THE SUITABILITY OF THE DIVISIA INDEX FOR THE MEASUREMENT OF ECONOMIC AGGREGATES

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This paper considers the properties of the Divisia, or chain-link, index, as they relate to the argument that this is the most appropriate index for use in studying the sources of economic growth. The great advantage of the Divisia index is alleged to be its "accuracy", that is, its capacity to combine time series of prices and quantities to give a true reflection of the height of a utility or production function over time. The paper shows that there are circumstances where the confidence in the accuracy of the Divisia index is justified, but that the conditions required are very restrictive and typically do not obtain in the contexts where the Divisia index is used. Misplaced confidence in the Divisia index has led to errors of interpretation that might otherwise have been avoided, and has given rise to a distorted view of the process of economic growth.

The Divisia, or chain-link, index is widely believed to be the best index measuring economic aggregates. In particular, the index is believed to be appropriate for studies of the sources of economic growth where the problem is to apportion observed economic growth into that which can be accounted for by increased availability of factors of production and that residual, commonly referred to as aggregate technical change, which must be attributed to invention, growth of knowledge, good luck, efficient management, etc.[1]

The great advantage that the Divisia index is alleged to have over all other indices is what we shall call "accuracy".[2] It is the capacity to seek out and faithfully represent the function in which we are interested. Thus, under certain conditions the Divisia can combine time-series of prices and quantities to give a true reflection of the height of a utility function or of a production function as the case may be.

In this paper we review the properties of the Divisia index to determine when it is likely to be accurate and when it is not. Then we consider how the index works out in practice in the measurement of aggregate technical change. We show that there are circumstances when the confidence in the accuracy of the Divisia index is justified but that the conditions required for the Divisia index to be accurate are very restrictive and typically do not obtain in the contexts where the Divisia index is used. In practice, the Divisia index number is no better

[1]Though the Divisia index has been used for some time in productivity studies, the clearest and most perceptive statement of this point of view appears in D. Jorgenson and Z. Griliches, "The Explanation of Productivity Change", *The Review of Economic Studies*, 1967, pp. 249–283. The results of this study have been modified and extended in subsequent papers by L. R. Christensen and D. Jorgenson entitled "The Measurement of U.S. Real Capital Input" (this journal, December 1969), and "U.S. Real Product and Real Factor Input, 1929–1967" (this journal, March 1970).

[2]"The main advantage of a chain index is in the reduction of errors of approximation as the economy moves from one production configuration to another. If weights could be changed continuously, errors of this type would be eliminated. This property of Divisia indexes, called 'invariance' by Richter, characterizes no other index number. Discrete chain-linked index numbers reduce errors of approximation to a minimum. For this reason chain indexes rather than a single base period should be used in real product accounting and productivity measurement." (D. W. Jorgenson and Z. Griliches, *Review of Income and Wealth*, (1971), pp. 227–229.

than other, simpler, index numbers. Misplaced confidence in the Divisia index has led to errors of interpretation that might otherwise have been avoided and has given rise to a distorted view of the process of economic growth.

The Properties of the Divisia Index

The Divisia index is defined as follows: Suppose that quantities $q_i(t)$ and corresponding prices $p_i(t)$ are all functions of time. The Divisia quantity index for the year t is given by

$$D(t) = D(0)\exp\left[\int_0^t \frac{d(\log D(t))}{dt}\,dt\right] \tag{1}$$

where

$$\frac{d[\log D(t)]}{dt} = \sum_{i=1}^{n} v_i(t)\frac{d(\log q_i)}{dt}, \tag{2}$$

where $v_i(t)$ is the value share of the i quantity defined as $(p_i(t)q_i(t))/(\sum_{i=1}^{n} p_j(t)q_j(t))$, and where $D(0)$ arbitrary. The corresponding Divisia price index does not concern us in this paper.

To say that the Divisia index number is accurate[3] is to assert that there is a true number, out in the world, and that the Divisia index tells us what it is. Let us designate that number as X. As we are concerned with quantity indices, we suppose that the value of X at time t is a function of quantities consumed

$$X(t) = X(q_1(t), \ldots, q_n(t)) \tag{3}$$

We shall refer to the function X as the price-generating function, and in working out the properties of the Divisia index, we shall suppose that prices are proportional to first derivatives of X.

$$p_i(t) = \lambda(t)\frac{\partial X(t)}{\partial q_i(t)} \tag{4}$$

where λ is independent of i, though it usually varies with t. The economics of the assumption in equation (4) is, of course, that, if X is utility and the p_i are prices of goods, the consumer makes equation (4) come true when he maximizes utility subject to his budget constraint; or, if X is aggregate output and the p_i are payments to factors of production, the firm makes equation (4) come true when it maximizes profit.

Given equation (4), we can establish three propositions about the accuracy of the Divisia index:

a. The Divisia index is constant if and only if X is constant, and the direction of change of the Divisia index at any time is the same as the direction of change of X.
b. If X is homogeneous in degree 1, and if we set $D(0) = X(0)$, then $D(t) = X(t)$ for all t and for all continuous functions $q_i(t)$.

