THE MEASUREMENT OF CAPITAL AND TOTAL FACTOR PRODUCTIVITY IN THE CONTEXT OF THE CAMBRIDGE THEORY OF CAPITAL*

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In this paper I set out the reasons why the measurement of total factor productivity must be conducted along Harrod–Robinson lines and why the traditional Hicks–Solow–Denison *et al.* approaches must be rejected as theoretically faulty. The basic point is simple. The traditional approach is based squarely on the fundamental distinction between technical change (a shift in production functions) and capital accumulation (movements along production functions). This distinction fails to take into account the essential intermediateness of capital goods. Once attention is focused on that point, the neoclassical distinction and all measures of total factor productivity based on neoclassical analysis are seen to be wrong.

Though it generalizes to many-commodity models, the analysis is drawn up in terms of one and two commodity growth equilibrium models and, particularly in the latter context, the validity of the Harrod–Robinson approach is clearly seen. The significance of the "re-switching controversy" for measurement of total factor productivity is briefly assessed. In the context of vintage models and a short discussion on the measurement of quality change, the Harrod–Robinson approach to the measurement of total factor productivity is again seen to be superior.

I. INTRODUCTION

In this paper, I set out what I believe to be the correct way of recording the national economic accounts in "real" terms to facilitate better understanding of the processes of economic growth. I shall concentrate on the measurement of the capital input and the rate of technical advance, total factor productivity, the "residual"—call it what you will. I shall pay attention to the very important problems of measuring labour and natural agent inputs only when they touch upon the "capital input" concepts under discussion.

In short-period analysis, the stock of capital is a given collection of heterogeneous goods (plants, machinery and equipment, etc.) which, depending on the state of aggregate demand, competition and the ease with which such goods can be substituted for labour, is more or less fully utilized in some rough sense. In long-period analysis, however, capital goods are not primary inputs independent of labour, natural agents and technology.[1]

"The question of whether there are two or three primary factors of production has been much debated. However, the answer seems to be fairly clear.

*This paper draws on and extends my *On Concepts of Capital and Technical Change* (Cambridge: Cambridge University Press; 1971) and two earlier papers: L. M. Read, "The measure of total factor productivity appropriate to wage-price guidelines," *Canadian Journal of Economics*, I, May 1968, 349–358 and T. K. Rymes, "Professor Read and the measurement of total factor productivity," *ibid.*, 359–367. My indebtedness to Read is very great. The paper was originally presented to the Twelfth General Conference of the International Association for Research in Income and Wealth, Ronneby, Sweden, 31 August 1971 and it has benefitted from comments by Mr. R. Hjerppe. I am also grateful to Professor R. J. Gordon for the very helpful comments he made in correspondence. Travel to the Conference was made possible by a Canada Council grant and my own University.

[1]J. Robinson, *The Accumulation of Capital* (3rd ed.), 311.

Considering any one period there are indeed three factors. But if economic development as a whole, past, present, and future is considered capital cannot be considered a primary factor."[2]

Technical advance is a long-period or dynamic phenomenon. It must be understood though that the long-period is not when Kingdom comes but is here and now—the long period works its way out, of course, in the short-period context of much turbulence. The dynamic forces of capital accumulation and technical change are, however, best understood by assuming long-period equilibrium conditions are holding in the short-period.[3] In dynamics, one deals with equilibrium rates of change at a point in time.[4] In this paper, I shall deal with the measurement of technical change in long-period equilibria. The argument applies outside such equilibria but the well-known intractable capital measurement problems[5] associated with disequilibria are liable to detract attention away from what I consider more fundamental problems.

In dynamic analysis, it is the essential non-primary intermediateness of the capital input which must be captured in the measurement of both capital and technical advance. When the fact that capital inputs are produced means of production is rigorously and logically incorporated in the measurement of capital and technical change, support is provided for Harrod–Robinson concepts of technical change. The Hicks–Meade–Solow concepts of technical change are shown to be theoretically faulty. This paper is largely concerned with demonstrating these two points.

II. The Basic Conceptual Framework

Consider the national accounts flow identity for a closed economy with no government

$$P_cC + P_KI \equiv WL + H_NN + H_KK \tag{II.1}$$

where P_c and C represent the nominal price and quantity of a consumption good, P_K and I the nominal price and quantity of a *new* capital good, W and L the nominal price and quantity of a labour input, H_N and N the nominal *rental* and quantity of a natural agent input, and H_K and K the nominal gross *rental* and quantity of a capital input.[6]

[2]E. Malinvaud, "Capital accumulation and efficient allocation of resources," *Econometrica*, XXI, 1953, 233–68, revised in AEA *Readings in Welfare Economics*, 684, n. 4.

[3]For an excellent discussion of the distinction between short- and long-period analysis, see J. Robinson, *Economic Heresies: Some Old-Fashioned Questions in Economic Theory*, especially chapters 1, 2 and 8.

[4]Sir Roy Harrod, "Replacements, net investment, amortisation funds", *Economic Journal*, LXXX, March 1970, 24.

[5]A review of such measurement problems is contained in my *Capital Flows and Stocks, Manufacturing, Canada, 1926–1960*, Ottawa, Queen's Printer for the Dominion Bureau of Statistics, 1967.

[6]It is understood that P_cC stands for

$$\sum_{i=1}^{n} P_{ci}C_i, \quad P_KI \text{ for } \sum P_{K_j}I_{K_j}$$

and so forth for the remaining components of the accounts. The identity II.1 encompasses, then, many kinds of commodity outputs and inputs and many kinds of labour and natural resource inputs.

These national accounts deal with average levels over some finite period of time. They can be expressed in terms of proportionate rates of change over the same arbitrary small period of time. Re-assessing identity II.1 accordingly, I have

II.2 $$\alpha(p_C + c) + \beta(p_K + i) \equiv \gamma(w + l) + \delta(h_N + n) + \epsilon(h_K + k)$$

where α, β, γ, δ and ϵ are respectively the shares of consumption, gross capital formation, labour income, returns to natural agents and gross returns to capital in national product and the small letters represent proportionate rates of growth. For example,

$$\alpha \equiv P_C C/[P_C C + P_K I]$$

and

$$p_C + c \equiv \frac{1}{P_C}\frac{dP_C}{dt} + \frac{1}{C}\frac{dC}{dt}.$$[7]

Clearly

$$\alpha + \beta \equiv \gamma + \delta + \epsilon \equiv 1.$$

If identity II.2 is rearranged, I have

II.3 $$[\alpha c + \beta i] - [\gamma l + \delta n + \epsilon_k] \equiv (\gamma w + \delta h_N + \epsilon h_K) - (\alpha p_C + \beta p_K) \equiv t$$

where t is what I shall call the Hicks–Meade–Solow or neoclassical concept of the proportionate rate of technical change. Identity II.3 states, then, that the weighted growth rates of output (consumption and investment) less the weighted growth rates of inputs (the labour, natural agent and capital inputs) will yield a measure of the growth rate in the efficiency with which inputs are being transformed into outputs. Identically, the weighted growth rates of *input prices* less the weighted growth rates of *output prices* yields the same measure of the growth rate of economic efficiency.

Inspection of the identity II.3 reveals that "capital," and "capital" alone, appears as both an input and an output both in terms of quantities and prices and is, incorrectly as I shall show, measured in the same terms. For any given capital good, i is measured in the same terms as k—i.e., for a particular type of capital good, i measures the rate of change of the output of such a capital good, while k measures the rate of change in the input stock of such a capital good. On the right hand side, where the rate of change of economic efficiency is measured in terms of prices, h_K measures the rate of change in the gross rental accruing to such a capital good and p_K the rate of change in its price as an output. As I shall show, these prices have components which are fundamentally the same. It is the similar measurement, in quantities or in prices, of "capital" as an output and as an input which is what is wrong with identity II.3. The similarity in measurement takes no account of the fact that "the capital input" is being produced with ever-increasing efficiency in an economy where technical progress is occurring. It takes no account of the logic of the long-period, namely, that "capital" is an intermediate means of production and is not a primary input

[7] Again, it is understood that $\alpha(p_c + c)$ stands for

$$\sum_i \alpha_i(p_{ci} + c_i) \text{ or } \sum_i \frac{P_{ci}C_i}{\sum_i P_{ci}C_i + \sum_j P_{kj}I_{kj}}\left(\frac{1}{P_{ci}}\frac{dP_{ci}}{dt} + \frac{1}{C_i}\frac{dC_i}{dt}\right)$$

and so forth for the remaining components of the accounts.

in the economic system in the same sense as are labour and natural agents. It is the working out of the logic of "capital" as an intermediate means of production in the measurement of capital and total factor productivity with which this paper is concerned.

The issue may be more clearly seen if simple examples are examined. This procedure also permits some recent theoretical literature on capital and growth to be joined.

III. One-Commodity Economies

If the economy being examined produced but one commodity under technical conditions of constant returns to scale, if perfect competition prevailed, if there were but one kind of labour, if natural agents are ignored and if the capital stock were subject only to a constant rate of "depreciation by evaporation," λ, then identity II.1 becomes

III.1 $$PQ \equiv WL + (R + \lambda)PK$$

where P and Q are the nominal price and quantity of the commodity being produced, and R is the real net rate of return to capital, i.e. $H_k \equiv R + \lambda$, or the gross rental on capital equals the net rate of return plus the constant rate of "depreciation by evaporation."[8] Reconstitution of identity II.1 into growth rate terms and rearrangement yields

III.2 $$q - [\gamma l + (\delta + \epsilon)k] \equiv \gamma(w - p)$$

where

$$\delta + \epsilon \equiv (R + \lambda)PK/PQ, \quad \text{and} \quad q = \frac{1}{Q}\frac{dQ}{dt} \quad \text{and} \quad P = \frac{1}{P}\frac{dP}{dt}.$$

Economists are generally more interested in the net rather than the gross product of society. The identity may also be re-expressed in net terms

III.3 $$PQ - \lambda PK \equiv WL + RPK$$

which, in growth rate terms with rearrangement, is

III.4 $$\frac{q}{1-\epsilon} - \frac{\epsilon k}{1-\epsilon} - \left(\frac{\gamma l}{1-\epsilon} + \frac{\delta k}{1-\epsilon}\right)$$
$$\equiv \frac{\gamma w}{1-\epsilon} + \frac{\delta(p + r)}{1-\epsilon} - \left(\frac{p}{1-\epsilon} - \frac{\epsilon p}{1-\epsilon}\right) \equiv t_N$$

where t_N is the net proportionate rate of technical change. The left hand side of identities III.3 and III.4 is precisely Meade's formulation of the Hicksian concept of technical advance.[9] It is also Solow's earlier concept[10] (Solow's later con-

[8]The use of the real, as distinct from the nominal, net rate of return eliminates capital gains from the identity.

