MEASUREMENT OF CAPITAL IN DENMARK

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In December 1973, the so-called 2nd Perspective Plan was published by the Danish Ministry of Finance. It included some 5 and 15 year forecasts of investments in the private sector, based on the projected development of production and labour. The forecasts were made by use of a simple Cobb–Douglas production function, taking as capital-input the stock of buildings and machinery, using the perpetual inventory method (assuming sudden death).

Since the publication of these forecasts, an attempt has been made to refine the capital concept, measuring its services as factor input. Thus, it has been necessary to introduce an exogenous rate of interest. Inspired by Danish findings for private cars, depreciation functions for stocks and utility of machinery are developed. These functions may not seem very realistic for the heterogenous class of durables called machinery, but other possibilities appear even less convincing.

Together with an assumption of exponential decay for buildings, it is possible to produce alternative time-series for changes in input of capital in the production process. Some of the resulting estimates of parameters in the Cobb–Douglas function give a better fit than the original version. But no value of the elasticity of production of capital is firmly established, e.g. it is obviously dependant upon the period of estimation, and therefore of no great value in forecasting. No firm connection between labour productivity and capital input (in short as well as the long run) has so far been revealed in Denmark, so no measure of capital is yet of great use in forecasting, except when future growth in production resembles that of the past fairly closely.

THE CAPITAL CONCEPT

If you are able to accept one figure, K_t, as an adequate expression of the value of all sorts of buildings, machinery, cars and equipment in a given economic sector, then measurement of capital is really quite simple.

All you need is knowledge about some actions, a set of prices to evaluate the actions, and a method. Of these there exist three to measure capital:

(1) The cost of producing the relevant stock of investment goods,

(2) The source of production and income to investors, and

(3) Some sort of combination of 1) and 2), e.g. the present evaluation of the market for capital.

Deciding between (1), (2) or (3) you also have to decide whether to use the concept of *capital stock* or the *flow* of its services. But knowing rates of interest and depreciation and/or the value of future services it ought to be easy to move from one concept to the other, in a determined system.

In relation to the time of measurement, t, the actions of investment or production (income-earning) might date from:

(a) the past, (b) the present (t), (c) the future or (d) all of these.

Choice of methods and data of course have to be consistent and make sense, dependent upon the use of the capital concept and the wider theory or model of which it is a part.

Among the more important uses are capital as a production factor in studies of production functions and productivity (usually adopting method (1), period (a)) and capital as a measure of wealth, in studies of distribution, e.g. (3) (b) or (2) (c).

Insurance companies, untroubled with theory, would happily settle for (3)(b). On the other hand, those untroubled with problems of empirical verification would choose (3)(d), in a theory embracing all markets at all times, known to all in the market of today. But even accepting the more humble aim of producing capital figures based on existing data, you run into serious problems of consistency between choice of methods, concepts, degree of aggregation, units and periods of measurement. This is true also if a capital study is made solely to find depreciation figures.

Danish Experience

Some of the difficulties encountered in making empirical estimates of the Danish capital stock are to be described on the next pages. Let it be admitted from the start that the methods contain no revolutionary news. The point of view has been capital as a production factor,[1] and the basic principle has been (1)(a), i.e. the perpetual inventory method. The main purpose was to produce investment forecasts for the private sector. This was part of a total public Danish plan[2] (or rather forecast) covering 1972–87.

Capital was defined as the simple sum of the value of buildings, machinery etc., not including stocks, nor land.

Machinery was assumed to last 15 years, as found in manufacturing industries in an earlier study. The capacity was assumed to remain fully intact until its "sudden death" at the end of the 15th year. In principle the same method was used for buildings, but the assumed life was less important, as annual investments before 1945 were small compared to the post-war period.[3] For manufactures we therefore used insurance figures from 1951 as the "benchmark" for the value of buildings. With the 1951 figures for manufactures in mind, we constructed a benchmark for other industries as the sum of investments since the beginning of the century (1905).

[1]The measured concept of capital in the private urban sector in year t, K_t, is supposed to be factor in a production function of the Cobb-Douglas type, i.e.

$$Y_t = K_t^a L_t^{(1-a)} e^{qt} \quad \text{or} \quad \dot{y}_t = a\dot{k}_t + q$$

where

Y_t is the gross domestic product of the private urban sector in year t,
L_t is the production factor labour in t, measured as the number of fully employed within the private urban sector,
a is the elasticity of production of capital and $(1-a)$ is accordingly the elasticity of labour, and
q is a measure of disembodied technical progress.
$\dot{y}_t = \Delta Y_t/Y_t - \Delta L_t/L_t$ and $\dot{k}_t = \Delta K_t/K_t - \Delta L_t/L_t$

$\dot{y}_t$ and $\dot{k}_t$ have been found for the period 1950–1972 and through a regression analysis the parameters a and q were estimated, i.e. we did not use the functional income distribution to determine a.

[2]The Second Perspective Plan (PP II), published December 1973 by the Danish Ministry of Finance. A detailed account of the capital study is given in: Bjerregaard, Groes, Schauby and Ramussen (1976).

[3]We did not contemplate reinvestments before 1985, which of course implied a fault, but of minor importance (presumably not more than 2 percent of fixed gross investment in any year). After 1985 life of buildings was predicted to be 40 years. As war-time investments were small, reinvestments will not be of any magnitude before scrapping of post-war buildings is supposed to start.