[3]Marcel Richter has referred to this property of the Divisia index as "invariance". See "Invariance Axioms and Economic Indexes", *Econometrica*, 1966, pp. 739–755. I prefer the term "accuracy" to the term "invariance" because I want to evaluate the performance of the Divisia index when the true number that the index is designed to reflect is changing over time.

c. If X is homothetic, the Divisia index is a perfect ordinal indicator of X in that the value of $D(t)$ depends only on the initial and terminal quantities, $q_i(0)$ and $q_i(t)$, and is independent of the path between.

Proposition (a) is derived by expressing the derivative of $\log X$ as a function of derivatives of all $\log q_i$.

$$\frac{d}{dt}(\log X) = \sum_{i=1}^{n} \left(\frac{(\partial X/\partial q_i)q_i}{X}\right)\frac{d}{dt}(\log q_i). \tag{5}$$

Combining equations (4) and (5), we see that

$$\frac{d}{dt}(\log X) = \left(\frac{\sum_j p_j q_j}{\lambda X}\right)\sum_i \left(\frac{p_i q_i}{\sum_j p_i q_i}\right)\frac{d}{dt}(\log q_i)$$

$$= \frac{\sum_j p_j q_j}{X\lambda}\frac{d}{dt}(\log D) \tag{6}$$

which establishes that X is an increasing function of D.

Proposition (b) is a consequence of the fact that

$$X = \sum_{i=1}^{n} \frac{\partial X}{\partial q_i} q_i \tag{7}$$

wherever X is homogeneous in degree 1. From equations (4), (6) and (7), it follows that

$$\log X(t) - \log X(0) = \int_0^t \sum_{i=1}^{n} \frac{\partial \log X}{\partial q_i}\frac{dq_i}{dt}dt$$

$$= \int_0^t \sum_{i=1}^{n} \frac{1}{X}\frac{\partial X}{\partial q_i}\frac{dq_i}{dt}dt$$

$$= \int_0^t \sum_{i=1}^{n} \left(\frac{\lambda}{\sum_{i=1}^{n} p_i q_i}\right)\left(\frac{p_i}{\lambda}\right)\left(q_i\frac{d\log q_i}{dt}\right)dt$$

$$= \int_0^t \sum_{i=1}^{n} v_i\frac{d\log q_i}{dt}dt$$

$$= \int_0^t \frac{d}{dt}(\log D(t))\,dt$$

$$= \log D(t) - \log D(0) \tag{8}$$

so that

$$\frac{X(t)}{X(0)} = \frac{D(t)}{D(0)} \quad \text{for all } t, \tag{9}$$

signifying that the Divisia index, D, is exactly like the function, X, it is intended to represent except for a change in scale, and that growth rates of X and D between any pair of years are the same.

Proposition (c) is established in a similar way. A function $X(q)$, where q is the vector $(q_1, \ldots, q_n)$, is said to be homothetic if X is a monotically increasing transformation of a function Y that is homogeneous in degree 1 in the arguments

$q_1, \ldots, q_n$, that is if

$$X = f(Y) \quad \text{and} \quad Y = Y(q) \quad \text{and} \quad f' > 0 \tag{10}$$

where Y is a function such that

$$Y(\alpha q) = \alpha Y(q) \tag{11}$$

for any positive scalar α.

By analogy with the argument leading to equation (9), it immediately follows that

$$\frac{Y(t)}{Y(0)} = \frac{D(t)}{D(0)} \tag{12}$$

in which case

$$\frac{X(t)}{X(0)} = \frac{f^{-1}[D(t)]}{f^{-1}[D(0)]} \tag{13}$$

proving that the Divisia index is a monotonically increasing function of $X(t)$.

It should be noted at this point that the Divisia index always possesses the proportionality property along rays from the origin. An index possesses the proportionality property with respect to a set of arguments $q_1, q_2, \ldots, q_n$, if the index grows by a factor α between the year 0 and the year t whenever $q_i(t) = \alpha q_i(0)$ for all i. It follows directly from the definition of the Divisia index in equations (1) and (2) that the Divisia index has the proportionality property whenever $q_i(\tau) = \alpha(\tau)q_i(0)$ for every i and for every τ between 0 and t because in that case all of the derivatives $d/dt\,(\log q_i)$ in equation (2) are the same and the value of $d/dt\,(\log D(t))$ is independent of the value shares $v_i(t)$.

It is a corollary to proposition (c) that the Divisia index possesses the proportionality property with regard to homothetic functions. Since the Divisia index always possesses the proportionality property along any line from the origin, and since proposition (c) guarantees that the Divisia index is independent of the path from $q(0)$ to $q(t)$, the Divisia index must possess the proportionality property regardless of the path from $q(0)$ to $q(t)$.

Though the Divisia index has some excellent features it is not robust and can perform very badly if the assumptions under which it is constructed are violated by the data to which it is applied. First, all of the nice properties we have discussed—accuracy, proportionality, and even independence of the path from $q_i(0)$ to $q_i(t)$—break down if the price-generating function in equation (3) is not homothetic. This happens because the Divisia index is a line integral and because the change in the value of a line integral between any two points is as a rule dependent on the path of integration. The case where $n = 2$ is illustrated in Figure 1; a line integral such as the Divisia index in equation (3) commencing at the point $q(0)$ and continuing to $q(t)$ along path 1 need not have the same value as the line integral commencing at $q(0)$ and continuing to $q(t)$ along path 2.[4] On each path

[4]Both Richter's article and Jorgenson and Griliches' article cited above contain footnotes, on pages 751 and 253 respectively, indicating that the dependence of the Divisia index on the path between $q(0)$ and $q(t)$ had been pointed out to the authors by W. M. Gorman. Richter goes on to suggest that the dependence of the Divisia index on the path might be a good reason for not using the Divisia index and for using a fixed-weighted index instead.