[9]See J. E. Meade, *A Neoclassical Theory of Economic Growth*, Chapters 2 and 6 and *The Growing Economy*, Chap. IV.

[10]R. M. Solow, "Technical change and the aggregate production function," *The Review of Economics and Statistics*, XXXIX, August 1957, 312–320. Identity III.2 may be re-expressed as

$$q - l - (\delta + \epsilon)(k - l) \equiv t$$

to put the left-hand side in Solow's per unit of labour input form.

cept,[11] which introduced "depreciation by obsolescence" will be dealt with in section VII). In index number form, it is also Kendrick's total factor productivity and in the growth rate form, Denison's "sources of growth."[12] It is a Divisia index of technological change.[13] The identities are also how Jorgenson and Griliches approach the measurement of the capital input and productivity change.[14]

All these formulations, which I call neoclassical, share one fundamental and fatal flaw. They do not take account of the fact that the "capital" input—being an intermediate input in the system—is being produced with ever-increasing efficiency in an economy subject to technical advance.

In his survey article, Nadiri touches upon some of the difficulties confronting traditional measures of total factor productivity but in no way gets to the heart of the matter.[15]

Underlying the accounts for such a simplified economy may be a standard neoclassical production function, $Q - \lambda K = Q(L, K) - \lambda K$, with the usual properties, showing net output as a function of labour and "capital." Differentiation of this production function with respect to time and rearrangement yields

$$\left[\frac{q}{1-\epsilon} - \frac{\epsilon k}{1-\epsilon}\right] - \left[\frac{\gamma l}{1-\epsilon} + \frac{(\delta+\epsilon)k}{1-\epsilon} - \frac{\epsilon k}{1-\epsilon}\right] \equiv t$$

or

$$\left[\frac{q}{1-\epsilon} - \frac{\epsilon k}{1-\epsilon}\right] - \left[\frac{\gamma l}{1-\epsilon} + \frac{\delta k}{1-\epsilon}\right] \equiv t$$

as illustrated by the left-hand side of identity III.4.

[11]R. M. Solow, "Investment and technical progress", ed. K. J. Arrow *et al.*, *Mathematical Methods in the Social Sciences, 1959.*

[12]The left hand side of III.3, in index number form, is in general

$$\frac{\Sigma P_0 Q_1}{\Sigma P_0 Q_0} \Big/ \left[\frac{\Sigma w_0 L_0}{\Sigma P_0 Q_0}\left(\frac{\Sigma w_0 L_1}{\Sigma w_0 L_0}\right) + \frac{\Sigma(R_0+\lambda_0)P_0 K_0}{\Sigma P_0 Q_0}\left(\frac{\Sigma(R_0+\lambda_0)P_0 K_1}{\Sigma(R_0+\lambda_0)P_0 K_0}\right)\right] \equiv \frac{T_1}{T_0}$$

which, in view of the simplifying assumptions, collapses to

$$\frac{Q_1}{Q_0} \Big/ \left[\gamma\frac{L_1}{L_0} + (\delta+\epsilon)\frac{K_1}{K_0}\right] \equiv \frac{T_1}{T_0}.$$

See J. W. Kendrick, *Productivity Trends in the United States*, and E. F. Denison, *The Sources of Growth in the United States and the Alternatives Before Us* and *Why Growth Rates Differ.*

[13]M. K. Richter, "Invariance axioms and economic indexes", *Econometrica*, XXXIV, October 1966, 739–755.

[14]D. W. Jorgenson and Z. Griliches, "The explanation of productivity change," *Review of Economic Studies*, XXXIV, July 1967, 249–283. The original Jorgenson–Griliches article showed a very low rate of technical change in the U.S. economy and reflected the theoretical argument, exemplified in T. W. Schultz, *Transforming Traditional Agriculture*, Chapter 9, that observed technical change merely implies that some "new factor of production" has not been specified and accounted for in the measurement procedures. The original Jorgenson–Griliches estimates were subjected to severe criticisms by Denison and revised estimates by Christensen and Jorgenson substantially change the earlier negligible role accorded to the rate of technical change. See E. F. Denison, "Some major issues in productivity analysis: An examination of estimates by Jorgenson and Griliches," *Survey of Current Business*, XLIX, May 1969, 1–27 and L. R. Christensen and D. W. Jorgenson, "U.S. Real product and real factor input," 1929–1967. *Review of Income and Wealth*, XVI, March 1970, 19–50. The paper presented by Jorgenson at the Conference does not, in my judgment, modify the substantial retreat which Jorgenson and Griliches have had to make from their earlier position.

[15]M. I. Nadiri, "Some approaches to the theory and measurement of total factor productivity: a survey," *Journal of Economic Literature*, VIII, December 1970, 1137–1177.

It is well-known that, if the highly simplified economy were postulated to be in a steady state long-period equilibrium, technical progress *must* be Harrod–Robinson neutral[16]—i.e., the capital-output ratio and the net rate of return must be constant. In my notation, the proportionate rates of growth of capital and output must be equal (i.e., $q = k$) and the proportionate rate of growth of the net rate of return to capital zero (i.e., $r = 0$). Insert these postulates into identities III.3 and III.4. Then,

III.3a $$\gamma(q - l) \equiv \gamma(w - p) \equiv t$$

and

III.4a $$\frac{\gamma(q - l)}{1 - \epsilon} \equiv \frac{\gamma(w - p)}{1 - \epsilon} \equiv t_N.$$

One immediate, if seemingly trivial, problem associated with the neoclassical concepts of capital and technical change may now be seen. For any simplified economy postulated to be in long-period equilibrium, the measured rate of technical progress will be greater in net terms than in gross terms. The logic of the neoclassical measure, in this case, produces strange results since it is clear that the underlying rate of technical progress must be the same, regardless of whether it is measured in net or gross terms.

Furthermore, if two economies, both experiencing the *same* rate of Harrod-neutral technical advance but exhibiting different partial labour elasticities of production (and the same *level* of real wage rates), were compared for the respective neoclassical rates of technical advance, the economy with the lower partial elasticity would be shown as recording the lower rate. There is no meaning to such a result. In the extreme case where capital earns all of national income (where labour is "human capital") in equilibrium growth, the observed neoclassical rate of technical advance would be zero—an equally meaningless outcome.

Fundamentally, neoclassical measures neglect the fact that capital is an intermediate input produced by the economic system, and is not exogenous to it, that in the context of technical advance, "capital" as output and input cannot logically be measured in the same units, and that the ever-increasing ability of the economic system to produce capital goods is not being taken into account. Is there a correct way to do this?

To capture the essential intermediateness of the "capital" input, to measure "capital" as input differently from capital as output and to take account of the ever-increasing efficiency of the economic system to produce output and new capital and to reproduce existing capital, the capital input measured in neoclassical terms must be "reduced" by technical change.[17] The identity III.4 for

[16]See, for example, R. M. Solow, *Growth Theory: An Exposition*, 35. Yet Solow fails to appreciate the fact that, with the adoption of Harrodian concepts of technical change, one must abandon the distinction between a shift in a production function and movement along it. For "with it goes and must go any possibility of dealing with technical change in traditional terms, for the traditional treatment is based squarely on this distinction." See R. M. Solow, "Comments" on L. L. Pasinetti, "On concepts and measures of changes in productivity," *Review of Economics and Statistics*, XLI, August 1959, 283.

[17]This is the important point made by Read. See L. M. Read, *op. cit.*

the highly simplified economy must be rewritten as

$$\text{III.5} \qquad q - [\gamma l + (\delta + \epsilon)(k - t^*)] \equiv \gamma(w - p) + (\delta + \epsilon)(r + t^*) \equiv t^*$$

In net terms, I would write

$$\text{III.6} \qquad \left[\frac{q}{1-\epsilon} - \frac{\epsilon(k - t^*)}{1-\epsilon}\right] - \left[\frac{\gamma l}{1-\epsilon} + \frac{\delta(k - t^*)}{1-\epsilon}\right]$$

$$\equiv \left[\frac{\gamma w}{1-\epsilon} + \frac{\delta}{1-\epsilon}(p + r + t^*)\right] - \left[\frac{p}{1-\epsilon} - \epsilon\frac{(p + t^*)}{1-\epsilon}\right] \equiv t_N{}^*$$

In identities III.5 and III.6, simultaneous account is being taken, as logically it must be, of the fact that it is the technical advance of the economy which is permitting increases in both output and capital per unit of labour. To see better the significance of the adjustment being made, I shall again suppose the simplified economy to be in long-period equilibrium exhibiting Harrod–Robinson neutrality in technical advance. Identities III.5 and III.6 will appear as

$$\text{III.5a} \qquad q - l \equiv w - p \equiv t^*$$

$$\text{III.6a} \qquad q - l = w - p \equiv t_N{}^*$$

The observed gross and net rates of technical change will now be the same, as they of course logically must be, and will exactly measure the rate of change of the Harrod–Robinson concepts of technical change.

It seems at first blush that a trivial result has been obtained. Having postulated the simple growing economy to be in long-period equilibrium, it is not surprising perhaps to "observe" Harrod–Robinson neutral technical advance. I have shown, however, that on the standard interpretation, the observed Harrod–Robinson rate of technical advance is broken into two parts: the observed rate of growth of capital per head and the observed "residual" or Hicks–Meade–Solow rate of technical advance. The two concepts of the rate of technical advance are, however, fundamentally different.[18] One tells us nothing about the rate at which real incomes per head (both workers and owners of capital) can rise, the other tells us precisely that rate. One is critically dependent on the static parameters of a given production function (which, in the dynamic context of technical change makes little sense), while the other is not. The fundamental difference, however, from which the above differences stem, is the fact that one takes no account of the ever-increasing efficiency with which the economic

[18]It is sometimes assumed that, if the underlying production function is Cobb–Douglas. "It is possible for technical progress to be both Hicks-neutral and Harrod-neutral" [F. H. Hahn and R. C. O. Matthews, "The theory of economic growth: a survey," *Economic Journal*, LXXIV, December 1964, 829]. If the underlying production function is of the CES type with an elasticity of substitution less than one, then in an economy exhibiting Harrod-neutral technical advance, the Hicks-type technical change is said to be biased. [cf. W. Fellner, "Measures of technological progress in the light of recent growth theories," *American Economic Review*, LVII, December 1967, 1073–1097]. Both of these standard arguments in the literature are, in my view, incorrect since they imply that somehow the two concepts of technical advance are of equal validity and may be compared. Of course, in a trivial mathematical sense they can be but in the deeper economic meaning they are strictly incomparable. For the standard exercise carried on in two sectors, see W. W. Chang, "The neoclassical theory of technical progress," *American Economic Review*, LV, December 1970, 912–923.

system produces its reproducible inputs while the other, absolutely clear on the point that capital is not a primary input, does.