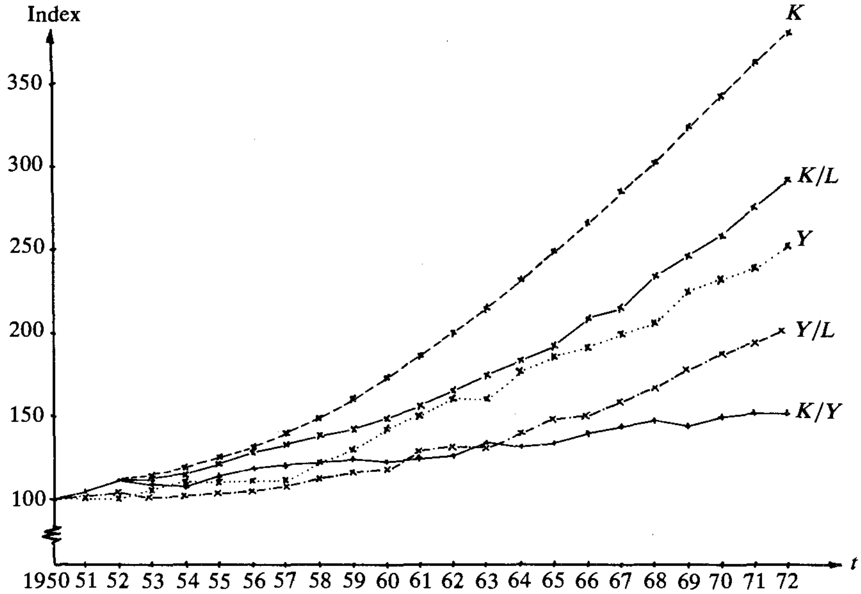


Figure 1. Index for Development of Capital and Production in the Urban Sector. (1950 = 100)

During the period under review, productivity of labour increased considerably. The first impression (Fig. 1) of exponential growth is not confirmed: the annual rate of growth is not constant, nor is the average level of relative growth constant, but apparently increasing. Of course we had hoped to explain this result by the increase in the capital-output ratio.

Unfortunately this is not so. As could be expected, the model was unable to explain annual fluctuations in the annual relative increase in output per head ($\dot{y}$), so we made moving 5 year averages of $\dot{y}_t$ and the annual relative increase in capital per head ($\dot{k}$). Now a distinct correlation is obvious, but the connection between $\dot{y}_t$ ahd k_t still is not linear.

Apparently q (the disembodied technical progress) has increased over the years.

As can be seen from Figure 2, $\dot{y}_t$ might be an exponential function of $\dot{k}_t$. We hesitated however to leave the well known and well behaving Cobb–Douglas function, especially as we had no ready theory to support a function like $\log \dot{y}_t = v + z\dot{k}_t$ and did not really believe that any such function was reversible.

Also, the unexplained increase in productivity apparently happened in one big jump. So, we took it for granted, that the production elasticity of capital (a) was constant in 1958–72, whereas the "disembodied" technical progress (q) made a jump in 1966, estimated by a dummy variable (Figure 2 and Table 1).

This model was used to predict the necessary investments in buildings and machinery (the relation between the two taken as given in advance). Y_t was assumed to grow at an annual rate of 4–$4\frac{1}{2}$ percent, whereas $\Delta L/L$ was -1 percent, and we expected continued high technical progress ($q_2 = 1.6$). The value and interpretation of q is of course very questionable, especially as it has been

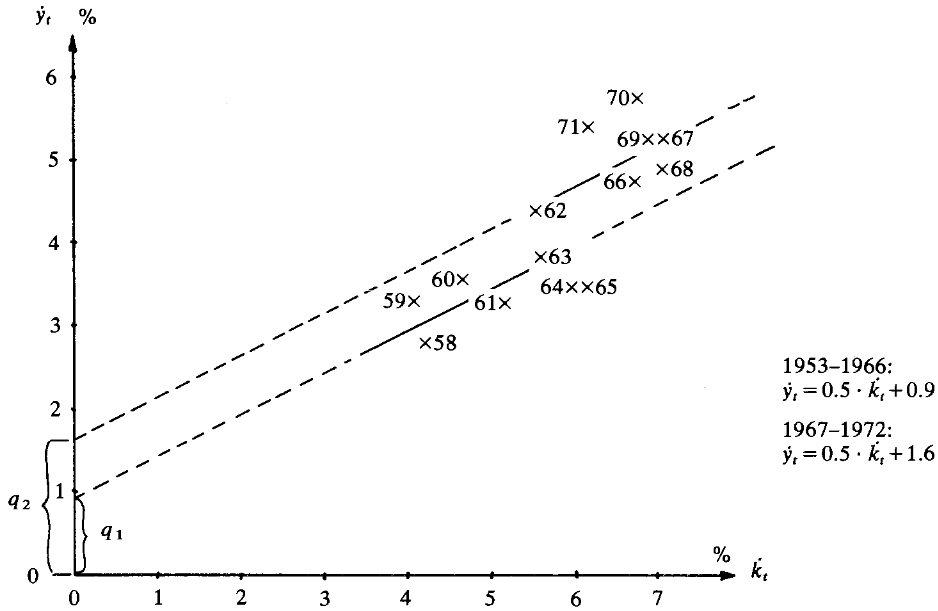


Figure 2. Connection between relative change in labour productivity ($\dot{y}_t$) and capital intensity ($\dot{k}_t$)

rising in the post-war period, where $\dot{k}_t$ and $\dot{y}_t$ have been rising as well. It is still to be seen whether q will remain at the high level in the mid-seventies with stagnating Y_t and very small investments.

