contrast among stationary states. Second, whatever bias is introduced into the measure of income by the divergence between OB and OD is maintained in the same direction over time. If it should happen that the proportion between OB and OD remains constant over time, as it may well do, then the measure of economic growth would be unbiased despite the bias in income itself.

Though the verbal definition is free of this defect, the definition in equation (39) has the unfortunate property that an increase in the propensity to save shifts *A* clockwise along the production possibility curve, raising the price of tractors and increasing OB, the value of income as apparent consumption foregone. One would not expect bias arising from changes in the rate of saving to have a large effect on the measure of economic growth unless the rate of saving itself had changed substantially in the course of time.

In a sense, the measure of real income in equation (39) is the Laspeyres approximation to the verbal definition of real income. So it is hardly surprising that they do not correspond in every detail.

G. Conclusion

We began with a simple model in terms of which real income was a unique concept containing within itself all the nice properties we could reasonably ask for. These properties would not hold together when we moved to the larger model with more realistic assumptions about our capacity to anticipate the future and about the constancy over time in the nature of the commodities consumed and invested. There are now several definitions of real income, each preserving some of the properties originally associated with income, losing others, and giving rise to its own measurement problems We cannot avoid having to pick and choose among the alternative definitions. We must weigh their strengths and weaknesses in the light of the purposes that measures of real income are intended to serve. And we must entertain the possibility that the preferred measure for one purpose can be quite unsuitable for another.

I think a case can be made that the measure of real income as the sum of real consumption and the output of new capital goods is the most appropriate of the measures we have considered for the purpose of budgeting, forecasting and economic planning in the short run. This is the sort of measure we want for comparing full employment output with actual output or for predicting the impact of taxes upon expenditure. The defects of this measure—that it may be a poor indicator of welfare, and the incomparability of capital goods over long periods of time—need not be of overriding importance if we are only concerned to correct for price level changes in the near future.

Things are entirely different if we are constructing a long time series of real income for the purpose of measuring a country's rate of economic growth. To decide which of the measures we have considered is most appropriate in this context, we must begin by asking ourselves why we want to measure economic growth at all. I think the answer is simply we wish to know how well off we are by comparison with our parents, grandparents or great-grandparents fifty or a hundred years ago. It follows immediately that the appropriate measure has to be an indicator of welfare rather than productive capacity. If the flow of income consisted of consumption goods alone—if there were no investment—we would want our measure to show for each year in the time series the dollar value at prices in some chosen base year of the level of utility attained in that year, in the manner described in section A. We would not be interested in a comparison of productive capacity except as a surrogate for a welfare comparison; and, as has already been demonstrated, any such measure we might construct would be so coloured by *ad hoc* statistical judgements about the comparability of goods over time as to be almost worthless.

We could make intertemporal comparisons of real consumption alone, but we would prefer to extend our measure each year to account for the fact that consumption is normally well below what it would be if we did not devote part of society's resources to saving, capital formation, or the augmentation of future consumption, depending on how one looks at the investment process. We want to make some allowance for real investment to avoid having to say, for instance, that people at time A are worse off than people at time B merely because people in time B choose to invest heavily, and in spite of the fact that people at time B could be better off than people at time A, both in the current year and in the future, if they chose to do so.

We therefore want a measure that tells us for each year how well off the people in that year might be. We do not want a measure that includes capitalized technical change as part of real income because future technical charge is inherently unpredictable. We wish to emphasize the alternative cost of investment rather than the output of capital goods *per se* because equally useful technical changes may or may not lead to an increase in real income depending on whether the technical change is disembodied or embodied in new capital goods and because it is not possible in practice to draw a clear line between an increase in the output of capital goods and an increase in the productivity of capital goods. After considering the alternatives, I judge the definition of real income in section F "the amount of consumption that could be obtained in the stationary state that would arise if technical change, net investment and population growth ceased today" to come closest to what I mean by the term and to what seems to be implied by the usage of the term in economic history and other contexts where long time series of real income are employed.