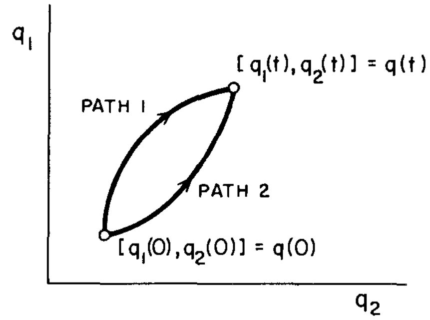

Figure 1

in Figure 1 the Divisia index rises or falls whenever $X(q_1, q_2)$ rises or falls, but the overall rise or fall of the Divisia index need not be the same.

A striking example of the dependence of the Divisia index on the history of the vector $q(t)$ when the price-generating function X is not homothetic is illustrated in Figure 2. Think of X as utility and let X depend on amounts consumed of two commodities, q_1 and q_2. A pair of non-homothetic indifference curves are designated as U^0 and U^1. The lines OAB and OCD are rays through the origin cutting U^0 at A and C and cutting U^1 at B and D. Because (and only because) indifference curves are not homothetic, we can suppose that $AB = 1/4OA$ and that $CD = OC$. Set the value of the Divisia index at 1 for the point A, and consider its value at the point B. It follows from the proportionality property of the Divisia index along rays from the origin that the value of the index at B assessed along the line AB is 1.25 because $AB = 1/4OA$. However the value of the Divisia index at B rises to 2 when assessed along the indirect route $ACDB$. By proposition (a), the Divisia index is constant along the paths AC and DB because both of these paths lie on indifference curves, and the value of the Divisia index doubles between C and D because $CD = OC$. Thus, the value of the Divisia index at B could be 1.25 or 2 depending on which path is followed between A and B. Dependence on the path disqualifies the Divisia index for service as an economic indicator except when we know that the price-generating function is homothetic.

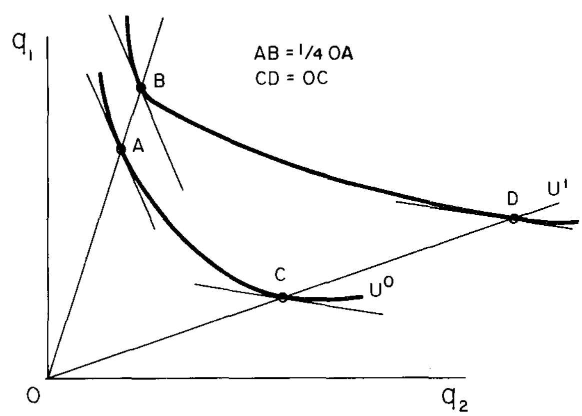

Figure 2

A disturbing implication of the dependence of the Divisia index on the path of integration is that the Divisia index need not possess the "circularity property"; an economy commencing at $q(0)$, proceeding to $q(1)$ by path 1, and returning to $q(0)$ by path 2 could experience a net change in its Divisia index. The Divisia index could increase, or decrease as the case may be, at each journey around the closed path and could be made as large or as small as we please without any net change in quantities consumed.

Second, the Divisia index is defined in equation (2) with respect to infinitesimal changes in q_i, but it can only be computed for finite increments in q_i in accordance with an approximation to equation (2) such as

$$\Delta(\log D^t) = \sum_i v_i^t \Delta(\log q_i^t) \tag{14}$$

where $\Delta(\log D^t)$ and $\Delta(\log q_i^t)$ are finite increments over the course of the accounting period.[5]

Third, and most important, an essential step in the proof that the Divisia index is accurate was the assumption, expressed in equation (4), that market prices reflect first derivatives of the function X. The assumption might prove false for a number of reasons. If we think of X as utility, the violation of equation (4) means that indifference curves are unstable over time, perhaps because taste itself is intrinsically unstable or perhaps because of habit formation. A sudden fall in the relative price of good A and good B may not have its full effect on consumption of good A all at once; consumption may increase gradually as people learn how to incorporate good A into their consumption patterns. Should this be the case, the relative price of good A and good B is not uniquely determined by quantities consumed and equation (4) is violated even though taste may still be considered invariant in some broad sense of the term. The situation on the supply side of the economy is generally worse, for technical change is constantly altering the shapes of the isoquants and of the production possibility curve. Though isoquants may be homogeneous at any moment of time, there is no reason to suppose a consistent relationship among the time-series of relative prices of factors of production and the time-series of amounts available such as would be required to impart nice properties to the Divisia index of total factor productivity.

This point can be expressed more intuitively as follows: The valuable property of the Divisia index is that, if the time-series of prices and quantities are generated by a homothetic function, then the Divisia index will record the height of that function. However, since the Divisia is trained to record values of homothetic functions, since prices and quantities are all that can be observed, and since the Divisia index cannot know whether the time-series of prices and quantities reflect a homothetic function or not—the Divisia index will, whenever possible, record the height of a real or imaginary homothetic function consistent with the time-series of prices and quantities regardless of whether that function makes any economic sense.