Although the measurement of capital as described above has been crude, the figures approximated fairly well the insurance value of buildings for manufactures in 1951 and the value of buildings given in the regular official evaluations. Both sets of figures were however 75–100 percent above the book values of the manufacturing companies (limited). These generally comply with tax regulations and are therefore in Denmark based on prices at the time of purchase. So they do not take price increases in investment goods into account, but allow on the contrary for a rapid depreciation of both buildings and machinery.[4]

The close relation between our constructed value of capital and estimates of trade value is of course not purely accidental. Fortunate as this may seem, it is however worth stressing once more, that the concepts of trade value and the capacity of capital as a production factor are different.[5] The depreciation allowable for tax purposes also plays a role in determining the managers' and owners' view of the value of real capital of their firm, at least as regards items for which there is no obvious market, e.g. specialized machinery. In this way tax allowances

[4]10 percent p.a. for building during the first 10 years of its lifetime, 30 percent for machinery plus possibilities of depreciation in advance of purchase.

[5]In fact no close correlation between trade value and the sum of investments can be found in manufactures. In agriculture, where figures go back to the 1920's, obviously earnings in agriculture and the urban sector are far more important in determining the annual values of capital than past investments (which actually are insignificant in determining capital prices).

influence estimates of the value of real capital, both from inside and outside any firm, thereby influencing trade value as well.

The production function has later been combined with a simple savings model (of the Kaldor–Passinetti type). Together with assumptions about foreign debt and public savings, capital accumulation and thereby Y_t will then be determined, i.e. growth is determined by the possibilities of saving and borrowing.

Rate of Interest and Depreciation

The measurement of production factors has evidently been very crude, counting labour in number of employed and capital as sum of past investments in fixed prices. In this rather clumsy way we tried to avoid an overdetermined model, whereby the prices of factor inputs were determined by the size of the factors.

The measurement of capital has been crude with regard to price indices, to hypothesis of survival and not least to the use of capital stock as factor instead of capital services. With this "puristic" approach, the value of capital should not be determined by future earnings, nor from the resulting distribution and interest rates, when on the other hand the magnitude of capital was to determine future production and earnings.

But the cost of our approach was that we were unable to measure any "quality" changes in the production factors. By adding stocks instead of services, the weight of capital items increases with expected lifetime, i.e. buildings dominate. By simply adding value of machinery and buildings we implicitly assume either limitationality or perfect substitution. As the first assumption is obviously not true in the Danish postwar case, we must stick to the second, although the substitution should really be between the services of machinery and buildings respectively.

One way out of this dilemma was to regard buildings and machinery as different production factors. We did try to estimate production elasticities of machinery and buildings separately, but without success. Even if it was an appropriate description of real production, the conditions would hardly ever be met in which the elasticities could be found empirically.

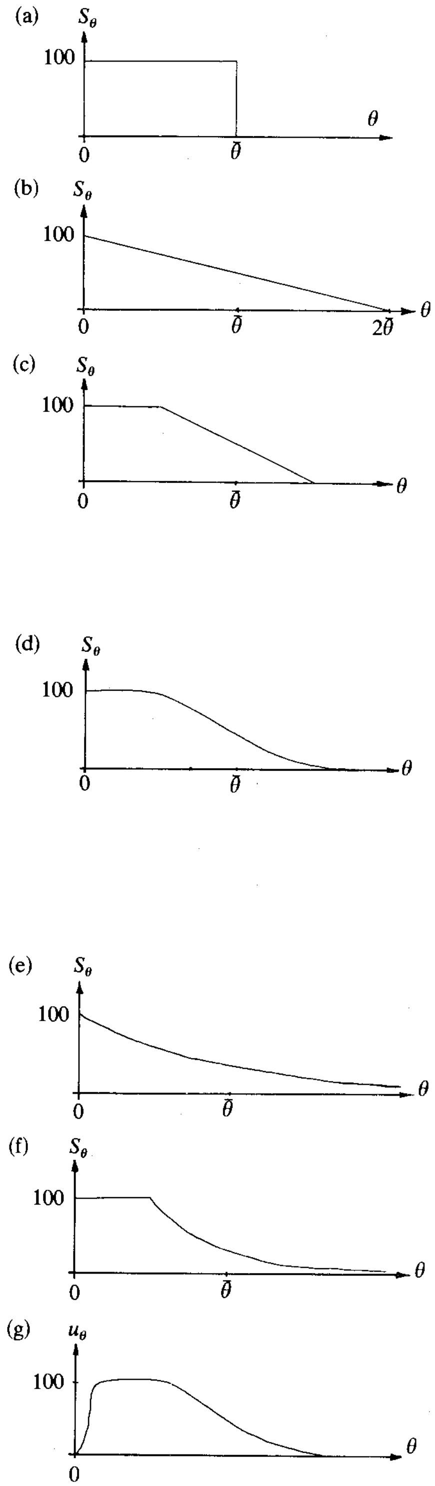
After publication of the first investment forecast we have tried to construct some more realistic capital-measures,[6] assuming that wages and interest rates are given outside the model, and are unaffected by the magnitude of capital accumulation.