In the simple long-period equilibrium economy being presently considered, what concept of capital emerges when technical change is measured along Harrod–Robinson lines? From identities III.5a and 6a, I have

$$q - [\gamma l + (\delta + \epsilon)(k - q + l)] \equiv [\gamma(w - p) + (\delta + e)(r + w - p)] \equiv t^*.$$

Under the *special* assumptions then, the rate of growth of commodity capital adjusted for the increasing efficiency with which the economic system is producing it, $k - t^* = k - (q - l)$ equals the rate of growth of the labour force. The rate of change in the price of such a capital concept $r + t^* = r + w - p$ is equal to the rate of change of the real wage rate. The capital concept is then, in this special case, Joan Robinson's "real capital" or Harrod's "average basket of waiting."[19]

The rate of change in the price of the derived capital concept is even more revealing. If it is argued that the fundamental primary inputs in an economic system are so much labour time and so much abstinence or "waiting time,"[20] then in an economy subject to steady *neutral* technical change, the prices of such primary inputs must be rising at the same rates. This is precisely what the alternative measures advocated here show. Indeed, this is precisely what Harrod and Robinson meant by neutrality.[21]

If the simple economy's technology exhibited increasing returns to scale, the two measures of technical change will incorporate such effects. (It can, of course, no longer be argued that weights reflect respectively competitive pricing of the partial elasticities of production.)

[19]In her *The Accumulation of Capital*, 121, Robinson defines real capital for economies in long-period equilibrium as follows:

> "We can divide the value in terms of commodities of the stock of capital in any economy by the wage per man hour in terms of commodities ruling in that economy and so obtain the quantity of capital in terms of labour time."

Thus, J ("real capital") $= K/(W/P)$ and the proportionate rate of change of "real capital" is $j = k - (w - p)$ which in our simple economy equals $j = k - (q - l) = l$. In his "The neutrality of improvements," *Economic Journal*, LXXI, June 1961, 303, Harrod states

> "If we define a unit of capital as so much waiting in respect of a unit of non-capital factors of production, then it should be supposed that the quantity of capital is growing at the same rate as the non-capital factors of production."

This is precisely the result derived above.

[20]Professor Solow, for example, while recognizing the need to make a conceptual distinction between the imputed return to capital and the income of capitalists, argues that, stripped of their moralistic overtones, the concepts of "abstinence" and "waiting" are economically useful descriptions of the non-labour primary input in the process of economic production. See R. M. Solow, *Capital Theory and the Rate of Return*, 10–11. It will be noticed that my formulation of the capital input meets Solow's requirement because it says nothing about the distribution of the "abstinence" or "waiting" amongst the population. On this point, see J. Robinson, "Harrod after twenty-one years," *Economic Journal*, LXXX, September 1970, 731–737.

[21]The Hicks–Meade definition of neutrality—namely that for given commodity–capital–labour ratios, the marginal physical product of such "factors" rise proportionately—is unhelpful because it imposes a static definition on an essentially dynamic, or long-period phenomenon. It misses the critical point that an economic system with technical change is producing its capital stock more efficiently.

Within the confines of the usual neoclassical assumptions, how would the respective measures reflect non-neutral technical advance? Harrod–Robinson neutrality means, within the context of the simple economy being discussed, that the primary input ratio—the ratio of labour to real capital or to "waiting time"—is remaining constant *and* the prices of such primary inputs are both experiencing the same proportionate rate of increase. (This is the essence of Harrod's concept of a constant commodity input–commodity output ratio and a constant marginal rate of transformation between commodity input and commodity output.) Non-neutral technical change can involve, then, constancy in the primary input ratios with changing primary input prices or constancy in relative primary input prices with changing primary input ratios or any combination of these two. Consideration of such cases indicates a change in relative shares and, as a consequence, in terms of constant price national accounts, index number problems invariably arise.

Index number problems, however, are well-known and entail no further discussion. More importantly, the concept of continuous non-neutral technical advance destroys the concept of steady equilibrium growth lying behind the simple theory usually set out. Exploration of the theories of induced technical change tending towards Harrod–Robinson neutrality would take me far beyond the context of this paper. The principle, however, remains clear—namely, that neoclassical concepts of technical change fail to reflect the intermediate nature of the commodity capital input. In a world of technical progress, even where there is only one commodity so that no aggregation problems in the measurement of capital arise, the neoclassical concept of the capital input is wrong.

Though the basic analytical point has now been made, it will serve to demonstrate the power of the Harrod–Robinson approach if it is examined in the more descriptive context of two and many-commodity economies.

IV. Two-Commodity Economies

I now assume that the simple economy is producing two goods in two sectors: one consumption good and one capital good. I shall continue to assume that the capital good is "putty" and is subject to an immutable rate of "depreciation by evaporation." The labour force is homogeneous and "land" does not exist.

The national accounts for such an economy, in my notation, will then be, at the aggregate level:

IV.1 $$P_cC + P_kI \equiv WL + (R + \lambda)P_kK$$

where

$$L = L_c + L_k \quad \text{and} \quad K = K_c + K_k,$$

and at the sectoral, or industrial, level,

IV.2a $$P_cC \equiv WL_c + (R + \lambda)P_kK_c$$

IV.2b $$P_kI \equiv WL_k + (R + \lambda)P_kK_k.$$

Again, imposing the usual growth rate formulation and rearranging terms, I have the following neoclassical measures of the rate of technical change:

At the aggregate level,

IV.3 $[\alpha c + \beta i] - [\gamma l + (\delta + \epsilon)k] \equiv [\gamma w + (\delta + \epsilon)(r + p_k)] - [\alpha p_c + \beta p_k] \equiv t_A$

and, at the sector level,

IV.3a $c - [\gamma_c l_c + (\delta_c + \epsilon_c)k_c] \equiv [\gamma_c w + (\delta_c + \epsilon_c)(r + p_k)] - p_c \equiv t_c$

and

IV.3b $i - [\gamma_k l_k + (\delta_k + \epsilon_k)k_k] \equiv [\gamma_k w + (\delta_k + \epsilon_k)(r + p_k)] - p_k \equiv t_k.$

From the fact that, for example,

$$L = L_c + L_k$$

$$l = \frac{L_c}{L} l_c + \frac{L_k}{L} l_k$$

$$= \frac{(WL_c/P_cC)/[P_cC/(P_cC + P_kI)]l_c}{WL/(P_cC + P_kI)} + \frac{(WL_k/P_kI)/[P_kI/(P_cC + P_kI)]l_k}{WL/(P_cC + P_kI)}$$

$$l = \frac{\gamma_c \alpha}{\gamma} l_c + \frac{\gamma_k \beta}{\gamma} l_k$$

it follows that identity IV.3 may be rewritten as:

$$\alpha[c - (\gamma_c l_c + (\delta_c + \epsilon_c)k_c)] + \beta[i - (\gamma_k l_k + (\delta_k + \epsilon_k)k_k)]$$
$$\equiv \alpha[(\gamma_c w + (\delta_c + \epsilon_c)(r + p_k)) - p_c] + \beta[(\gamma_k w + (\delta_k + \epsilon_k)(r + p_k)) - p_k]$$
$$\equiv t_A$$

or,

IV.4 $$\alpha t_c + \beta t_k \equiv t_A.$$

In neoclassical terms, then, the aggregate rate of technical change will be equal to the rates of technical change in the two sectors weighted by their relative importance in the total national product.

Following the basic idea and formulation set out in section III, I now derive the correct Harrod–Robinson measures of capital and technical change. I shall deal with the sectoral measures first because there are important problems associated with the idea of the aggregate rate of technical change outside the one-commodity context. It will be shown that the Harrod–Robinson approach can again be applied while even for the simple case where the neo-classical and Harrod–Robinson sectoral measures are the same the aggregate rate of technical change must be carefully interpreted.

The Harrod–Robinson sectoral measures are, then

IV.5a $$c - [\gamma_c l + (\delta_c + \epsilon_c)(k_c - t_k^*)]$$
$$\equiv [\gamma_c w + (\delta_c + \epsilon_c)(r + p_k + t_k^*)] - p_c \equiv t_c^*$$

and

IV.5b $$i - [\gamma_k l_k + (\delta_k + \epsilon_k)(k_k - t_k^*)]$$
$$\equiv [\gamma_k w + (\delta_k + \epsilon_k)(r + p_k + t_k^*)] - p_k \equiv t_k^*$$

It is important to notice that capital being used as an input in the consumption good sector is being adjusted for the improvement in the economic efficiency of the capital good sector. The rate of technical advance in the consumption good sector cannot be calculated independently of the same calculation for the capital good sector.

Thus, a fundamental characteristic of the Harrod–Robinson measures is immediately brought to light. The measures are general equilibrium measures and they reflect, as it is important to do, the *technological interdependence* of complex growing economic systems.