[5]The possibility that small errors which arise when equation (2) is approximated by equation (14) may cumulate, rather than cancel out, has been raised by J. M. Keynes in *A Treatise on Money*, Vol. 1, p. 118. This possibility is sometimes referred to in statistical literature as the tendency of the Divisia index to "drift" over time. See B. Mudgett, *Index Numbers*, pp. 65–79.

As we are concerned with properties of the Divisia index rather than with the full explanation of the sources of economic growth, it is sufficient to limit our attention to a model represented by equation (15) in which a homogeneous output Q is produced by means of m types of labour $L_1, L_2, \ldots, L_m$, and to ignore capital completely. The production function is

$$Q = F(L_1, L_2, \ldots, L_m, t) \tag{15}$$

where each L_i is a function of time. Time, t, itself is included as an argument in the production function to signify that F is actually a sequence of production functions which differ in form from year to year. In the year 0, an output $Q(0)$ was produced with inputs of labour $L_1(0), \ldots, L_m(0)$, and in the year t, an output of $Q(t)$ was produced with inputs of labour $L_1(t), \ldots, L_m(t)$.

Total technical change between year 0 and year t may be defined as the ratio of the output in the year t to the maximum output that could have been produced if the inputs available in the year t were combined in accordance with the production function of the year 0. Total technical change is the ratio $Q(t)/\hat{Q}(t)$ where, by assumption,

$$\hat{Q}(t) = F(L_1(t), \ldots, L_m(t), t) \tag{16}$$

and, by definition,

$$\hat{Q}(t) = F(L_1(t), \ldots, L_m(t), 0) \tag{17}$$

The claim that the Divisia index is accurate amounts to saying that the ratio $Q(t)/\hat{Q}(t)$ can be discovered by a Divisia index of output per unit of input computed exclusively from time-series of quantities, prices, wages, and inputs of the different types of labour. The rate of technical change may be defined as r_1 such that

$$e^{r_1 t} = Q(t)/\hat{Q}(t). \tag{18}$$

When there are only two types of labour, the model may be illustrated on an ordinary isoquant diagram. Suppose output is produced with uneducated labour, L_1, and educated labour, L_2. In Figure 3, input of L_1 is indicated on the vertical axis and input of L_2 is indicated on the horizontal axis, and the two curved lines are isoquants. It is assumed that all isoquants are convex, and that the production function is homogeneous in degree one.

In his study of the sources of economic growth, Denison[6] combined amounts of different kinds of labour into an index of the total input of labour by weighting numbers of workers of different educational attainments by their relative wages in some base year. If there are only two kinds of labour, educated and uneducated, if we take the labour supplied by an uneducated person to be the numeraire of labour, and if we choose the year 0 to be the base year, the real input of labour as assessed by Denison's method is:

$$L = L_1 + w^0 L_2 \tag{19}$$

[6]E. F. Denison, *The Sources of Economic Growth in the United States and the Alternatives Before Us*, Committee for Economic Development, 1962.

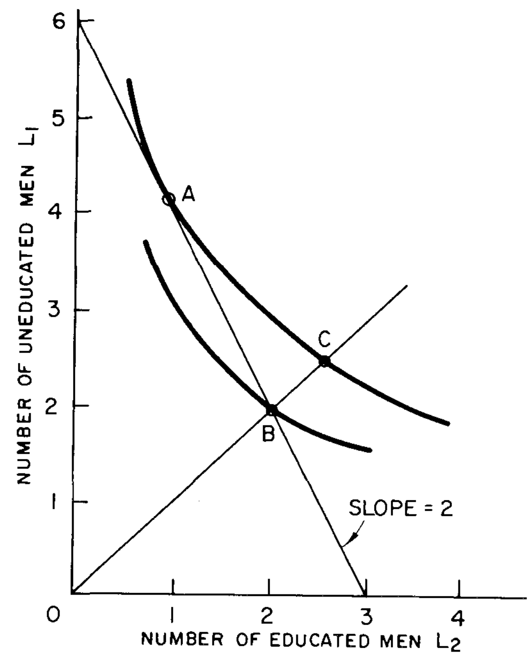

Figure 3

where

L is the total input of labour,
L_1 is the number of uneducated workers,
L_2 is the number of educated workers,

and

w^0 is the relative wage of educated and uneducated workers in the base year.

Equal quantities of labour as assessed by Denison's method may be illustrated in Figure 3 as combinations of L_1 and L_2 lying on a straight line of slope w^0.

Suppose that in the year 0, a unit of output is produced by the combined efforts of four uneducated men and one educated man, and the relative wage of educated and uneducated labour is 2. This combination of L_1 and L_2 is indicated by the point A, the corresponding value of L according to equation (19) is 6 units of labour, and the locus of all combinations of educated and uneducated labour yielding 6 units of labour is the straight line through A of slope 2. In the year 1, a unit of output is produced by two uneducated men and two educated men as indicated by the point B.

