NOTES AND COMMUNICATIONS

TERMS OF TRADE EFFECT, PRODUCTIVITY CHANGE AND NATIONAL ACCOUNTS IN CONSTANT PRICES: A FURTHER COMMENT

BY RAYMOND COURBIS

University of Tours and Institut National de la Statistique et des Etudes Economiques, Paris, France

1. It is with great interest that I have read the comment [1] of Mr. Y. Kurabayashi in reply to my previous comment [2]. It was deliberately that I had treated only the terms of trade effect formulation but I agree with him that an important point of his paper [3] concerns the problem of the relation between the terms of trade effect arising from relative changes in prices between outputs and inputs on the one hand, and changes in factor productivity on the other hand. As Kurabayashi's reply concerns essentially this problem, I think now that it is also of importance to discuss, more especially as I had treated it in 1967 [4] and 1968 [5].

This note will therefore be relative to this problem. First, in the following section, I shall briefly review the total factor productivity problem in the framework of national accounts in constant prices. Then, I shall discuss the link which exists between terms of trade gain and factor productivity gain (section 3).

2. According to Kurabayashi's decomposition, let us note by U, W and V the value in current prices of intermediate inputs, labour inputs and other inputs expenditures; by $\tilde{U}$, $\tilde{W}$ and $\tilde{V}$ the corresponding volumes (values in constant prices); by Y and $\tilde{Y}$ the value and the volume of output; by X and $\tilde{X}$ the value and the volume of total factor inputs; by p_u, p_w, p_v, p_x and p the price indexes relative respectively to U, W, V, X and Y; and by q_u, q_w, q_v, q_x and q the corresponding volume indexes.

With this notation, the index of total factor productivity from the base year $t = 0$ to the current year t is by definition:

$$\pi = \frac{\tilde{Y}/\tilde{Y}_0}{\tilde{X}/\tilde{X}_0} \tag{1}$$

where $\tilde{Y}$ and $\tilde{X}$ relate to the current year and $\tilde{Y}_0$ and $\tilde{X}_0$ to the base year.

The unit productivity gain $\tilde{g}$ (i.e. total diminution of factors required by production per unit of output) will be equal to:

$$\tilde{g} = 1 - \frac{1}{\pi} = 1 - \frac{\tilde{X}/\tilde{X}_0}{\tilde{Y}/\tilde{Y}_0} \tag{2}$$

and consequently the total productivity gain $\tilde{G}$ equal to:

$$\tilde{G} = \tilde{Y}\tilde{g} = \tilde{Y} - \tilde{Y}_0\frac{\tilde{X}}{\tilde{X}_0} \tag{3}$$

If we suppose that *all* components of Y correspond to production factors (i.e. $Y = U + W + V$), we have $\tilde{X}_0 = X_0 = Y_0 = \tilde{Y}_0$ and so:

$$(4) \qquad \tilde{G} = \tilde{Y} - \tilde{X}$$

$\tilde{Y}$ and $\tilde{X}$ are the deflated values (the values at constant prices) of output and total factor inputs. As $\tilde{X} = \tilde{U} + \tilde{W} + \tilde{V}$, we have:

$$(5) \qquad \tilde{G} = \tilde{Y} - [\tilde{U} + \tilde{W} + \tilde{V}].$$

At constant prices, the production account $Y = U + W + V$ is no longer balanced and *it is necessary to introduce an adjustment variable which*—as L. A. Vincent ([6] or [7]) has pointed out (see also Courbis [4], pp. 58–59)—*is equal to the productivity gain* $\tilde{G}$, a result which has since been rediscovered by Y. Kurabayashi [3], p. 296.

The consideration of the productivity gain $\tilde{G}$ as an adjustment variable of the production account in constant prices is thus logically introduced when we analyze the problem of economic evolution in physical terms, but the balancing of the production account in constant prices can evidently be done in a direct way in terms of prices. We have therefore in constant prices:

$$(6) \qquad \tilde{Y} + \tilde{T} = \tilde{U} + \tilde{W} + \tilde{V}$$

where $\tilde{T}$ is the terms of trade variable.