Any measure of the improvement in the efficiency with which any economic system is producing consumption goods must take into account not only the increases in efficiency occurring directly in the production of consumption goods but, as well, the increases in the efficiency of those sectors which are supplying it with produced means of production. To see this point, identities IV.5a and IV.5b may be arranged as

$$c - [\gamma_c l_c + (\delta_c + \epsilon_c)k_c] \equiv [\gamma_c w + (\delta_c + \epsilon_c)(r + p_k)] - p_c$$
$$\equiv t_c^* - (\delta_c + \epsilon_c)t_k^*$$
$$i - [\gamma_k l_k + (\delta_k + \epsilon_k)k_k] \equiv [\gamma_k w + (\delta_k + \epsilon_k)(r + p_k)] - p_k \equiv \gamma_k t_k^*.$$

From identities IV.3a and IV.3b,

$$t_c \equiv t_c^* - (\delta_c + \epsilon_c)t_k^*$$
$$t_k \equiv \gamma_k t_k^*$$

or, in matrix notation

$$\begin{bmatrix} t_c \\ t_k \end{bmatrix} = \begin{bmatrix} 1 & -(\delta_c + \epsilon_c) \\ & \gamma_k \end{bmatrix} \begin{bmatrix} t_c^* \\ t_k^* \end{bmatrix}$$

or

$$\begin{bmatrix} 1 & \dfrac{\delta_c + \epsilon_c}{\alpha_k} \\ 0 & \dfrac{1}{\alpha_k} \end{bmatrix} \begin{bmatrix} t_c \\ t_k \end{bmatrix} = \begin{bmatrix} t_c^* \\ t_k^* \end{bmatrix}$$

This formulation of the relationship between the standard neoclassical measures and the Harrod–Robinson measures reveals a number of important points. First, given the data and information necessary to construct sectoral measures of neoclassical technical change, sectoral measures of Harrod–Robinson change can be constructed *providing the to-whom from-whom information provided by modern input-output accounts is also available.*

Second, the sectoral (or industrial) neoclassical and Harrod–Robinson measures are equal if and only if technical change is occurring solely in the production of the consumption good. From the matrix formulation, if $t_k = 0$, then I have

$$t_c^* = t_c$$
$$t_k^* = t_k = 0.$$

This case reveals how arbitrary the neoclassical measures are and illuminates the problem hidden to some extent by the one-commodity formulation. This case is precisely where the intermediateness of the capital input does not affect the sectoral measures of technical advance for no technical advance is occurring in the production of the capital good.[22] It is only taking place in the production of what Sraffa calls "non-basics." He defines non-basics as

> ". . . "luxury" products which are not used, whether as instruments of production or as articles of subsistence, in the production of others."[23]

and what is most relevant for the point I seek to make, Sraffa goes on immediately to say

> "These products have no part in the determination of the system. Their role is purely passive. If an invention were to reduce by half the quantity of each of the means of production which are required to produce a unit of a "luxury" commodity of this type, the commodity itself would be halved in price, but there would be no further consequences; the price-relations of the other products and the rate of profits would remain unaffected. But if such a change occurred in the production of a commodity of the opposite type, which *does* enter the means of production, all prices would be affected and the rate of profits would be changed."[24]

In the case of technical advance solely in the consumption good sector, the respective measures of technical advance will then be identical:

$$c - [\gamma_c l_c + (\delta_c + \epsilon_c)k_c] = [\gamma_c w + (\delta_c + \epsilon_c)(r + p_k)] - p_c = t_c$$

$$i - [\gamma_k l_k + (\delta_k + \epsilon_k)k_k] = [\gamma_k w + (\delta_k + \epsilon_k)(r + p_k) - p_k] = 0.$$

If I take the price formulation of the measures of technical advance and solve for the proportionate rate of change of commodity prices, I have

$$p_k - p_c = t_c - [\gamma_c w + (\delta_c + \epsilon_c)r] + \frac{\gamma_c}{\gamma_k}(\gamma_k w + (\delta_k + \epsilon_k)r).$$

If long-period equilibrium holds, r = zero, $p_k - p_c = t_c$ and the proportionate rate of change of relative prices is exactly "predicted" by the differences between the sectoral rates of technical change. This is Sraffa's simple result and it is provided by both measures of technical change in the context of the simple general equilibrium economy *only* for the case of technical progress taking place solely in the production of the consumption good.

Suppose technical advance were occurring in both sectors. In this more general case, the neo-classical and Harrod–Robinson measures of technical change part company for the fundamental reason I have outlined. Suppose I take the price formulation and solve again for the proportionate rate of change

[22]C. Kennedy, "The character of improvements and of technical progress," *Economic Journal*, LXXII, December 1962, 899–911.

[23]P. Sraffa, *Production of Commodities by Means of Commodities*, 7.

[24]*Ibid.*, 7–8.

in relative prices. In terms of the neoclassical measures

$$p_k - p_c = t_c - \frac{\gamma_c}{\gamma_k}t_k + \frac{\gamma_c}{\gamma_k}[\gamma_k w + (\delta_k + \epsilon_k)r] - [\gamma_c w + (\delta_c + \epsilon_c)r].$$

In long-period equilibrium again, where $r = 0$,

$$p_k - p_c = t_c - \frac{\gamma_c}{\gamma_k}t_k.$$

In terms of the Harrod–Robinson measures

$$p_k - p_c = t_c^* - t_k^* + \frac{\gamma_c}{\gamma_k}[\gamma_k w + (\delta_k + \epsilon_k)r] - [\gamma_c w + (\delta_c + \epsilon_c)r]$$

and, again, where $r = 0$,

$$p_k - p_c = t_c^* - t_k^*.$$

Given the equilibrium assumptions, I would suppose that all economists would argue that the proportionate rate of change in relative prices should be "predicted" by intersectoral differences in the rates of technical change. This is precisely what the Harrod–Robinson measures show. The neoclassical measures will not—if I exclude the trivial case where $\gamma_c = \gamma_k$ or, where the commodity capital-labour factor intensities of the two sectors are identical.[25] Indeed, it is perfectly possible for the relative prices to be remaining unchanged and for the Harrod–Robinson measures to record correctly identical rates of technical advance in the two sectors while the neoclassical measures could show higher (or lower) rates of technical advance in either sector. Only fortuitously will the neoclassical measures show even the correct *direction* of change in "predicting" the course of relative prices. This critical point shows, in my judgment, how arbitrary are the neoclassical measures of technical change at the sectoral level.

The third point, which follows from the point that Harrod–Robinson measures are general equilibrium measures and that they "predict" logically the movement of relative commodity prices, is to cast further light on the Harrod conception of neutral-technical change. If the rates of technical change in the two sectors are the same and neutral, the *aggregate* commodity capital-output ratio, $V^* \equiv P_k(K_c + K_k)/(P_cC + P_kI)$, will be constant. If the aggregate commodity capital-output ratio is differentiated with respect to time, and re-expressed in proportionate growth rate form, I have

$$v^* \equiv \frac{K_c}{K}k_c + \frac{K_k}{K}k_k + p_k - (\alpha c + \beta i) - (\alpha p_c + \beta p_k)$$

and since $k_c = k_k = c = i$ and $p_k = p_c$, $v^* \equiv 0$. The Harrodian concept is more robust. The rates of technical change, while remaining neutral in both sectors, could be different. If the overall technical advance is neutral, the equilibrium

[25]This was Samuelson's special assumption which permitted a "rigorous" defense of the neoclassical aggregate production function. See P. A. Samuelson, "Parable and realism in capital theory: the surrogate production function," *Review of Economic Studies*, XXIX, June 1962, 193–206.

net rate of return remains unchanged and the aggregate commodity capital-output ratio expressed in terms of consumption goods will remain constant. Take the trivial case of technical change solely in the consumption good sector. Then,

$$v^* \equiv \frac{K_c}{K}k_c - \alpha c + \frac{K_k}{K}k_k - \beta i + p_k - p_c - \beta[p_k - p_c]$$

and since $t_c = p_k - p_c = c - k_c$ and $k_k = i = k_c$, it follows that

$$\begin{aligned} v^* &\equiv \frac{K_c}{K}k_c - \alpha(t_c + k_c) + \left(\frac{K_k}{K} - \beta\right)k_c + (1 - \beta)t_c \\ &\equiv \left[\frac{K_c}{K} + \frac{K_k}{K} - (\alpha + \beta)\right]k_c + [1 - (\alpha + \beta)]t_c \\ &\equiv \text{zero}. \end{aligned}$$

The aggregate capital output ratio, then, when expressed in terms of the *numéraire* consumption good, remains unchanged. Thus, given the constant net rate of return (the marginal rate of transformation of present into future consumption goods) and the constant consumption good aggregate commodity capital-output ratio (the marginal rate of transformation of commodity input into commodity output), the Harrodian conception of neutral technical advance is seen to hold. The aggregate commodity capital–output ratio expressed in physical terms or in terms of constant capital and consumption good prices, however, *does not remain constant.* Such a concept, in proportionate growth rate terms, is

$$v \equiv \frac{(K_c/K)k_c + (K_k/K)k_k}{\alpha c + \beta i}$$

and, again since $k_c = k_k = i$,

$$v \equiv \frac{i}{\alpha c + \beta i}.$$

Only in the case where $i = c$ will v be zero. In the trivial case being discussed $c > i$ and hence $v < 0$. The same argument applies when different rates of technical change are occurring in both sectors. Thus, those statistical studies which examine the stability of the aggregate commodity capital-output ratio in terms of physical units or constant capital and consumption good prices are seen to be misguided.[26] There is nothing in Harrod's formulation (nor in Kaldor's and Robinson's growth theory) which would imply that such a ratio should remain constant. Those studies which showed such ratios unstable, both at the aggregate and sectoral level, in no way counter Harrod's presumption—in fact, they do not deal with his analysis at all.

The basic assumption in the foregoing, that difference in sectoral rates of technical changes and changes in the relative prices of the consumption and capital commodities is consistent with long-period equilibrium, is, of course,

[26]See, for example, E. Domar, "The capital-output ratio in the United States: Its variations and stability," ed. F. A. Lutz, *The Theory of Capital.*

highly questionable from the theoretical viewpoint.[27] In addition, if the underlying technical advances are non-neutral, the assumption of Harrod equilibrium becomes even more tenuous. From the measurement viewpoint, a host of index number ambiguities arise. However, since the neoclassical distinction between "movements along" and "shifts in the production function" which rests on the analytical attempt to treat commodity capital as a primary input is shown to be faulty in an assumed world where such analysis is most comfortable, it may be assumed that the error holds with added force in a world of disequilibrium growth.

The demonstration that Harrod–Robinson measures of technical change are better "predictors" of changes in relative prices is what I need to show how *adjusted* aggregate Harrod–Robinson measures of technical change may be produced. The aggregate measures would appear to be

IV.5 $$[\alpha c + \beta i] - [\gamma l + (\delta + \epsilon)(k - t_k^*)] \equiv [\gamma w + (\delta + \epsilon)(r + p_k + t_k^*)] - [\alpha p_c + \beta p_k] = t_A^*.$$

One immediately evident difficulty is that two rates of technical change must be solved from one identity. Moreover, even if t_k^* were known (say, from the previous sectoral analysis), what meaning can be attached to this aggregate measure of technical change?

By means of the movement in relative prices, the identity may be re-expressed in terms of consumption goods, in which case it is

IV.6 $$[\alpha c + \beta(i + p_k - p_c)] - [\gamma l + (\delta + \epsilon)(k + p_k - p_c - \bar{t}_A^*)] \equiv [\gamma(w - p_c) + (\delta + \epsilon)(r + \bar{t}_A^*)] \equiv \bar{t}_A^*.$$

This recorded rate of technical change I call an *adjusted* measure (to be denoted by $\bar{t}_A^*$).

Expressed in terms of consumption goods, the identity no longer represents solely differences in the proportionate rates of growth of outputs and inputs in the neo-classical sense of "shifting" *aggregate* production function. As I have shown however, there is no meaning to such an idea *even when no aggregation problems arise.*

To explore the idea of an aggregate measure of the rate of technical change, consider again the special case where the neo-classical and Harrod–Robinson sectoral measures yield identical results—the case of technical progress only in the consumption good sector.