Perhaps it can be justified to say that the real interest rate is determined outside Denmark (in the European capital market). In any case, it does not matter much what value is chosen within a realistic range of 3 to 6 percent p.a. (actually we chose 4).

The next problem was to get some realistic depreciation figures, i.e. an idea of the development over time of the value of services of any vintage.

One possible way to tackle the problem is first to find a "survival function" of any category of fixed investment and secondly to estimate the value of services over time for any remaining unit of capital.

[6]Bjerregaard and Groes (1976).



(a) sudden death

$S_\theta = 100$ for $\theta < \bar{\theta}$

$S_\theta = 0$ for $\theta > \bar{\theta}$

$d_\theta = f_\theta = 100$ for $\theta = \bar{\theta}$

(b) linear depreciation

$S_\theta = 100 - c \cdot \theta$

$d_\theta = -c$

$$f_\theta = \frac{-c}{100 - c \cdot \theta}$$

(c) combination of (a) and (b)

(d) z-shape

Three versions are mentioned:

i. Logistic function:

$$S_\theta = \frac{h}{1 + i\, e^{jh\theta}}$$

$f_\theta = j(S_\theta - h)$ (linear with regard to S)

ii. "IOWA-function":

$$f_\theta = n\left(1 - \frac{\theta^2}{l^2}\right)^m \text{ (bell-shaped)}$$

$$S_\theta = 100 - \int_0^\theta f_\theta\, d_\theta$$

iii. "Kaergaard-function"

$S_\theta = 100\, e^{-(p\theta - q)\theta}$

$f_\theta = 2p\theta - q$ (linear with regard to θ)

(e) exponential decay

$S_\theta = 100\, e^{-n\theta}$

$f_\theta = r$

(f) combination of (a) and (e)

(g) concerns the value of services in any year (U_θ) and might be constructed out of two logistic functions.

Figure 3. Possible survival curves*

Many assumptions about survival curves have been tried. Apart from the above mentioned "sudden death", the most commonly used have been linear and exponential depreciation. None of them appear very likely, but they are easy to handle. The exponential method especially has the advantage that investment dates of a given capital stock are unnecessary.

This is however hardly adequate for our purpose. For example, Figure 4 illustrates that "death" frequency does not develop horizontally. And if you believe that frequency is dependent upon age of the machinery, fluctuations in gross investments will result in later fluctuations in the need for reinvestments (the echo effect).

In Denmark we have experienced considerable changes in investments since the thirties. The small wartime investments might have been expected to mean small reinvestments in the fifties. On the other hand gross investments rose three-fold from the mid-fifties to the beginning of the sixties, which should result in a considerable increase in the necessary reinvestments in the seventies (believing that average life of machinery is around 15 years).

Fluctuations in investments also differ much from industry to industry. Even within the relatively stable period of 1960–72, investments in some industries were very unstable. As lifetime and homogeneity of fixed capital also differs, the choice whether to disregard the age structure depends much upon the industry in question.

Actually we did try "linear depreciation" (Figure 3b and 3c), but the resulting estimates of a and q were not much different from the original version, nor was the explanation better. So we concluded that linear depreciation gave no better description than sudden death.

The most likely survival curve is usually believed to be some sort of a z-curve (Figure 3d). In estimating this z-function, it is common to try a logistic function, where death frequency is a linear function of the stock (3d(i)).

In Danmarks Statistik[7] work is under way, using survival curves originated by Robley Winfrey in Iowa in the 1920s and 1930s and later developed in Sweden and Denmark.[8] A simple symmetric version of this "Iowa-function" is shown in 3d(ii). Elaborated versions produce Left and Right modes.

Instead of using a function found in another part of the world 40 years ago, it might be possible to use newer findings about cars in Denmark. Quite a number of survival functions for cars have been found, as the statistics on this field are more complete and easier to handle than those of more elusive concepts like machinery or equipment. Niels Kaergaard has published a survival curve for Denmark.[9] An

[7]The Central Statistical Bureau in Denmark.
[8]Winfrey (1935), Cederblad (1971) and Larsen (1974).
[9]Kaergaard (1970 and 1975) (See Figure 4).

*The following variables are used:

S_θ: number of remaining machines at year θ in percentage of the total number of machines purchased in year zero (I_0). S_θ is never negative.

d_θ: ($=\Delta S_\theta$) fall in the number of machines at year θ in percentage of I_0. d_θ is never positive.

f_θ: frequency of death: $\frac{d_\theta}{S_\theta} \sim \frac{dS/d\theta}{S_\theta}$ Like d_θ, f_θ is never positive.

u_θ: value of services at year θ in percentage of maximum value (full utilization).

c, h, i, j, l, m, n, p and q are constants.

attempt was made to use this for machinery in Denmark. Now cars obviously have one way of dying not common to other pieces of machinery—they smash. This is presumably why death frequency is above zero the first 3–4 years. If we ruled out automobile accidents, it looks like the frequency curve was a linear function of the age (θ) of the car, something like:

$$f_\theta = 2\tfrac{1}{3}\theta - 9\tfrac{1}{3} \quad (f_\theta \geq 0) \quad \text{or} \quad S_\theta = 100\exp[-(1\tfrac{1}{6}\theta - 9\tfrac{1}{3})\theta]$$

Inspired by this we have chosen a frequency function as follows:

$$\text{for} \quad \begin{array}{lll} f_\theta = 0 & f_\theta = 1.5\theta - 7.5 & f_\theta = 30 \\ 0 \leq \theta \leq 5 & 5 < \theta \leq 25 & 25 < \theta \end{array}$$

where the frequency (or death probability) is expressed in percentages.