As I have pointed out in [4], p. 59, it results from (5) and (6) that:

$$(7) \qquad \tilde{T} = -\tilde{G}.$$

Algebraically, this relation means that the gain in productivity $\tilde{G}$ is "distributed" by the way of price variations (on output or production factors).

But this equality between $\tilde{G}$ and $-\tilde{T}$ is not always true as indicated in my paper of 1967 [4], pp. 65–68 and 70–71 and also by Y. Kurabayashi [3], pp. 295–296. The equality (7) supposes that all the components of Y—and especially the *profit—can be considered as production factors*[1] and it is on this point that I shall now comment in this note.

3. It is evident that in reality all the components cannot be (see [5] and [4]) considered as production factors. Pure profit is in fact not a factor resulting from market mechanisms: it is not fixed *ex-ante*[2] but is determined *ex-post* as the difference between the value of output and total production expenditures. It is only under the assumption of perfect competition and in the long term, when pure profit vanishes, that we can consider that all components of Y correspond to production factors.

Consequently, we must decompose V into two elements: the first, which we shall denote K, corresponds to expenditures (and especially the cost of physical capital utilisation) and is directly *linked* to the production level; the second, B, which is a residual, is the net benefit or pure profit.

[1]It is such an assumption which is explicitly made by Y. Kurabayashi in [1].

[2]If this was true, we should have (see [4] or [5]):

$$p = \frac{p_z}{\pi}$$

The output price would be in these conditions *only* determined by factor prices and factor productivity, which is difficult to suppose.

With this notation, the productivity gain $\tilde{G}$ (which corresponds to the factor saving at a given production level) is equal to:

$$\tilde{G} = \frac{\tilde{Y}}{\tilde{Y}_0}[\tilde{U}_0 + \tilde{W}_0 + \tilde{K}_0] - [\tilde{U} + \tilde{W} + \tilde{K}] \tag{8}$$

where $\tilde{K}$ is the constant price value of K.

As by definition $Y_0 = U_0 + W_0 + K_0 + B_0$, we can rewrite equation (8), in the following form:

$$\tilde{G} = \frac{\tilde{Y}}{\tilde{Y}_0}(\tilde{Y}_0 - \tilde{B}_0) - (\tilde{U} + \tilde{W} + \tilde{K})$$

or:

$$\tilde{G} = \tilde{Y} - \left[\tilde{U} + \tilde{W} + \tilde{K} + \tilde{B}_0\frac{\tilde{Y}}{\tilde{Y}_0}\right]. \tag{9}$$

It appears from relation (9) that for calculating the productivity gain $\tilde{G}$, if we consider conventionally profit as a factor, we must also consider conventionally *that the rate of margin (i.e., $b = B/Y$) remains unchanged* between the base year and the considered current year.[3]

To make explicit the productivity gain $\tilde{G}$ and the terms of trade gain $\tilde{T}$ in the framework of national accounts, we can then (as I have proposed in 1967 [4], pp. 65–67) proceed as indicated in table 1. In this table,[4] we interpolate four accounts between the account of the base year and that of the current year:

—an account in constant prices with constant returns to scale (with constant average factor productivity);

—an account for the current year in constant prices with effective factor productivity but with a constant rate of margin (and so the same profit as in the previous account);

—an account for the current year in constant prices where the profit is deflated by a convenient price index p_M ($1/p_M$ represents the "real" value of the current money unit in the framework of the base year's price system);

—an account for the current year with constant value of money (i.e. in relative prices).

Both the second and third accounts are not balanced and it is necessary to introduce an adjustment variable.

In the constant price account (the third intermediate account) the adjustment variable—which is considered in resources—is by definition the terms of trade variable $\tilde{T}$. For the second account in constant prices but with a constant rate of margin (i.e. the rate of margin b_0 of the base year), it is easy to see that—

[3]I had already noted this in my study of 1964 (see [8] p. 64 footnote 3). My 1967 paper [4] develops it in its second part.

[4]Kurabayashi [3], p. 294, gives such a table again, only with some presentation modifications.