It would appear, then, that the neo-classical and Harrod–Robinson rates of technical change are the same. The aggregate neo-classical measures are

$$\alpha[c - (\gamma_c l_c + (\delta_c + \epsilon_c)k_c)] + \beta[i - (\gamma_k l_k + (\delta_k + \epsilon_k)k_k)] \equiv \alpha[\gamma_c(w - p_c) + (\delta_c + \epsilon_c)(r + p_k - p_c)] + \beta[\gamma_k(w - p_k) + (\delta_k + \epsilon_k)(r)] \equiv t_A$$

[27]F. Hahn, "On two sector growth models," *Review of Economic Studies*, XXXII, October 1965, 339–346 and "Equilibrium dynamics with heterogeneous capital goods," *Quarterly Journal of Economics*, LXXX, November 1966, 633–646.

which, under the assumptions reduce to

$$\alpha[c - (\gamma_c l_c + (\delta_c + \epsilon_c)k_c)] \equiv \alpha[\gamma_c(w - p_c) + (\delta_c + \epsilon_c)(p_k - p_c)] \equiv t_A$$

Since, for the Harrod–Robinson measure, one inserts $t_k^* = 0$ into identity IV.5, one gets exactly the same results.

What are these aggregate measures supposed to tell us? Under the assumptions, from the quantity side, the aggregate measure of technical change is αt_c. Thus, if output per man and per machine in the consumption good sector were growing at the rate of (say) 2 percent and the overall rate of *gross* saving were (say) 20 percent, the aggregate rate of technical change would be 1.6 percent. (If the aggregate accounts were expressed in net terms so that the net rate of saving were (say) 10 percent, the aggregate net rate of technical change would be 1.8 percent—again, variance of the conceptual rate of technical change occurs as depreciation is taken into account.) Suppose p_c = zero.[28] Then it must be the case that w and $p_k = t_c = t_c^*$. Returns to labour and capital are both rising at a rate equal to the rate of technical change in the consumption good sector.

Under the conditions postulated, it would appear then that money wages and returns to capital were rising at a rate which exceeded the observed aggregate rate of growth in the efficiency of the economy. The "inflation" which is the logical counterpart of this result appears as the rising price in terms of the consumption good of the capital good. What interpretation then can be placed on the aggregate measure of technical change? It says: Given the recorded rates of change of labour and "waiting" (equal only to commodity capital in the special case of no technical change in the capital good sector), the efficiency of the economy is measured in terms of the ability of such inputs to be transformed into a flow of output—conceived as some weighted average of consumption and capital goods. This interpretation merely puts into words the arithmetic of the aggregate identity. When it is remembered that the capital goods being produced are essentially intermediate inputs, however, such an interpretation has limited meaning.

The aggregate rate of technical change, expressed in terms of consumption goods, is, however, a more meaningful concept. If identity IV.6 is re-examined under the special assumption of technical advance only in the consumption good sector, I have

$$[\alpha c + \beta(i + t_c^*)] - [\gamma l + (\delta + \epsilon)(k + t_c^* - \bar{t}_A^*)]$$
$$\equiv [\gamma(w - p_c) + (\delta + \epsilon)(r + \bar{t}_A^*)] \equiv \bar{t}_A^*$$

so that $c \equiv w - p_c \equiv \bar{t}_A^*$, precisely the result the Harrod–Robinson concept of neutrality would suggest.

I have suggested that an aggregate rate of technical change in an economy of two commodities—one consumption good and one capital good—where the concept is the rate of change of transformation between aggregated physical output and primary inputs is of limited meaning. Transformed into the changing rate of transformation between output and inputs where the outputs and *reproducible* inputs are expressed in terms of the *numéraire* consumption good, the

[28]This is purely an expository assumption. Impose whatever rate of change on p_c—all of the other rates of change of nominal prices (w and p_k) must be adjusted accordingly.

aggregate rate of technical change becomes both meaningful and important for economic analysis.[29] However, because relative prices are incorporated in the adjustment, it follows that the *adjusted* Harrod–Robinson aggregate measures of technical change can in no way be interpreted as an aggregate production function being shifted by advances in technology or a measure of changing technical efficiency since a statement of purely technical arrangements must be derivable independently of the very relative prices the technology is supposed partially to determine.

The most general formulation for the aggregate Harrod–Robinson technical change for the two sector economy under consideration is

IV.7 $$[\alpha c + \beta i] - [\gamma l + (\delta + \epsilon)(k - t_k^*)] \equiv [\gamma w + (\delta + \epsilon)(r + p_k + t_k^*)] - [\alpha p_c + \beta p_k] \equiv t_A^*$$

or

$$\alpha[c - \{\gamma_c l_c + (\delta_c + \epsilon_c)(k_c - t_k^*)\}] + \beta[i - (\gamma_k l_k + (\delta_k + \epsilon_k)(k_k - t_k^*))] \equiv \alpha[\gamma_c w + (\delta_c + \epsilon_c)(r + p_k + t_k^*)] + \beta[\gamma_k w + (\delta_k + \epsilon_k)(r + p_k + t_k^*) - p_k] \equiv t_A^*$$

or

$$\alpha t_c^* + \beta t_k^* = t_A^*.$$

Again, since I know the relationship at the sectoral level between the neoclassical and Harrod–Robinson formulations, given the neoclassical formulation I can derive the Harrod–Robinson versions.

Suppose

$$t_c^* = t_k^*.$$

Then

$$t_A^* = t_c^* = t_k^*.$$

Under the equilibrium conditions postulated, where the relative prices are remaining constant, the aggregate rate of Harrod–Robinson technical change is the same as for both sectoral rates. In this case, re-expression of the accounts in terms of consumption goods will not change the observed rate of technical change. The observed *adjusted* rate of Harrod–Robinson technical change, when the accounts have been transformed into consumption goods, is, from the quantities side,

$$[\alpha c + \beta(i + p_k - p_c)] - [\gamma l + (\delta + \epsilon)(k + p_k - p_c) - \bar{t}_A^*] = \bar{t}_A^*$$

and must be unchanged, since

$$p_k - p_c = t_c^* - t_k^* = 0.$$

[29]Professor Dan Usher, of Queen's University, has also been struck by the limited meaning to be attached to the neoclassical aggregate measures of technical change and stresses that, when transformed into consumption good terms, such measures not only make sense but fit as well the requirements of modern growth theory. See D. Usher, "Two concepts of aggregate technical change" (mimeo) and "How to measure real income, economic growth, and aggregate technical change" (mimeo). I must record my debt to Professor Usher for stimulating comments on some of my earlier work.

Again, for the neoclassical measures, since there is no *a priori* reason for $t_c = t_k$ or for $p_k - p_c = t_c - t_k$, there is no reason for the aggregate neoclassical measure to equal t_c or t_k.

However, what about those cases where $t_c^* \neq t_k^*$ (i.e., where $p_k - p_c = t_c^* - t_k^* \neq 0$)? It follows immediately that $t_A^* \neq t_c^*$ or t_k^*. As I have shown, however, when transformed into consumption goods, I have

$$[\alpha c + \beta(i + t_c^* - t_k^*)] - [\gamma l + (\delta + \epsilon)(k + t_c^* - t_k^* - \bar{t}_A^*)] = \bar{t}_A^*.$$

It is easy to show that this identity collapses to $\bar{t}_A^* = t_c^*$. Indeed, this was what was found when the trivial case of technical advance only in the production of consumption goods was found. It is clear that these two concepts of aggregate Harrod–Robinson technical change are both meaningful *but they are different though related in the way I describe.*

To see this most clearly, consider again the simple case of technical advance solely in the consumption good sector. It was shown that the neoclassical and Harrod–Robinson measures are the same. If the sectoral measures are themselves re-expressed in terms of consumption goods, I will have for the Harrod–Robinson measures

$$c - [\gamma_c l_c + (\delta_c + \epsilon_c)(k_c + p_k - p_c) - \bar{t}_k^*]$$
$$\equiv \gamma_c(w - p_c) + (\delta + \epsilon_c)(p_c + \bar{t}_k^*) \equiv \bar{t}_c^*$$
$$(i + p_k - p_c) - [\gamma_k l_k + (\delta_k + \epsilon_k)(k_k + p_k - p_c - \bar{t}_k^*)]$$
$$\equiv \gamma_k(w - p_c) + (\delta_k + \epsilon_k)(r + \bar{t}_k^*) \equiv \bar{t}_k^*.$$

Solution of these identities reveals that

$$\bar{t}_c^* \equiv \bar{t}_k^* \equiv c - l_c \equiv i + p_k - p_c - l_k.$$

Inspection of these identities shows that $\bar{t}_k^*$ is positive which shows that activity in the capital good sector is transforming present consumption into future consumption with ever-increasing efficiency. Thus $\bar{t}_k^* > t_k^*$. That is, the adjusted measure exceeds the unadjusted one. Similarly, the recorded *adjusted* rate of technical change in the consumption goods sector is also raised so that $\bar{t}_c^* > t_c^*$. Now, however, the two *adjusted* sectoral rates of technical change are equal to one another. Both sectors would be recorded now as showing equal increases as the efficiency with which present consumption goods are being transformed into future consumption goods.

Here, now, the problem thrown up by measures of technical change when re-expressed in terms of consumption goods may be seen. Because changes in relative prices are introduced, the measures no longer reflect changing conditions between outputs and inputs in physical units—as I have said, prices should not enter into statements about technology. As a consequence, a further difficulty arises in connection with such adjusted measures. In our simple economy, if no technical progress at all were occurring but a change in the rate of saving were to occur, the relative prices of the two commodities would, in the general case, change. If changes in relative prices were used, adjusted measures would incorrectly reflect this as technical change—and this must be a fatal objection to using movements in relative prices to arrive at adjusted aggregate measures of technical change.[30]

It is extremely important, however, to note one fundamental fact at this point. In long-period equilibrium, re-expression of the aggregate measure of technical change in terms of the consumption good *either by means of relative prices or by means of the relative unadjusted Harrod–Robinson rates of technical change in the two sectors amounted to the same thing.* Outside of long-period equilibrium, however (say where a change in the rate of saving is occurring), re-expression of the aggregate measure of technical changes in terms of the consumption goods either by means of relative prices or by means of the sectoral Harrod–Robinson rates of technical change will *not* yield the same result. The former would take into account non-technical change while the latter would take into account only the changing *technical* ability of the system to transform intermediate inputs into a flow of the consumption good. Thus, the adjusted aggregate measure of technical change in the latter sense would remain a meaningful measure of such technical change because changing relative prices would *not* be employed in its construction.