Has there been technical change? The answer to this question depends upon how we measure the real input of labour. If we follow Denison's method, we are obliged to say that there has been no technical change, for the combination of four uneducated men and one educated man and the combination of two uneducated men and two educated men are both counted as six units of labour in equation (19) when the relative wage is 2, so that output per unit of aggregate input is the same in the two years.

On the other hand, it is evident that the mix of inputs available in the year 1 would not be sufficient to produce one unit of output in the year 0 because the isoquant through the point A lies above the point B. Some technical change must have occurred between the year 0 and the year 1 to enable inputs of labour in the year 1 to produce what they did. The rate of technical change can be assessed as a relation between the isoquant through A and the isoquant through B, where the isoquant through B is understood to reflect the technology of the year 0 exclusively. As we have assumed that the production function in the year 0 is homogeneous in degree 1, the isoquants are necessarily homothetic and the measure of the reduction in real input from the year 0 to year 1 is CB/OB. As inputs of labour represented by the points A and B are requirements per unit of output, the measure of the increase in output per unit of input is CB/OB as well. These measures are entirely consistent with the rate of technical change as defined in equation (18). Only if C and B coincide, as would be the case if the elasticity of substitution between grades of labour were infinite, would the true measure of output per unit of input remain constant from year 0 to year 1. Otherwise, Denison's method of correcting the measure of labour input for the educational composition of the labour force overstates the increase in real input as assessed in relation to the set of base year isoquants, and understates the rate of technical change. The bias in what we are calling Denison's method is a special case of the well-known upward bias in any Laspeyres quantity index. Denison is fully aware of the bias and has stated that he would have attempted to correct for it by changing the weights in his indices from time to time if the data reqired for the change were available.

Jorgenson and Griliches attempt to eliminate the bias by measuring total labour input by a Divisia index. Though their computations are undertaken in a much broader context than is considered in this paper—they consider capital as well as labour and they take special care in the measurement of capital—their method of aggregating inputs can be illustrated with reference to a simple production function in equation (15) where output of a single commodity is produced with m kinds of labour. A Divisia index of total labour input, L, is constructed in accordance with equation (2) as

$$\text{(20)} \qquad \frac{d}{dt}[\log L(t)] = \sum_{i=1}^{m} v_i(t)\frac{d}{dt}[\log L_i(t)]$$

where $v_i(t)$ is the factor share of the ith type of labour

$$\text{(21)} \qquad v_i(t) \equiv [w_i(t)L_i(t)]\Big/\left[\sum_{j=1}^{m} w_j(t)L_j(t)\right]$$

and where $w_i(t)$ is the wage of the ith type of labour at time t. The rate of aggregate technical change, r_2, is now defined as the growth rate of Q/L.

$$\text{(22)} \qquad e^{r_2 t} \equiv \left(\frac{Q(t)}{L(t)}\right)\Big/\left(\frac{Q(0)}{L(0)}\right).$$

The Divisia index is correct in this context if and only if the rate of technical change r_2 in equation (22) is equal to the rate r_1, as defined in equation (18).

It is not difficult to construct a case in which these two rates are equal. Suppose technical change is Hicks neutral so that

$$Q = F(L_1, \ldots, L_n, t)$$
$$= A(t)L(L_1, \ldots, L_n) \tag{23}$$

where the function L, which may be thought of as the input of labour, is homogeneous in degree 1 and invariant over time. Suppose also that the wage of each type of labour in each year is equal to the value of the marginal product of labour

$$w_i(t) = P(t)\frac{\partial Q(t)}{\partial L_i(t)} = [P(t)A(t)]\frac{\partial L(t)}{\partial L_i(t)} \tag{24}$$

where $P(t)$ is the price of output in the year t. Thus equation (4) holds in this case with respect to the appropriate change of variables. On these assumptions, it follows from proposition b above that the Divisia index of total labour input in equation (20) is an accurate measure of total labour input in equation (23), except for a change in scale. The right-hand sides of equation (22) and equation (18) are, therefore, both equal to the ratio $A(t)/A(0)$ and the rates r_1 and r_2 are identical.

In justifying their use of the Divisia index, Jorgenson and Griliches state that "if price ratios are identified with marginal rates of transformation of a production function with constant returns to scale, the index will remain constant if the shift in the production function is zero".[7] This proposition is correct as we have shown, but it is not strong enough to guarantee that the Divisia index supplies an automatic correction for the bias in Denison's method. The impression that the Divisia index is in some sense ideal for productivity measurement is created by a confusion between necessary and sufficient conditions. While adherence of prices and quantities to an isoquant is sufficient to maintain a Divisia index constant, the constancy of the Divisia index is no guarantee that prices and quantities conform to an isoquant, and the Divisia index may perform very badly if the function it is intended to represent is changing its shape in the course of time.