TABLE 1

ANALYSIS OF THE PRODUCTIVITY GAIN AND OF THE TERMS OF TRADE GAIN*

	(1) Base Year *No.* Values	(2) Output volume index	(3) Year *N* Volumes with constant productivity (3) = (1) × (2)	(4) Indices of relative factor consumption	(5) Year *N* Volumes with effective productivity and constant rate of margin (5) = (3) × (4)	(6) Index of "gain" incorporation	(7) Year *N* Account in "constant prices"	(8) Relative price indices	(9) Year *N* Account in relative prices and constant money (9) = (7) × (8)	(10) General price level index (**)	(11) Year *N* Current account (11) = (9) × (10)
Output	Y_0	q	$\tilde{Y}$	1	$\tilde{Y}$	1	$\tilde{Y}$	p/p_M	$\bar{Y}$	p_M	Y
Net gain arising from relative prices variation	—		—		—		$\tilde{T}$		—		—
Total	Y_0	q	$\tilde{Y}$	1	$\tilde{Y}$		$\tilde{Y} + \tilde{T}$		$\bar{Y}$	p_M	
Intermediate product consumption	U_0	q	$\hat{U}$	q_U/q	$\tilde{U}$	1	$\tilde{U}$	p_U/p_M	$\bar{U}$	p_M	U
Wages and labour expenditures	W_0	q	$\hat{W}$	q_W/q	$\tilde{W}$	1	$\tilde{W}$	p_W/p_M	$\bar{W}$	p_M	W
Capital expenditures	K_0	q	$\hat{K}$	q_K/q	$\tilde{K}$	1	$\tilde{K}$	p_K/p_M	$\bar{K}$	p_M	K
Total of inputs	X_0	q	$\hat{X}$	q_X/q	$\tilde{X}$	1	$\tilde{X}$	p_X/p_M	$\bar{X}$	p_M	X
Net profit	B_0	q	$\hat{B}$	1	$\hat{B}$	$\tilde{B}/\hat{B}$	$\tilde{B} = \hat{B} + \tilde{S}$	1	$\bar{B} = \tilde{B}$	p_M	B
Productivity gain	—	—	—		$\tilde{G}$		—		—		
Total	$X_0 + B_0 = Y_0$	q	$\tilde{Y}$	1	$\tilde{X} + \hat{B} + \tilde{G} = \tilde{Y}$		$\tilde{Y} + \tilde{T}$		$\bar{Y}$	p_M	$Y = X + B$

*Courbis [4] p. 65 (but the notations used in the original paper are different).

**p_M is the deflator which corresponds to the "general price level" for producers and which is used for deflating the net profit B; $1/p_M$ corresponds to the real value of the money unit.

considered as expenditure[5]—the adjustment variable is the productivity gain $\tilde{G}$ which we have previously calculated.[6]

As by definition $\tilde{Y} + \tilde{T} = \tilde{U} + \tilde{W} + \tilde{K} + \tilde{B}$ (where $\tilde{B}$ is the value of B in terms of constant prices, i.e. $\tilde{B} = B/p_M$) we have:

$$(10) \qquad \tilde{T} = -\tilde{G} + \left[\tilde{B} - \tilde{B}_0 \frac{\tilde{Y}}{\tilde{Y}_0}\right]$$

or:

$$(11) \qquad \boxed{\tilde{T} = -\tilde{G} + [\tilde{B} - \hat{B}]}$$

where $\hat{B}$ is the fictive benefit of current year with the same rate of margin as for the base year (i.e. $\hat{B} = \tilde{B}_0\ \tilde{Y}/\tilde{Y}_0$).

It appears in this relation[7] that $\tilde{G}$ and $-\tilde{T}$ are not in general equal, contrary to the case where profit is considered as a production factor.

The reason is the following: the difference between $\tilde{G}$ and $-\tilde{T}$ is due to the fact that producers do not distribute completely the gains which result for them from an amelioration of factor productivity and of the terms of trade.

If we note:

$$(12) \qquad \tilde{T} = \tilde{T}_x - \tilde{T}_y$$

[5]It should be noted here that in his table [3], p. 294, Kurabayashi considers this correction term $\tilde{G}$ on the resource side of the account. It is easy to see that $\tilde{G}$ is then a productivity *loss*. Mr. Kurabayashi interprets this term as a productivity gain term but it is due (see following note) to a mistake in his relation (4.6).