It might be correct to let f_θ fall again at some point,[10] but as there is no more than 2 percent of the stock left after 26 years, any great speculation about the unknown seems not worth the trouble. Fitting some of the more well-known functions to Kaergaard's frequencies has been tried without great success.

Using the survival pattern of cars as a pattern for that of total machinery is of course dangerous. Car accidents have been discussed; it could be added that cars belong to the class of "moving" machinery. Also the well organized market for second hand cars, public control and high turnover taxes are special features. But more than anything else, all cars belong to one mass-produced, relatively homogeneous category. Machinery and equipment consist of an enormous number of different categories—each with its own life-time.

Throughout our study it has to be assumed that life-time was given in advance and unaffected by economic conditions. This is not so; at least in Denmark total repairs and maintenance are above total reinvestments and make it possible to lengthen lifetime[11] when liquidity or profit is low.

So far we have discussed the life-time of machinery only. Another problem is the development of utility with age. Also here, Kaergaard has come to some interesting results. Using the sales price at a particular time on the Danish market for Volkswagens of different ages, and presuming that all buyers of used cars knew the existing frequency curve, Kaergaard could find the annual value of the use of an existing car at different ages. Prices of used cars, C, followed a logarithmic pattern:

$$\log C_\theta = \underset{(0.01)}{4.33} - \underset{(0.001)}{0.069} \cdot \theta \quad \text{or} \quad C_\theta = 21.500 \cdot 1.17^{-\theta} \text{ (Danish kr.)}$$

$$R^2 = 0.998$$

Combining this with the probability (p_θ, given in Figure 4) that a car will survive in year θ, the price should be

$$C_\theta = u_\theta + p_\theta \cdot C_{\theta+1}(1+r)^{-1}$$

[10]Like the "Iowa-function".

[11]Repairs are also of importance for the magnitude of q; see Bjerregaard and Groes (1976).

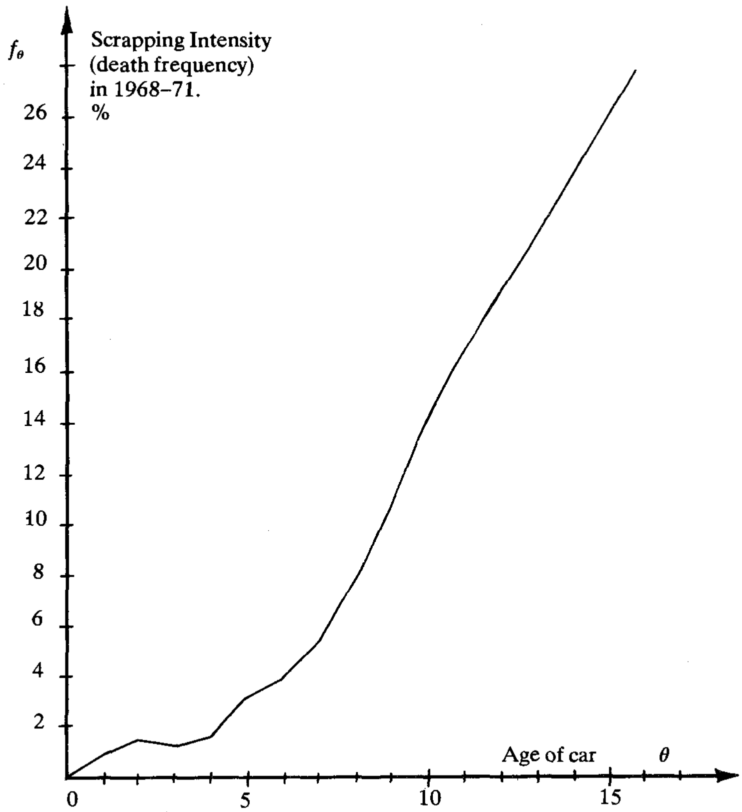


Figure 4. Death frequencies of cars in Denmark.

where u_θ is the utility of the car in year θ and r is the real interest rate (here 5 percent per annum).

Knowing prices and probabilities, utility can be found as

$$\log u_\theta = 3.61 - 0.049 \cdot \theta \quad \text{or} \quad u_\theta = 4100 \cdot 1.12^{-\theta} \text{ (Danish kr.)}$$

Inspired by this, we have claimed exponential decay of 10 percent p.a. of utility of surviving machinery, i.e. $u_\theta = u_1 \cdot 1.10^{-\theta}$.

The objections against the use of life expectancy of a Volkswagen could well be raised also against the use of its "utility-function" for all machinery. Furthermore, for machinery there is no equivalent to the social prestige a new car gives. Fortunately, the old Volkswagens might resemble other machines more than other cars in this respect.

But a car is a finished product when leaving the salesman, the value of its services being more or less at a peak at that very moment. On the other hand, a machine or set of machinery to be fitted into a production line does not run smoothly from the start. Interviews with managers in Danish industry leave you with the impression that quite some time and expenses will be spent before new machinery functions as designed (Figure 3g).