In particular suppose technical change is not Hicks neutral. The function $\hat{Q}(t)$ in equation (17) is homogeneous in degree 1 in the inputs $L_1(t), \ldots, L_n(t)$ and would be correctly represented by a Divisia index if observed wages each year reflected first derivatives of the function as required by equation (4), that is, if

$$w_i(t) = \lambda(t)\frac{\partial}{\partial L_i(t)}[F(L_1(t), \ldots, L_m(t), 0)]. \tag{25}$$

But this equation will not be satisfied. What actually happens is that relative wages reflect marginal products of labour in the production function of the *technology of the year t.*

$$w_i(t) = \lambda(t)\frac{\partial}{\partial L_i(t)}[F(L_1(t), \ldots, L_n(t), t)] \tag{26}$$

[7]"The Explanation of Productivity Change," *op. cit.*, p. 253.

in which case none of the nice properties of the Divisia index can be expected to hold.[8]

The possibility of bias in the Divisia index of total labour input is illustrated with the aid of Figure 4 which reproduces the unit isoquant of the year 0 from Figure 3. It is convenient to suppose that technical change causes the unit isoquant to shift gradually from the unbroken curve through A in the year 0 to the broken curve through B in the year t. The shift occurs in such a way that the unit isoquant is always tangent to the chord AB and the relative price of educated and uneducated labour remains constant throughout the process. The Divisia index shows no change from year 0 to year 1 because it does not "know" that total labour input assessed with regard to the original set of isoquants has decreased, and it "thinks" that the isoquant is the chord AB itself.

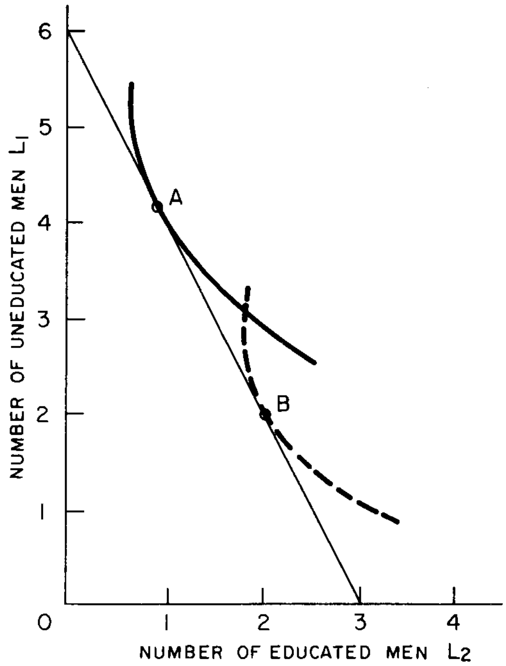

Figure 4

One might question the significance of this example on the grounds that productivity changes as assessed by the isoquants through A and B are symmetric; if the passage from A to B entails a productivity increase as assessed with regard to the unit isoquant through A, it is also the case that it entails a productivity decrease as assessed with regard to the unit isoquant through B. The cases are not symmetric with respect to time. If the proportion of educated labour is to be

[8]The bias that arises from using actual wages to construct weights in a Divisia index instead of wages as they would be if technical change had not occurred has been discussed by R. R. Nelson in "Recent Exercises in Growth Accounting: New Understanding or Dead End", *American Economic Review*, June 1973. Nelson points out a family resemblance between this criticism of the Divisia index and the well-known Diamond-McFadden theorem concerning the impossibility of inferring the form of technical change.

increased between year 0 and year 1, come what may, the shift in the isoquants is unambiguously beneficial in forestalling the decline that would otherwise have occurred in the marginal product of the factor of production that is becoming relatively more abundant. Consider the extreme case: if the isoquants were *L*-shaped as illustrated in Figure 5, the change in the composition of the labour force from four uneducated men and one educated man in year 0 to two uneducated men and two educated men in year 1 would cause a 50 percent reduction in output unless technical change intervened to preserve the productivity of educated labour.

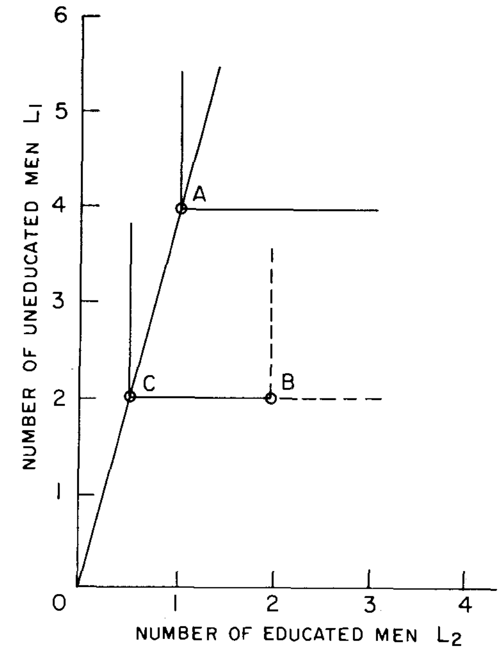

Figure 5

The bias in the Divisia index is not limited to the case where the Divisia index shows no growth of labour input at all. In the case described in Figure 6, the Divisia index picks up part of the technical change but fails to catch that part embodied in a tilting of the isoquants. Again, the unbroken curve represents the technology of the year 0, the broken curve represents technology of the year 1, and the relative price of educated and uneducated labour is the same in both years. This time, however, the labour force is held constant while the proportion of educated to uneducated men increases as indicated by a movement from *A* to *B* along the 45 degree line, and the isoquants through *A* and *B* are not necessarily unit isoquants. The Divisia index records the growth of total labour input to be *BD*/*OD* when the true growth as assessed by the isoquant system of the year 0 is only *BC*/*OC*.