The unadjusted sectoral Harrod–Robinson measures remain valid and meaningful. They reflect the changing efficiency by which primary inputs are being transformed into outputs. The aggregate *unadjusted* Harrod–Robinson measures are also of validity and their meaning is clear provided relative prices are remaining unchanged. When relative prices are changing, the aggregate *unadjusted* measure will not adequately reflect the rate at which real incomes in terms of the consumption good are rising and their meaning in such cases is thus less clear. This reflects, in my judgement, the great care which must be employed in interpreting them as measures of *aggregate* technical change.

I have shown that at the sectoral and aggregate levels, the neo-classical attempt to distinguish between shifts in and movements along the production function breaks down. One is left with two choices. One produces either Harrod–Robinson measures at the sectoral and aggregate level in the manner described or the adjusted measures. The former concentrates and rigorously takes into account the essential intermediateness of the non-primary commodity capital *input*. The second takes it into account as well on the *output* side. The former provides the means by which the unadjusted measures may be re-expressed in terms of the consumption good—because as has been demonstrated, reliance upon changing relative prices alone in measuring adjusted aggregate technical change confuses technical change with a possible movement in relative prices owing to some development in the economy not associated with technical change.

[30]The same objection must be raised against Hick's sophisticated aggregate production function. See J. R. Hicks, *Capital and Growth*, Chapter XXIV and my *On Concepts of Capital and Technical Change*, Chapter VIII. This was also the objection raised by Solow against Pasinetti's concept of productivity. See L. L. Pasinetti, "On Concepts and measures of changes in productivity," *Review of Economics and Statistics*, XLI, August 1959, 270–282 and accompanying comments by Solow.

V. Multi-Sectoral Multi-Commodity Economies

The principle difference between neo-classical and Harrod–Robinson concepts of technical change is now clear. In the long-period context of technical change, the former makes no allowance for the essential intermediateness of the capital inputs while the latter does. The Harrod–Robinson approach takes into account the changing economic efficiency of inputs which are truly primary to the economic system. The former is a hybrid. It takes into account the changing economic efficiency of inputs which are both primary and intermediate. It measures the outputs and inputs of such intermediate reproducible inputs in the same terms and hence is an incomplete measurement of the rate of technical change.

In the case of a simple economy in long-period equilibrium again with homogeneous labour but now with many consumption and many capital goods (some appearing as intermediate inputs), the sectoral accounting identities will appear as

V.1 $$Pc_iC_i \equiv wL_i + R\sum_j Pk_jK_{ji} + \sum_j \lambda_j Pk_jK_{ji} \qquad i = 1, \ldots, m$$
$$Pk_jI_j \equiv wL_j + R\sum_j Pk_jK_{jj} + \sum_j \lambda_j Pk_jK_{jj} \qquad j = m+1, \ldots, z.$$

For some of the capital inputs the rate of "using-up" is one. (By a rate of "depreciation by evaporation" equal to one, I mean that the commodity inputs are fully used up or transformed during the "year" in the process of production.) They are intermediate inputs. If production is fully integrated in long-period equilibrium, there will be no stocks of raw materials nor final goods but, if consideration is paid to the gestation period of capital goods, there will, of course, be many kinds of commodity capital stocks appearing as goods-in-process.

Transforming identities V.1 in the familiar way, I have, for the neo-classical measures,

V.2 $$c_i - \left[\gamma_i l_i + \sum_j (\delta_{ji} + \epsilon_{ji})(k_{ji})\right]$$
$$\equiv \left[\gamma_i w + \sum_j (\delta_{ji} + \epsilon_{ji})(r + pk_j) - p_i\right] = t_i \qquad i = 1, \ldots, m$$
$$i_j - \left[\gamma_j l_j + \sum_j (\delta_{jj} + \epsilon_{jj})(k_{jj})\right]$$
$$\equiv \left[\gamma_j w + \sum_j (\delta_{jj} + \epsilon_{jj})(r + pk_j) - pk_j\right] = t_j \qquad j = m+1, \ldots, z$$

For Harrod–Robinson measures, I have

V.3 $$c_i - \left[\gamma_i l_i + \sum_j (\delta_{ji} + \epsilon_{ji})(k_{ji} - t_j^*)\right]$$
$$\equiv \left[\gamma_i w + \sum_j (\delta_{ji} + \epsilon_{ji})(r + pk_j + t_j^*) - p_i\right] \equiv t_i^*$$

$$i_j - \left[\gamma_j l_j + \sum_j (\delta_{ji} + \epsilon_{ji})(k_{jj} - t_j^*)\right]$$

$$\equiv \left[\gamma_j w + \sum_j (\delta_{jj} + \epsilon_{jj})(r + pk_j + t_j^*) - p_j\right] \equiv t_j^*$$

$$i = 1, \ldots, m$$

$$j = m + 1, \ldots, z$$

Again, the intermediate nature of the capital goods is being rigorously taken into account in the Harrod–Robinson formulation and the technological interdependence of the economy is fully considered. Indeed, inspection of identities V.3 shows that there are $m\ t_i^*$'s and $(z - m)\ t_j^*$'s to be determined from the z identities.

All of the discussion pertinent to the two commodity case carries over to the more general multi-sectoral case. For example, if all the $(z - m)\ t_j^*$'s were zero, showing no technical advance in *any* of the capital good sectors, the neo-classical and Harrod–Robinson measures for all the sectors will be identical.[31] The multi-sectoral formulation shows how important the Leontief-type information on the technological interdependence of the economy is for the measurement of Harrod–Robinson technical change.

There is one further point which can be made in the multi-sectoral context. The aggregate neo-classical measure of technological change becomes

V.4 $$\left[\sum_i \alpha_i c_i + \sum_j \beta_j k_j\right] - \left[\gamma l + \sum_j (\delta_j + \epsilon_j) k_j\right]$$

$$\equiv \left[\delta w + \sum_j (\delta_j + \epsilon_j)(r + pk_j)\right] - \left(\sum_i p_{c_i} + \sum_j \beta_j pk_j\right) \equiv t_A.$$

How are the various sectoral rates of technical change aggregated up to this result? Each sector's output can be expressed in net terms [gross output less intermediate inputs including capital consumption on durable commodity capital], the neo-classical sectoral rates of technological change calculated in net terms and weights equal to the net output in each sector attached to derive the aggregate measure.

For each sector, I have the values of net output

V.5 $$P_{ci} C_i - \sum_j \lambda_j Pk_j K_{ji} = wL_i + R \sum_j Pk_j K_{ji} \qquad i = 1, \ldots, m$$

$$Pk_j I_j - \sum_j \lambda_j Pk_j K_{jj} = wL_j + R \sum_j Pk_j K_{jj} \qquad j = m + 1, \ldots, z.$$

[31] If the economic system were decomposable such that the production of some consumption goods were carried on by means of capital inputs, *directly or indirectly*, in the production of which no technical change was going on, then the neoclassical and Harrod–Robinson measures of technical change for these sectors would again be the same.

The neo-classical sectoral measures of net technical change are, then[32]

$$\text{V.6} \qquad \frac{c}{1-\Sigma_j \epsilon_{ji}} - \frac{\Sigma_j \epsilon_{ji}(k_{ji})}{1-\Sigma_j \epsilon_{ji}} - \left[\frac{\gamma_i(li)}{1-\Sigma_j \epsilon_{ji}} + \frac{\Sigma_j \delta_{ij}(k_{ji})}{1-\Sigma_j \epsilon_{ji}}\right]$$

$$\equiv \frac{\gamma_i(w)}{1-\Sigma_j \epsilon_{ji}} + \frac{\Sigma_j \delta_{ij}(r+pk_j)}{1-\Sigma_j \epsilon_{ji}} - \left[\frac{1(p_c)}{1-\Sigma_j \epsilon_{ji}} - \frac{\Sigma_j \epsilon_{ji}(pk_j)}{1-\Sigma_j \epsilon_{ji}}\right]$$

$$\equiv t_{i_N} \qquad i = 1, \ldots, m$$

and

$$\frac{i}{1-\Sigma_j \epsilon_{jj}} - \frac{\Sigma_j \epsilon_{jj}(k_{jj})}{1-\Sigma_j \epsilon_{jj}} - \left[\frac{\gamma_j(l_j)}{1-\Sigma_j \epsilon_{jj}} + \frac{\Sigma_j \delta_{jj}(k_{jj})}{1-\Sigma_j \epsilon_{jj}}\right]$$

$$\equiv \frac{\gamma_j(w)}{1-\Sigma_j \epsilon_{jj}} + \frac{\Sigma_j \delta_{jj}(r+p_k)}{1-\Sigma_j \epsilon_{jj}} - \left[\frac{(pk_j)}{1-\Sigma_j \epsilon_{jj}} - \frac{\Sigma_j \epsilon_{jj}(pk_j)}{1-\Sigma_j \epsilon_{jj}}\right] = t_{j_N}$$

$$j = m+1, \ldots, z.$$

For the reasons outlined in the one and two commodity cases there is no reason to expect the neo-classical net measures to equal the gross measures of technical change.[33]

The aggregate neoclassical net measure of technical change will then be

$$\text{V.7} \qquad \sum_{i=1}^{m} \psi_i(t_{i_N}) + \sum_{j=m+1}^{z} \psi_j(t_{j_N}) = t_{AN}$$

where ψ_i is the share in total output of the ith consumption good sector and ψ_j is the share of the jth capital good sector.

The one further point thrown up by consideration of these net neoclassical multi-sectoral measures is the questionable nature of the net output concept at the industry or sectoral level. The identities V.6 show the rate of change of net output as a weighted average (positive and negative weights) of the rate of change of gross outputs and the rate of change of intermediate inputs. Intermediate inputs are commodity inputs and by their subtraction, the neoclassical measures eliminate from the measures of gross output the effect of shifting along the production functions with respect to intermediate inputs.

Consider a simple case where

$$Q = Q(L, K, M) \quad \text{and} \quad Q - \epsilon M = Q(L, K).$$

Gross output is a function of labour and the services of commodity capital in stock form and in flow form. The neoclassical gross measure of the rate of technical change will be

$$q = [\gamma l + \delta k + \epsilon m] \equiv t_G$$

[32]It will be noted that, in the identities which follow, the left hand terms in the quantities approach are Divisia indexes of the net outputs of the various sectors while the right hand terms in the prices approach are Divisia indexes of the price of net output. In more standard terminology, they are respectively quantum and price indexes of net output produced by the so-called "double deflation" method.