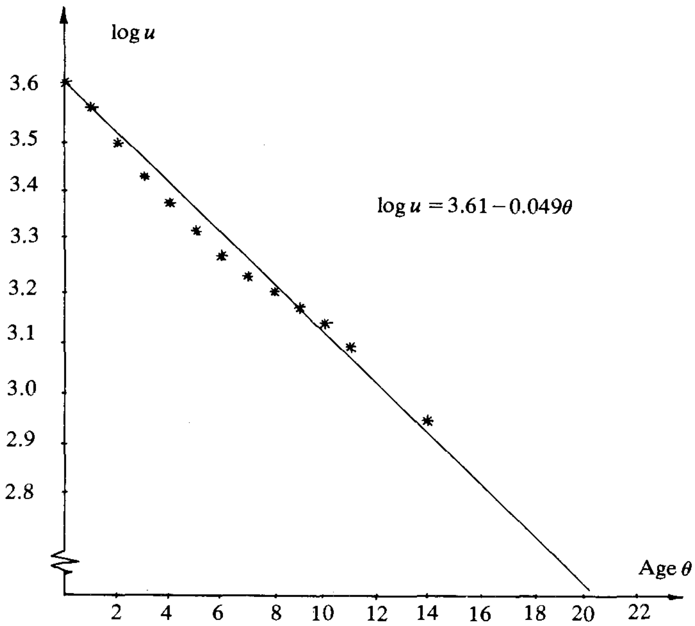


Figure 5. Utility of a car (u) as a function of its age (θ).

Alternative Estimates of Parameters

The original version included the implicit assumption that prices of capital goods were determined solely by suppliers of these goods. Now we further have to assume that investors buy *machinery* until the discounted value of a machine's services at the time of purchase equals the price of that machine, i.e. that the investment at year 0 is:

$$I_0 = \sum_{\theta=1}^{30} u_1 S_\theta (1+d)^{-\theta}(1+r)^{-\theta} = \sum_{\theta=1}^{30} u_1 S_\theta \cdot 1.144^{-\theta}$$

where d is the annual rate of fall in utility of the surviving stock (10 percent) and r the rate of discount (4 percent). Given I_0 and S_θ we can find u_1 and thereby u_θ. With machine investments since 1920 we are able to calculate the annual value of machine services since 1950, assuming a constant fall in utility. Another possibility is to assume no such fall, i.e. $I_0 = \sum u_1 S_\theta (1+r)^{-\theta}$. As a by-product the calculation of the value of services also produces the value of stocks (the value of future services, which equal past investments).

Economic conditions play a considerable role in the death frequency of existing *buildings* (through repairs etc.) The age structure of buildings therefore is of less interest, especially for the older part of the building stock. This is rather fortunate, as knowledge about the ages of buildings is limited. Reinvestments in buildings are small in the urban sector, and mostly dependent upon alternative use of the land, not upon the age of existing buildings. So if you are not willing to ignore the reinvestment problem (as we first did) the exponential method seems just as good as anything else, to decide the decrease both in stock and in value of use (as we did later). Using the benchmark figures of 1950 we have assumed a constant death frequency of $1\frac{1}{2}$ percent p.a. and an annual fall in the value of the services of 2 percent of the stock. Of course, we could also assume no fall in the value of services.

Given the depreciation functions described above, it is possible to construct four different capital concepts, i.e.:

α services with an annual fall in utility of the remaining stock,
β services without any such fall in utility
γ stock with fall in utility (equivalent to α),
∂ stock without fall in utility (equivalent to β).

In the earlier calculations we assumed that all investments in any year were fully productive in that same year. This of course implies a fault; as an alternative we now assumed that all investments in any year were fully productive the next year, but not at all before. So on average we got a lag of $\frac{1}{2}$ year from the time of investment to the time when the investment good took part in production (Figure 3g).

Finally, we estimated the parameters for two periods: 1953–72 including the years of stagnation 1953–57, and 1958–72—a period of high relatively constant growth.

The result has been 16 regression estimates of the parameters a and q in the equation: $\dot{y}_t = a \cdot \dot{k}_t + q$. They are shown in Table 1 together with the original version. We still assumed a jump from q_1 in 1965 to q_2 in 1966, estimated by a dummy in 1966–72 (illustrated in Figure 2). Below the estimated coefficients in Table 1 are given the t-values.

The first impression of Table 1 is the great variation in the estimated parameters, from 0.69 to 0.24.

Three of the estimates *without lag* in *1953–72* produce a negative q_1, which is hard to explain. The exception is (3) where the explanation also is the most acceptable, especially as regards the t-value for q_1.

Estimating for *1958–72 without lag* makes q_1 positive, which is not strange, as we avoid 1953–58, a group of years with low $\dot{y}_t$. No wonder DW is better in the short period.

If we estimate for *1953–72 with lag*, q_1 is always positive, but once more the "stock with fall in utility" (11) proves the best (but not as good as (3)).

Only estimates for *1958–72 with lag* show coefficients based on capital services ((13) and (14)) that fit just as well as those related to stocks. From an economic and methodological point of view these results are preferred, and the value of a is consistent with distribution theory. On the other hand it is not easy in any way to distinguish among the last 4 equations.