Though Jorgenson and Griliches' method of measuring total labour input permits them to adjust their index to account for changes over time in the relative earnings of educated and uneducated people, their data displays virtually no

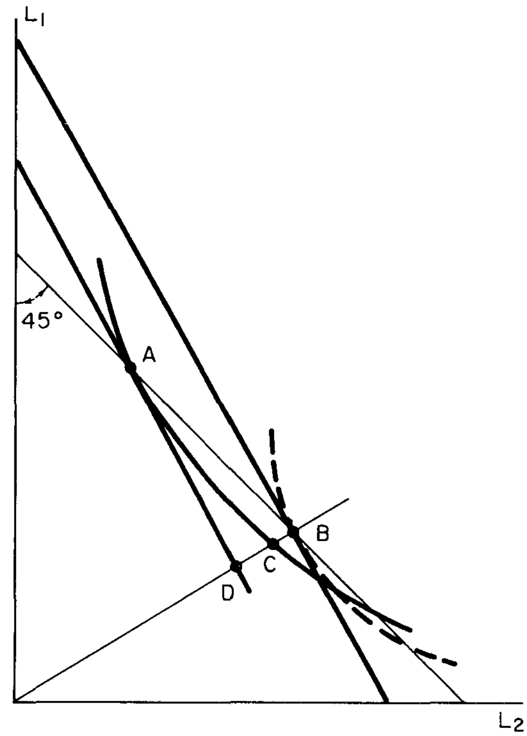

Figure 6

change in relative earnings, with the result that their method and Denison's method turn out to be the same in practice. Their data on numbers of educated and uneducated workers and on relative earnings are presented as Tables 1 and 2, which are their Tables XI and XII on Page 279. From these data, one must conclude either that there is an infinite elasticity of substitution in use between the different educational classes of labour (that the isoquants at any moment of time are flat), or that real input of labour is substantially overestimated when technical change is measured in accordance with equation (26)—overestimated in the sense that the increase over time in the proportion of educated labour would have caused a substantial decline in the relative marginal product of educated labour and would have resulted in a slower growth of output per head but for technical change of the kind illustrated in Figures 4, 5 and 6.

Two polar explanations of the data in Tables 1 and 2 are illustrated in Figures 7a and 7b. For the purpose of demonstrating a point, we suppose that an uneducated man may transform himself into an educated man instantly by studying part-time; an educated worker is one who acquires a complete education by studying for a few hours in the morning, who employs his knowledge in the afternoon and who forgets what he learned overnight. In a competitive economy, the relative wage of educated and uneducated labour depends on the number of hours of study required, and this may depend in turn on the number of educated workers supplied. We imagine a pseudo-production possibility curve of educated and uneducated

TABLE 1

CIVILIAN LABOUR FORCE, MALES 18 TO 64 YEARS OLD, BY EDUCATIONAL ATTAINMENT
PERCENT DISTRIBUTION BY YEARS OF SCHOOL COMPLETED

School Year Completed	1940	1948	1952		1957	1959	1962	1965
Elementary 0–4	10.2	7.9	7.6	7.6	6.3	5.9	5.1	4.3
5–6 or 5–7	10.2	7.1	6.6	11.6	11.4	10.7	9.8	8.3
7–8 or 8	33.7	26.9	25.1	20.1	16.8	15.8	13.9	12.7
High School 1–3	18.3	20.7	19.4		20.1	19.8	19.2	18.9
4	16.6	23.6	24.6		27.2	27.5	29.1	32.3
College 1–3	5.7	7.1	8.3		8.5	9.4	10.6	10.6
4+ or 4	5.4	6.7	8.3		9.6	6.3	7.3	7.5
5+	—	—	—		—	4.7	5.0	5.4

Source: Table XI, D. Jorgenson and Z. Griliches, "The Explanation of Productivity Change."

TABLE 2

MEAN ANNUAL EARNINGS OF MALES 25 YEARS AND OVER BY SCHOOL YEARS COMPLETED, SELECTED YEARS

School Year Completed	1939	1949		1956	1958	1959	1963
Elementary 0–4	665	1724		2127	2046	2935	2465
5–6 or 5–7	900	2268		2927	2829	4058	3409
7–8 or 8	1188	2693	2829	3732	3769	4725	4432
High School 1–3	1379	3226		4480	4618	5379	5370
4	1661	3784		5439	5567	6132	6588
College 1–3	1931	4423		6363	6966	7401	7693
4+ or 4	2607	6179		3490	9206	9255	9523
5+	—	—		—	—	11136	10487

Source: Table XII, D. Jorgenson and Z. Griliches, "The Explanation of Productivity Change."

labour. In both figures, pseudo-production possibility curves in year 0 and year 1 are combined with sets of isoquants which may be thought of as derived indifference curves for the two types of labour. By assumption, the proportion of educated to uneducated labour is greater in year 1 than in year 0, but the relative wage is the same. The two figures differ in their explanations of the constancy of the relative wage. The explanation in Figure 7a is that the elasticity of substitution in production of educated and uneducated labour is infinite, that a carpenter can always transform himself into a brain surgeon by studying for seven out of eight hours in the working day. The alternative explanation in Figure 7b is that the elasticity of substitution in use of educated and uneducated labour is infinite, that eight carpenters working together can perform brain surgery.[9]

[9]Of course, eight marginal carpenters can perform brain surgery in the sense that there is a chain of substitutions through different types of labour beginning with a brain surgeon and ending with eight carpenters. The question is whether marginal products of different types of labour are independent of the composition of the labour force.