[33]On this point, see E. D. Domar, "On the measurement of technological change," *Economic Journal*, LXXI, December 1961, 709–729.

and in net terms will be

$$\frac{q}{(1-\epsilon)} - \frac{\epsilon m}{(1-\epsilon)} - \left[\frac{\gamma l}{(1-\epsilon)} + \frac{\delta k}{(1-\epsilon)}\right] \equiv t_N.$$

Thus, a distinction is being drawn up between technical change with the production function expressed in net terms and commodity capital stock accumulation with commodity capital flow accumulation netted out.

I have shown that the neoclassical attempt to distinguish between technical change and capital accumulation is a fallacious one. In the measurement of *net* output by sector, the fallacy shows up with a vengeance. In an economy where multi-sectoral technical change is going on, because the net measures of sectoral output incorporate the neoclassical distinction, they are, in my judgment, meaningless measures of output.

It has long been known that net measures of actual output can yield peculiar results. If a sector's intermediate inputs are falling relatively in prices owing to technical advance taking place *directly or indirectly* in their production so that, in constant price terms, the ratio of intermediate inputs to gross output is rising, the net measure of output can show small changes. Indeed, the *level* of the *index* of net output can even fall to negative levels. In such an extreme form, this phenomenon has always been brushed aside as a severe index number problem.[34] Everyone admits that in such extreme cases the measures have no meaning. Even in less extreme cases, however, the measures defy meaningful interpretation for the reasons I have advanced.

Thus, one of the more important practical conclusions which follows from the analysis of this paper is that the standard measures of net output, produced by the so-called "double deflation" approach, are seen to be invalid.[35] The basic reason, again, is that accumulation of intermediate inputs are netted out or, in other words, that a separation of changes in output into two components, accumulation and technical change, is attempted and, as I have tried to show, such a separation is without theoretical foundation. It also follows from the analysis that Harrod–Robinson measures of sectoral change continue to be valid and aggregate measures of Harrod–Robinson technical change can be prepared, both in their unadjusted and adjusted forms.

Given the multi-sectoral measures of the Harrod–Robinson sectoral rates of technical change, the aggregate Harrod–Robinson measure can be produced in the manner illustrated in the two commodity two sector case. Again, if the sectors were all experiencing the same rate of technical change in steady balanced long-period equilibrium, the aggregate measure is meaningful and invariant for any aggregation procedures. Again, if the sectors are not experiencing the same rate of technical change, the aggregate measure has the same meaning outlined in the discussion of the two commodity two sector case.

[34]See the articles by P. David, "The deflation of value added," *Review of Economics and Statistics*, LXIV, May 1962, 148–155 and "Measuring real net output: a proposed index," *ibid.*, XLVII, November 1966, 419–425.

[35]The same conclusion is arrived at by C. A. Sims, "Theoretical basis for a double deflated index of real value added," *Review of Economics and Statistics*, LI November 1969, 470–471.

VI. The "Reswitching" Controversy

In recent years, a discussion has broken out on the question of whether, comparing economies with the same technology in long-period equilibrium, there is any reason to expect an inverse monotonic relationship between the net rate of return to capital and the *value* of capital per unit of labour.[36] In the one commodity "smooth" production function neoclassical case, larger stocks of commodity-capital per unit of labour are associated with a lower marginal physical product of commodity capital and, under the assumption of competitive factor pricing, a lower net rate of return to capital. In the many capital goods case, where production functions are "smooth"—and gestation periods of capital goods are ignored, there will continue to be the traditionally assumed relationship between the value of capital per unit of labour—*expressed in terms of a constant set of relative prices of capital goods*—and the net rate of return.[37] However, for linear technologies and for cases where the gestation periods of capital goods are taken into account,[38] there need not be. What are the implications of the debate for this paper?

If comparisons amongst economies are drawn up on the assumption that they employ the same technology, then can one seek to explain differences in real income per head in terms of differences in inputs per head?[39] I shall assume two economies with the same homogeneous labour force, producing one homogeneous consumption good with different bundles of the same heterogeneous capital goods (including goods-in-progress)—each bundle pertaining to the particular technique selected by each economy from the same available technology. Both economies are deemed to be in long-period stationary state equilibrium. The aggregate economic accounts per unit of labour of Economy A will be

VI.1 $$C_A \equiv W_A + R_A \sum_j P_{K_J} K_{J_A}$$

and for Economy B will be

VI.2 $$C_B \equiv W_B + R_B \sum_j P_{K_{j_B}} K_{j_B}$$

where the W's and P_{K_J}'s are expressed in terms of the homogeneous consumption good. To compare these two economies, their accounts must be expressed in a constant set of prices. Re-expressing Economy B in terms of A's prices and comparing them by subtraction, I have

VI.3 $$C_B - C_A \equiv R_A \sum_j P_{K_{jA}}(K_{jB} - K_{jA})$$

[36]For a review of aspects of the controversy, see G. C. Harcourt, "Some Cambridge controversies in the theory of capital" *Journal of Economic Literature*, VII, June 1969, 369–405.

[37]It is important to note that in such a valuation a constant set of relative prices is being employed.

[38]See, for example, D. M. Nuti, "Capitalism, socialism and steady growth," *Economic Journal*, LXXX, March 1970, 32–57.

[39]For an example of a study which sought to explain the difference in income per head between the U.S.A. and Canada in terms of differences in inputs per head *and* differences in technology, see D. Walters, *Canadian Income Levels and Growth. An International Perspective* Economic Council of Canada Staff Study No. 23 (Ottawa: Queen's Printer, 1968).

In the Divisia index number approach, I transform the aggregate economic accounts

$$C \equiv WL + R\sum_j P_jK_j$$

into proportionate growth rate form to get

VI.4 $$c - \sum_j \delta_j k_j \equiv \gamma w + \sum_j \delta_j(r + p_{K_j}) \equiv 0$$

where again $\delta_j = RPK_jK_j/C$, etc.

Identity VI.3 states that the economy with the higher consumption per unit of labour will have *identically* the higher capital per head but, in the light of the "reswitching" discussion, no causal significance at all can be attached to the measures.

It can no longer be assumed that, in a comparison of countries using techniques drawn from the same technology, differences in the constant price value of capital per unit of labour "explain" differences in the constant price value or output or consumption per unit of labour.

Whether or not "reswitching" does occur would appear to be an empirical phenomenon.[40] However, the basic reason why "reswitching" *can* occur, in my view, is that capital goods are produced means of production. As a consequence, no aggregate stock of capital can be used to determine the rate of interest (as has, of course, always been known) and no aggregate stock of capital can be used, along marginal productivity lines, to explain differences amongst economies in consumption or real income.

The "reswitching" controversy, though related to this paper, is not essential to it. For the "reswitching" problem is one of aggregation. As I have tried to show in this paper, however, the invalidity of the neoclassical distinction, in explaining differences in output per unit of labour input over time or across economies between capital accumulation and technical change, has been demonstrated for economies where no aggregation problems arise. I have shown that such a distinction falls down even in simple one-commodity economies where by assumption (e.g., a smooth production function, competitive factor pricing, etc.) the neoclassical analysis is most at home. Thus, the fact that the stock and flow forms of commodity capital are produced means of production appears to be the critical phenomenon which must be taken into account in explaining inter-country comparisons in consumption and output and "sources of growth" for economies over time.

VII. The Problem of Quality Change

Up to this point in my paper I have been making the assumption that, over time, the commodity capital inputs retain their identifiable physical characteristics. This is unrealistic. In the case of an economy altering its techniques of production in response to variations in the net rate of return, different techniques are liable to involve different commodity capital goods with different

[40]The important theoretical question thrown up by the debate is: What are the determinants of the net rate of return to capital. For a more extensive review of the "reswitching" problem, see my *On Concepts of Capital and Technical Change*, Chapters 2 and 4.

physical characteristics. In the dynamic case, technical change may appear as new commodity capital goods again with new physical characteristics. Indeed, it is this latter phenomenon which gives rise to "depreciation by obsolescence."

Old capital goods fall in price relative to new capital goods as time goes by because the introduction of new capital goods associated with new techniques cause higher real wage rates to be paid to labour associated with old capital goods which, depending on the ease with which labour may be taken away from them, causes the flow of returns to capital earned by old capital goods to fall To preserve the competitive position and use of such old capital goods their price must fall. For the oldest capital good just passing out of existence real wages exhaust the product to be produced with such goods and their competitive market price falls to zero.

Consider again the simple "one commodity" economy. The stock of capital in such an economy will be made up of various "vintages" of capital goods. In steady equilibrium, the average age of the capital stock is constant and a set of relative prices of the stock of such vintage capital goods will exist which permits the computation of the value of the *net* stock of capital. In steady growth equilibrium, the growth rate of the constant price net stock of capital will be equal to the growth rate of output, itself equal to the growth rate of the homogeneous labour input and the rate of Harrod–Robinson neutral technical change.

The accounts for the "one-commodity" economy will be

VII.1 $$Q \equiv WL + R\sum_i P_{K_i}K_i - \Sigma_i pk_i P_{k_i}K_i$$

where $P_{K_i}K_i$ is the value of the ith vintage component of the stock of capital, $\Sigma_i P_{K_i}K_i$ is the value of the net stock of capital and $-\Sigma_i pk_i P_{k_i}K_i$ is the value of "depreciation by obsolescence," where pk_i is the proportionate rate of change owing to depreciation in the price of the ith vintage commodity capital, all relative prices being expressed in terms of the latest commodity being produced. [Thus, even when nominal inflation is proceeding, pk_i will be negative if technical advance is causing "depreciation by obsolescence."][41]

[41]Portfolio balance across the vintages of the capital goods requires, for one period returns, that

$$-P_{K_i}\frac{(+\delta Q/\delta K_i) + P_{K_i}(1 + p_{K_i})}{1 + R} = 0$$

where PK_i is the relative price of vintage i, $\delta Q/\delta K_i$ is the marginal physical product of vintage i, p_{K_i} is the proportionate rate of change in the ith vintage's relative price and R is the *real* rate of interest. Re-expressed, I have

$$1 + R = \frac{\delta Q}{\delta K_i}/P_{K_i} + 1 + p_{K_i}$$

or

$$R = \frac{\delta Q}{\delta K_i}/P_{K_i} + p_{K_i}$$

or

$$RP_{K_i} = \frac{\delta Q}{\delta K_i} + p_{K_i}\cdot P_{K_i}$$

that is, the net rental on the ith vintage capital good equals its marginal physical product less any capital loss owing to "depreciation by obsolescence." For all capital goods of the ith

[*Continued on next page*

If the standard manipulations on identity VII.1 are performed, I have

VII.2
$$q - \left[\gamma l + \sum_i (\delta_i + \epsilon_i)k_i\right] \equiv \gamma w + \sum_i \delta_i(\bar{p}k_i + r) + \sum_i \epsilon_i(\bar{p}k_i + \dot{p}k_i/pk_i) \equiv t$$

where t is again the neoclassical rate of technical change, k_i is the rate of growth of capital of the ith vintage, $\bar{p}k_i$ is the proportionate rate of change in the relative price of the ith vintage and $\dot{p}k_i/pk_i$ is the proportionate rate of change in the proportionate rate of change of the ith vintage *as it changes to the i + 1th vintage.*

It is necessary to be clear on the distinctions raised. In a steadily growing system, the amount of commodity capital of the ith vintage will be growing at the rate q. For example, the amount of commodity capital twenty "years" old will be constant. "Twenty years" ago, one hundred machines will have been constructed, "nineteen" years ago, one hundred and ten. "Today" the number of machines being twenty years old will be increasing from one hundred to one hundred and ten—a 10 percent growth rate. The growth rate $\bar{p}k_i$ is to be similarly interpreted. The growth rate $\dot{p}k_i/pk_i$ deals with any acceleration or deceleration in the decline in value to which commodity capital of the ith vintage would be subject owing to changes in the rate of "depreciation by obsolescence" brought about (say) by a change in the rate of Harrod–Robinson neutral technical change.