Obviously, the choice of estimation period is more important than methods of depreciation and capital concepts. Also choice of lag is important, but mostly because it involves the estimation period of $\dot{k}_t$. By refining the methods we have improved the tests of fit but also demonstrated more clearly that the assumed (and admittedly very crude) production function is not stable over time. You cannot

TABLE 1.

		a	q_1	Dummy	q_2	R^2/SE	D.W./F-test
A. Original version: stock, sudden death, estimation period: 1958–72.							
		0.52	0.86	0.79	1.65	0.61	2.61
		2.018	0.623	1.521		0.667	12.05
B. No lag, estimation period: 1953–72							
1)	α	0.6637	−0.1207	1.2892	1.17	0.7622	1.2283
		4.4757	−0.1701	3.2152		0.6990	31.4428
2)	β	0.6915	−0.5104	1.1236	0.61	0.6995	0.9816
		3.5089	−0.5072	2.2631		0.7857	23.1169
3)	γ	0.5420	0.9034	1.0785	1.98	0.7949	1.3499
		5.0937	2.0705	2.7693		0.6491	37.8220
4)	∂	0.6430	0.0272	1.0064	1.03	0.7339	1.0761
		4.0118	0.0361	2.1281		0.7394	27.1959
C. No lag, estimation period: 1958–72							
5)	α	0.3848	1.6293	1.2231	2.85	0.6688	1.7971
		2.3806	1.8926	3.5772		0.5935	15.1350
6)	β	0.4110	1.4260	1.0391	2.47	0.6744	1.7703
		2.4440	1.5493	2.7712		0.5885	15.5009
7)	γ	0.3084	2.2041	1.1542	3.36	0.6537	1.7365
		2.2130	3.2762	3.1454		0.6069	14.2138
8)	∂	0.3624	1.8023	1.0385	2.84	0.6645	1.7422
		2.3326	2.2362	2.7039		0.5974	14.8643
D. Lag, estimation period: 1953–72							
9)	α	0.4729	0.8124	1.3611	2.17	0.6942	1.0614
		3.3929	1.2331	2.9128		0.7942	22.5670
10)	β	0.5122	0.4487	1.2295	1.68	0.6566	0.9124
		2.8966	0.5069	2.2751		0.8416	19.1661
11)	γ	0.4347	1.3877	1.0878	2.48	0.7483	1.1969
		4.1994	3.3492	2.4335		0.7206	29.2397
12)	∂	0.5056	0.7385	1.0433	1.78	0.6972	0.9976
		3.4334	1.0991	2.0018		0.7904	22.8697
E. Lag, estimation period: 1958–72							
13)	α	0.2861	2.1765	1.1828	3.36	0.6658	1.8647
		2.3404	3.3933	3.2883		0.6044	14.9440
14)	β	0.3307	1.8953	1.0221	2.92	0.6733	1.8101
		2.4253	2.5919	2.6308		0.5975	15.4287
15)	γ	0.2429	2.5476	1.1191	3.67	0.6563	1.7891
		2.2353	4.9573	2.9316		0.6129	14.3674
16)	∂	0.2976	2.1731	1.0048	3.18	0.6658	1.7712
		2.3423	3.3833	2.5085		0.6042	14.9548

conclusively tell whether a is 0.5 or 0.3. One dummy has been introduced since 1966, but one may well ask why we did not put in more dummies, e.g. one since 1958.

Forecasting Investments

Although the Cobb–Douglas production function does not produce a convincing picture of the production process, it might be a useful tool for prognosis. Table 2 show 16 gross investment forecasts, based on the 16 relations of Table 1. In all of them Y is expected to grow at 4.2 percent p.a. and L to fall 1 percent p.a. These assumptions are taken from the public plan, and are very near the actual development of the sixties.

TABLE 2

No.	a	q_2	K_{72}	K_{87}	NI_{87}	GI_{87}
			bill. Danish kr.—1960 prices			
1)	0.66	1.17	58.8	142.7	7.8	18.4
2)	0.69	0.61	86.2	219.8	12.9	20.6
3)	0.54	1.98	58.8	133.0	7.0	18.0
4)	0.64	1.03	86.2	207.8	11.7	19.4
5)	0.38	2.85	58.8	151.9	8.8	20.0
6)	0.41	2.47	86.2	230.5	14.1	22.0
7)	0.31	3.36	58.8	141.6	7.9	19.6
8)	0.36	2.84	86.2	221.5	13.3	21.3
9)	0.47	2.17	56.4	219.1	13.0	22.0
10)	0.51	1.68	82.1	221.5	13.8	22.7
11)	0.43	2.48	56.4	137.3	8.2	20.2
12)	0.51	1.78	82.1	207.3	13.0	21.0
13)	0.29	3.36	56.4	154.2	9.5	22.1
14)	0.33	2.92	82.1	233.0	15.1	24.5
15)	0.24	3.67	56.4	231.4	9.9	23.1
16)	0.30	3.18	82.1	222.2	14.9	23.3

K_{72}: the value of capital in 1972. NI_{87}: net investments in 1987. GI_{87}: gross investments in 1987.

The original version led to gross investment of 19.6 bill. Danish kr. in 1987 (1960 prices), but we published a forecast of 18–23 bill. Danish kr. Table 2 indicates that the new estimates do not alter this interval. On the contrary, the 16 forecasts yield rather uniform gross investment (not surprisingly they differ much less than capital and net investment figures).