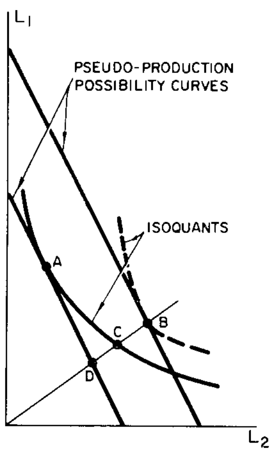

Figure 7a

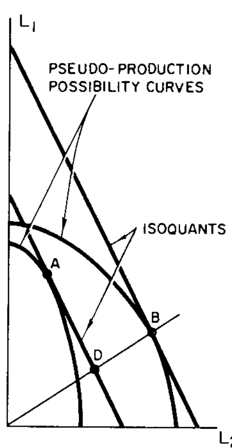

Figure 7b

The reason for emphasizing the contrast between the explanations of the constancy of the relative wave in Figures 7a and 7b is to draw attention to the fact that the Divisia index of the total input of labour in equation (20) is intended to measure the location of the vector $(L_1, \ldots, L_n)$ in a field of isoquants, not the outward shift over time in the pseudo-production possibility curve. When input is measured correctly, the set of value shares ought to reflect the relative usefulness of carpenters and brain surgeons at the base year technology, not the cost of converting one into the other. In the circumstances of Figure 7a, the true measure of the growth of labour input is BC/OC, which is less than that indicated by the Divisia index; in the circumstances of Figure 7b, the true measure is BD/OD which is correctly reflected in the Divisia index. The Divisia index reflects the growth of labour input correctly if there is an infinite elasticity of substitution in use between educated and uneducated labour, and the Divisia index overestimates the growth of labour input if the elasticity of substitution in use within the technology available in the base year is less than infinite.

One's suspicion that the observed constancy of relative wages is not a demand phenomenon but is instead an aspect of technical change, so that the use of the Divisia index in equation (20) leads to an overestimate of the growth of the labour input and an underestimate of the rate of technical change, is reinforced when one considers the composition of the labour force within any educational category. From 1940 to 1965, the proportion of college graduates in the work force increased by over 150 percent, from 5.4 percent to 12.9 percent of the total labour force. In applying equation (20), the number of college graduates is treated as one of the inputs, L_j, as though each college graduate were a perfect substitute for every other college graduate and as though the typical college graduate in 1965 were a perfect substitute for the typical college graduate in 1940. This assumption may well be true of some fields of study; the 1940 vintage of Doctor of Divinity or Ph.D. in Medieval Literature may be a perfect substitute for the 1965 vintage. This assumption is obviously not true of the sciences; the 1940 vintage physicist or chemist is a very different person from the 1965 vintage, and the computer specialist and biophysicist of 1965 have no counterparts in 1940. The difference between the 1965 college graduate and the 1940 college graduate is not merely that the 1965 graduate is more productive in the sense that he can do some given multiple of work of his 1940 counterpart. The 1965 graduate is equally productive in some occupations, more productive in others, and he possesses skills that were unknown in 1940 because they depend upon technology developed in the intervening period. The point I am making is that the relative wage of college graduates has been preserved because, and only because, technical advance has brought forth new skills and has made it profitable for people to acquire these skills, so that what we measure as labour input contains a very large component of technical change. Inputs with the same name are not the same inputs at different periods of time.

To determine the true measure of labour input with respect to base year isoquants, we would have to know what the economy would be like today if there had been no technical change since the base year. The 1965 vintage college graduate trades his skill for the skill of a 1940 vintage college graduate, and all industrial processes learned since 1940 are abandoned. Then we observe how the market responds to the altered skill-composition of the labour force and use the new observed wage structure in computing the terms v_i for equation (20). I think it very unlikely that the premium on educated labour would be preserved in the face of changes in the composition of the labour force as large as those observed from 1940 to 1965. Indeed, it is doubtful whether there would be any premium at all, for many college graduates would have to work at jobs for which their skills are not required.

These considerations suggest that the use of the Divisia index coupled with the practice of treating factors of production with identical names as though they were identical factors of production may be leading us to attribute a disproportionate share of observed economic growth to the mere replication of factors of production, and may conceal the vital role of invention. Much depends on what one really means by aggregate technical change. Our assertion that the Divisia index leads to an underestimate of the rate of technical change as represented by the term r_1 in equation (18) is only of interest if one agrees that the rate r_1 is what should be measured in studies of total factor productivity. I think that it is. I think that the economic historian, for instance, is interested in aggregate technical change because and only in so far as that statistic tells him what part of economic growth is attributable to the development of new technology and how much greater is actual economic growth than it would be if resources devoted to invention had instead been allocated to the purchase of capital goods of the kinds available in the first year of the time-series and if the economy were forbidden to utilize any technology not available at that time. It is my view that our sophisticated methods of measuring total factor productivity have led to sophisticated errors which tend to obscure the simple fact that invention of new processes is the *sine qua non* of economic growth.