In this condition, my formulation as proposed in 1967 [4] appears the only correct one.

[6]The variable adjustment $\tilde{G}$ of the second intermediate account is, as results from (8), equal to:

$$\tilde{G} = \tilde{U}\left(\frac{q}{q_U} - 1\right) + \tilde{W}\left(\frac{q}{q_W} - 1\right) + \tilde{K}\left(\frac{q}{q_K} - 1\right)$$

where $\tilde{U}$, $\tilde{W}$ and $\tilde{K}$ are the volumes of the three groups of factors and q_U, q_W, q_K, the corresponding volume indexes, and q the volume index of output.

If we have only two groups of factors U and W (as considered by Kurabayashi in [3]), we have:

$$(8\text{ bis}) \qquad \tilde{G} = \tilde{U}\left(\frac{q}{q_U} - 1\right) + \tilde{W}\left(\frac{q}{q_W} - 1\right).$$

This is different from the relation (4.9) given by Kurabayashi [3], p. 296, which, with *our* notations, is:

$$\tilde{G} = \tilde{U}\left(1 - \frac{q_U}{q}\right) + \tilde{W}\left(1 - \frac{q_W}{q}\right)$$

which—as a result of relation (8bis)—is in fact incorrect. The mistake issues from mistakes in Kurabayashi's relation (4.6).

[7]This relation is similar to the relation (4.11) of Kurabayashi [3], p. 296, but we must observe that—independently of the incorrect value of $\tilde{G}$ (see above footnote 6)—there is a mistake if we consider, as does Kurabayashi, that in the account in constant prices all the inputs different from intermediate inputs U are deflated by the same price index as B. We have in fact:

$$\tilde{T} = -\tilde{G} + \left(\frac{W}{p_M} - \tilde{W}\right) + (\tilde{B} - \hat{B})$$

if we have only two factors U and W.

Moreover in this case, $\tilde{G}$ is not the total productivity gain and $\tilde{T}$ is not only the terms of trade gain *but incorporates a part of the total productivity gain.*

where $\tilde{T}_x$ is the (algebraic) gain arising from a diminution of relative prices of factors and $\tilde{T}_y$ the (algebraic) loss arising from a diminution of the relative price of output, using relation (12), we can ([4], pp. 70–71) rewrite the relation (11) in a symmetric way:

$$\boxed{\tilde{G} + \tilde{T}_x = \tilde{T}_y + (\tilde{B} - \hat{B})} \tag{13}$$

$\tilde{G} + \tilde{T}_x$ is the *total* gain of the producers (productivity gain + gain by diminution of relative prices of factors), and relation (13) describes the *utilisation* of this total gain: a part ($\tilde{T}_y$) is distributed to the customers (by way of a diminution of the relative price of output); *the surplus* ($\tilde{B} - \hat{B}$) *is retained by the producers* (and increases the self-financing possibilities).

We can also give another form to the relationship between $\tilde{G}$ and $-\tilde{T}$. As $\tilde{B} = B/p_M$, we can rewrite relation (10) thus:

$$\tilde{T} = -\tilde{G} + \left[\frac{B}{p_M} - \tilde{B}_0 \frac{Y}{\tilde{Y}_0}\right] \tag{14}$$

or, if we note by b_0 and b the rate of margin (i.e., $b = B/Y$) for the base year and the current year:

$$\tilde{T} = -\tilde{G} + \left[b\frac{Y}{p_M} - b_0\tilde{Y}\right]$$

or as $Y = p\tilde{Y}$

$$\begin{cases} \tilde{T} = -\tilde{G} + \tilde{S} \\ \text{with } \tilde{S} = b\tilde{Y}\left[\dfrac{p}{p_M} - \dfrac{b_0}{b}\right] \end{cases} \tag{15}$$

where $\tilde{S}$ is the gain retained by the producers.