In steady state equilibrium, $\bar{p}k_i$, r and $\dot{p}k_i/pk_i$ = zero, and identity VII.2 collapses to

VII.3
$$q\left[\gamma l + \sum_i (\delta_i + \epsilon_i)k_i\right] \equiv \gamma w \equiv t$$

This is fundamentally no different from the one commodity neoclassical measurement of technical change examined in Section II of this paper. It is equally faulty and must be replaced by the Harrod–Robinson equivalent

VII.4
$$q - \left[\gamma l + \sum_i (\delta_i + \epsilon_i)(k_i - t^*)\right] \equiv \gamma w + \sum_i (\delta_i + \epsilon_i)t^* \equiv t^*.$$

Again, the essential intermediateness of the commodity capital inputs are being

vintage, the net rentals must be

$$RP_{K_i}K_i = \frac{\delta Q}{\delta K_i} K_i + pk_i \cdot P_{K_i} K_i$$

for all vintages

$$R \sum_i P_{K_i}K_i = \sum_i \frac{\delta Q}{\delta K_i} K_i + \sum_i pk_i \cdot P_{K_i}K_i$$

the total gross returns to capital must be then

$$R \sum_i P_{K_i}K_i - \sum_i p_{K_i}P_{K_i}K_i = \sum (\partial Q/\partial k_i)K_i$$

as was shown in identity VII.1. These gross returns to capital are, of course, Jorgenson's gross rentals where $RP_{K_i} - p_{K_i}P_{K_i}$ is the price of the service of the ith capital good. See Christensen and Jorgenson, *op. cit.*

rigorously taken into account in the Harrod–Robinson version. All of the argument, therefore, of the preceding sectors of the paper can be carried over to this case, *mutatis mutandis.*[42]

The *value* of the net stock of capital is $K_N \equiv \Sigma_i P_{K_i} K_i$. The rate of growth of the net stock of capital in constant prices is

$$\dot{K}_N/K_N - \sum \frac{Pk_i K_i}{\Sigma Pk_i K_i}(\bar{p}_{k_i}) \equiv \sum \frac{P_{k_i} K_i(k_i)}{\Sigma Pk_i K_i}.$$

The value of depreciation is $D \equiv \Sigma_i pk_i Pk_i K_i$. The rate of growth of depreciation or capital consumption allowances is

$$\frac{\dot{D}}{D} - \sum \frac{\dot{p}k_i Pk_i K_i(\bar{p}k_i + \dot{pk_i}/pk_i)}{\Sigma_i pk_i Pk_i K_i} \equiv \sum \frac{pk_i Pk_i K_i}{\Sigma_i pk_i Pk_i K_i}(k_i).$$

The rate of growth of the homogeneous labour input is l. If these proportionate growth rates are weighted by their relative shares and subtracted from the rate of growth of output, I have

$$q - \left[\gamma l + \Sigma_i \delta_i \cdot \sum \frac{Pk_i K_i(k_i)}{\Sigma Pk_i K_i} + \Sigma_i \epsilon_i \cdot \sum \frac{pk_i Pk_i K_i}{\Sigma_i pk_i Pk_i K_i}(k_i)\right] = t.$$

Since $\Sigma_i \delta_i = R\Sigma Pk_i K_i/Q$ and $\Sigma_i \epsilon_i = \Sigma_i pk_i Pk_i K_i/Q$ the above expression reduces to

$$q - [\gamma l + \Sigma_i(\delta_i + \epsilon_i)k_i] = t$$

exactly identity VII.3.

As I have shown, this neoclassical formulation of technical change is wrong. Again the Harrod–Robinson correction must be made to yield

$$q - [\gamma l + \Sigma_i(\delta_i + \epsilon_i)(k_i - t^*)] = t^*.$$

The same remarks pertain to the measurement of technical change in terms of prices.

In steady equilibrium,

$$q = \sum \frac{P_{k_i} K_i(k_i)}{\Sigma P_{k_i} K_i}$$

and, consequently, the neoclassical rate of technical change reduces to

$$\gamma(q - l) = t$$

and the Harrod–Robinson measure to $q - l = t^*$ exactly as was seen before.

[42]In Solow's original formulation of the "vintage" model of capital accumulation, by ensuring that the homogeneous labour force was so distributed to equalize the marginal product of labour over all vintages, it was possible to move from

$$Q(i, t) = f(L(i, t), K(i, t))$$

—the production function appropriate "today" for the ith vintage—to an aggregate production function

$$Q(t) = F(L(t), J(t))$$

where $J(t)$ represented the stock of "efficient" or embodied capital. (See R. M. Solow, "Investment and technical progress," ed. K. J. Arrow, *Mathematical Methods in the Social Sciences.*) It is well known that, under equilibrium conditions, Solow's J concept will show the same growth rate as the constant price net stock of capital. (See, for example, M. Brown, *On the Theory and Measurement of Technological Change.*)

Sectoral vintage measures of technical change may be constructed. One would refer to the growth rate of the *net* stock of capital and depreciation in the consumption and capital goods sectors. The analysis contained in previous sections would be repeated.

In the foregoing it has been assumed that q, which, it will be remembered, is related to Q_1/Q_0 or, in general, to the Laspeyres index number, $\Sigma P_0 Q_1/\Sigma P_0 Q_0$, can, in fact, be measured. How is that to be done when, in fact, Q_1 is physically different from Q_0?

In discussions on quality change it has been established that where physical characteristics of commodities alter—in short, where models change—a price index for such commodities can be maintained if model prices overlap.

In the case of an economy *not* experiencing technical change but "switching" over from one technique to another because of changes in the net rate of return, at the "switch" point the relative prices for *all* models associated with the different techniques will be available. In the real world the relative prices will not, of course, be the long-period equilibrium prices they are in the literature dealing with the "re-switching" phenomenon. The usual national accounting conventions about when (market shares, etc.) the price overlap information pertaining to the different models can be introduced will apply. Quarrels about the timing can take place but the principle is clear.

Where no overlap information is available—where the new model entirely supplants the old—the overlap must be constructed. Two methods are available. The well-known hedonic price indexes, where the price of the new model in the period to be compared with that of the old model is constructed by means of assessing the price weights of the characteristics of the new model on the basis of information pertaining to the old model, are just one such device.

If the characteristics (not just their rearrangement) of the models have changed, the hedonic price index approach will not work.[43] Resort must be had to an attempt to produce the required overlap information on the basis of comparing unit costs of production of the two models in the same time period. Thus, if the unit input requirements of producing the new model in the same time period in which the old model was produced could be estimated, then the base period *input* prices would be assigned to estimate the base period price for the new model.

Since the estimated price of a model using the characteristics approach must be the same as that derived from the cost of production approach, they must in principle yield the same result. It is sometimes argued that the cost of production approach fails to capture any "costless" improvements in the quality of the capital good. This argument, advanced by Professor R. J. Gordon,[44] attempts to incorporate into the commodity capital input measures the profits earned in disequilibria when new capital goods are introduced. It is, in my judgment, an attempt to measure capital inputs in terms of the output they produce and hence

[43]An excellent survey of hedonic price indexes is given in R. J. Gordon, "Recent developments in the measurement of price indexes for fixed capital goods," a paper presented to the Business and Economic Statistics Section of the American Statistical Association, Detroit, December 29, 1970.

[44]See R. J. Gordon, *ibid.*, III-21ff.

eliminate *all* technical change.[45] In the one commodity "vintage" model, "costless" improvements in the commodities being produced will appear as technological change which is surely where such improvements should show up.

I conclude that the conventions followed by national accountants in handling "quality change" are in principle correct, though debates can occur about the timing and statistical precision of such conventions. However, the important point is this: Whatever the constant price measures of commodity capital input used in measuring technical change, the Harrod–Robinson procedures *must* be followed. Whatever the constant price measures are conventionally accepted to be, the neoclassical concepts and measures are erroneous.

VIII. Conclusion

In this paper, I have examined the measurement of technical change for economies evidencing increasing complexity from one to multi-commodity worlds. In *all* cases, one result seems clear. Neoclassical measures are founded on a basic error—they neglect the fact that capital goods of all kinds—whether they appear as flows (intermediate inputs, rentals of capital equipment, etc.) or stocks (fixed capital goods, inventories, etc.) are produced means of production and proper measures of technical change *must* take into account the fact that in technically progressive economies, such capital goods themselves are being produced with ever-increasing efficiency. I have tried to show that this is precisely what is involved in the Harrod–Robinson conceptions of technical change and such measures can be made operational. Moreover, I have argued that the Harrod–Robinson measures, at the disaggregated level, capture the technological independence of modern economies in a precise and meaningful way. One need not operate at the aggregate level—indeed, for a world of heterogeneous capital goods, aggregate concepts such as production functions, aggregate measures of technical change etc., break down. The basic point is that the fundamental neoclassical distinction between shifts in a "production function" and movements along that "function" is seen to be logically and theoretically at fault. To measure the "national accounts" in "real" terms along Harrod–Robinson lines, given the need for information on the sectoral independence of the economy, is a big task but that is the direction in which empirical research in the measurement of technical change should now turn.

[45] I should point out that in correspondence with Professor Gordon, he denies this is his intention. But see T. W. Schultz, *Transforming Traditional Agriculture*, Chap. 9, Factors of Production concealed under "Technological Change." I think the logic of Professor Gordon's approach will lead to the results postulated by Schultz—viz., that evidence of technical change is evidence that some (capital) factor of production has not been adequately taken into account.