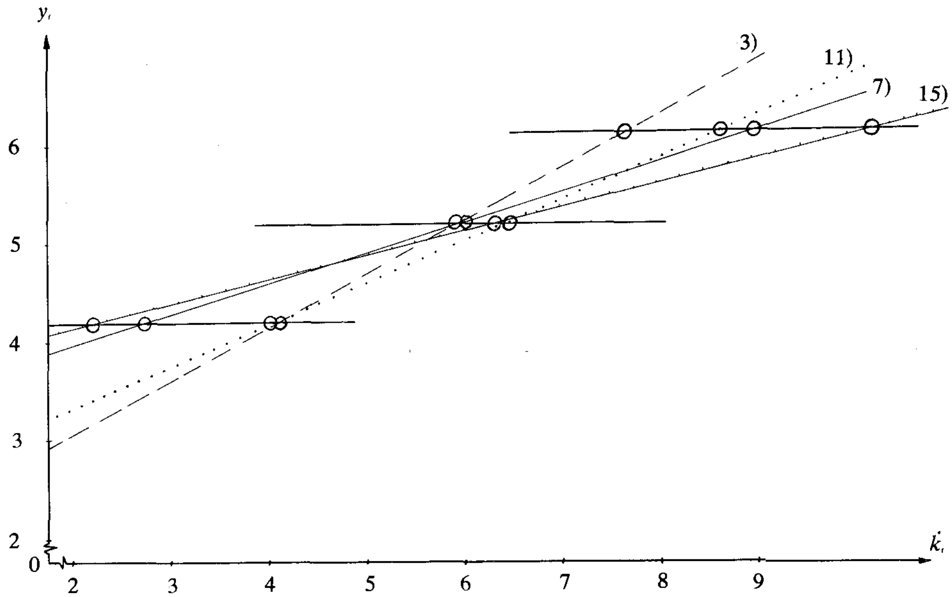


Figure 6. Four possible connections between relative change in labour productivity and capital intensity.*

*The numbers of the sloping lines refer to the numbers of the CD-functions in Table 1 and Table 2.

The fact that these forecasts appear rather robust is of course to a great extent explained by the underlying assumptions. As long as we choose $\dot{y}_t$ to be around 5 percent as it was in the sixties (Figure 2) the value of k_t will not differ much, as illustrated in Figure 6. But as our assumption of growth changes, the differences increase.

TABLE 3

No.	a	K_{72}	$\Delta Y_t/Y_t$	K_{81}	NI_{81}	GI_{81}	K_{87}	NI_{87}	GI_{87}
3)	0.54	58.8	3.2	74.9	2.0	8.5	87.7	2.3	9.9
			3.7	85.1	3.4	10.6	108.6	4.3	13.6
			4.2	96.3	5.1	13.0	133.0	6.9	18.0
			4.7	108.4	7.0	15.7	161.4	10.3	23.4
7)	0.31	58.8	3.2	64.7	0.6	6.5	68.5	0.6	6.8
			3.7	81.0	2.8	9.7	99.8	3.3	12.0
			4.2	100.4	5.7	13.9	142.2	7.9	19.7
			4.7	123.3	9.6	19.2	198.6	15.0	30.8
11)	0.43	56.4	3.2	70.5	1.7	8.1	81.0	1.9	9.3
			3.7	82.7	3.6	10.9	106.5	4.5	14.0
			4.2	96.5	5.8	14.2	137.3	8.2	20.2
			4.7	111.9	8.6	18.3	174.7	13.3	28.4
15)	0.24	56.4	3.2	58.1	0.14	5.5	58.8	0.07	5.5
			3.7	70.0	2.5	8.7	95.7	3.3	11.9
			4.2	102.4	6.9	15.7	150.9	9.9	23.1
			4.7	133.2	13.0	24.2	231.4	22.0	41.6

All forecasts assume $\frac{\Delta L_t}{L_t}$ to be -1 percent p.a. $\Delta Y_t/Y_t$ is measured in percent p.a.

All capital and investment figures are in bill. Danish kr. (1960 prices).

Table 3 shows in more exact terms that in case of low elasticity (a) a low growth ratio ($\Delta Y_t/Y_t = 3.2$ percent p.a.) leads to very small net investment, while a high growth (4.7) demands very high investment. On the other hand a high elasticity leads to relatively high net investment during slow growth, but rather low investments when growth is high. It is no wonder that the low slope means relatively great change in $\dot{k}_t$ for a given change in $\dot{y}_t$. But Table 3 also illustrates that the connection is not all that simple as regards gross investment, because the size of K_t and reinvestments enter the picture.

Summary

No way of depreciation proved itself obviously correct or useful. More important than refining these survival functions should be distinguishing between capital goods of different industries and of different categories such as buildings, cars, ships, as well as different types of machines (e.g. moving, non-moving) and equipment (metal, non-metal). But even more important would be putting the disaggregated capital figures into a wider production function, including more factors than are mentioned above.

Using the crude Cobb–Douglas function has not enabled us to find any stable connection between crude measures of production and inputs of capital and labour. In this function, *all* capital is supposed to be homogeneous, *all* technical progress is disembodied. This has not been the case in post-war Denmark, so embodied technical progress has influenced a, possibly so that q measures only part of the technical progress, dependent upon period and technique of estimation (dummies).

The general possibility of forecasting capital needs in the long run is therefore modest. Only if the relative growth of production and labour in the forecast period is close to that of the estimation period might the forecasts be of some value.

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