We can also write $\tilde{S}$ in the following way:

$$\boxed{\tilde{S} = b\tilde{Y}\left(\frac{p}{p_M} - 1\right) + b\tilde{Y}\left(1 - \frac{b_0}{b}\right)} \tag{16}$$

If we choose for the p_M deflator of B the value I have proposed in [8] and [4] (and which is also admitted by Mr. Kurabayashi, see [2]), we have:

$$p_M = \frac{X + Y}{\tilde{X} + \tilde{Y}}$$

and consequently:

$$\frac{p}{p_M} - 1 = p\frac{\tilde{X} + \tilde{Y}}{X + Y} - 1 = \frac{p\tilde{X} - X}{X + Y}$$

or, as by definition $X = p_x\tilde{X}$:

$$\frac{p}{p_M} - 1 = \frac{X}{X + Y}\left(\frac{p}{p_x} - 1\right).$$

We have finally:

$$\tilde{S} = b\theta \tilde{Y}\left(\frac{p}{p_x} - 1\right) + b\tilde{Y}\left(1 - \frac{b_0}{b}\right) \tag{17}$$

with

$$\theta = \frac{X}{X + Y} \tag{18}$$

In [3], p. 296, Kurabayashi relation (4.11) gives an apparently similar expression for the term $\tilde{S}$, i.e. the difference between[8] $\tilde{T}$ and $-\tilde{G}$. But Kurabayashi's formula implicitly supposes $b = b_0$,[9] so the second term of (17) vanishes. Relation (17) given above is consequently more general.

The interpretation of this relation (17) is interesting. It shows that the gain $\tilde{S}$ which is retained by producers is the sum of two elements: the first is positive if $p > p_x$, i.e. if there is a relative increase of the output price; the second is positive if $b > b_0$, i.e. if there is an increase of the benefit rate b (which can arise from the productivity increase or from an amelioration of the terms of trade).

Relations (13) and (17) thus describe the origin and use of the total gain of producers and the elements which contribute to the formation of the "retained" gain. They can therefore be useful for analysis of the formation and distribution of income.

August, 1972

REFERENCES

[1] Y. Kurabayashi, "Terms of Trade Effect, Productivity Change and National Accounts in Constant Prices. Reply and Further Comment". *Review of Income and Wealth*, Series 18, No. 3, Sept. 1972, pp. 327–331.

[2] R. Courbis, "Comment on Y. Kurabayashi: The Impact of Changes in Terms of Trade on a System of National Accounts", *Review of Income and Wealth*, Series 18, No. 2, June 1972, pp. 247–250.

[3] Y. Kurabayashi, "The Impact of Changes in Terms of Trade on a System of National Accounts: An Attempted Synthesis", *Review of Income and Wealth*, Series 17, No. 3, September 1971, pp. 285–297.

[4] R. Courbis, "Comptabilité nationale à prix constants et à productivité constante", *Review of Income and Wealth*, Series 15, No. 1, March 1969, pp. 33–76.

[5] R. Courbis, "Productivité globale des facteurs, prévisiont des prix et politique des revenus", *Revue Economique*, Vol. XIX, No. 4, July 1968, pp. 563–606.

[6] L. A. Vincent, "Productivité et prix relatifs dans 15 branches de l'économie française (1949–1963)", *Etudes et Conjoncture*, February 1966, pp. 17–84.

[7] L. A. Vincent, "Les relations entre productivité et prix relatifs au niveau des branches", *Revue Economique*, May 1967, pp. 374–395 and 402–407.

[8] R. Courbis, "Comptes économiques nationaux à prix constants". *Etudes de Comptabilité Nationale*, No. 6, 1964 (Paris, Imprimerie nationale) and *Etudes et Conjoncture*, July 1964, pp. 5–76.

[8]Or between $\tilde{T}$ and Kurabayashi's variable $\tilde{G}$ which is different. If $\tilde{G}$ is our productivity gain and $\tilde{G}_K$ Kurabayashi's variable, we have: $\tilde{G}_K = -\tilde{G}$, but we must interpret $\tilde{G}_K$ as a loss (see above footnote 5).

[9]Mr. Kurabayashi ([3] p. 296, footnote) writes indeed (with my notations) that:

$$B = \hat{B}p$$

Or we have in fact:

$$B = \frac{b}{b_0} p\hat{B}.$$

Kurabayashi's relation (4.11) in [3] is consequently only true if $b = b_0$.