NOTES AND MEMORANDA

THE CASE AGAINST DIVISIA INDEX NUMBERS AS A BASIS IN A SOCIAL ACCOUNTING SYSTEM*

BY W. J. MERRILEES
University of Toronto

The purpose of this paper is to explore the question of at what stage should Divisia index numbers be introduced—within the social accounting schema proper or in the explanatory stage following? Three major reasons are given to support the case that Divisia index numbers have no role to play in measurement, but should and do provide a powerful tool in the explanation of productivity change.

Dale Jorgenson, Zvi Griliches and colleagues have expressed discontent with the fact that the economic theory underlying measurement of real product and real factor input has not been fully exploited. They forward the hypothesis that when the relevant variables are properly measured nearly all of the observed change in real product should be explained by the observed change in real factor input. Any minor discrepancy could be accounted for by intractable measurement errors or by a lack of correspondence between private costs and social costs.

The major innovation proposed by Jorgenson and Griliches (1967)[1] incorporates Divisia index numbers into the definitions of real output change and real input change and hence productivity change. Other adjustments to established indexes were made including correction for varying rates of stock utilization, but with these changes the qualitative nature of the errors is not in dispute. Of course the actual magnitudes of these admitted errors can be debated and in fact have since been revised by Christensen and Jorgenson (1970). Table 1 below gives a breakdown as to how an orthodox rate of productivity for the U.S. private sector, 1945–1965, of 1.6 percent p.a., is transformed by Jorgenson and Griliches into an estimate of only 0.10 percent p.a., with the remaining 1.5 percent denoted as an upward bias in the productivity estimate, a bias which relocates itself in the input side of the equation. The thesis of this paper rejects the notion that the first three "errors" in Table 1 can be regarded as such.

In a later article Christensen and Jorgenson (1970) attempt further refinements, partly in terms of the period covered, partly in terms of the categories into which the variables fit, but mainly in providing more reasonable estimates of capital rates of utilization. Their 1967 conclusions concerning the relative proportion of product growth explained by real input change were considerably altered; from 97 percent in the 1967 study to about 70 percent in the 1970 study[2], though plainly the explanatory burden remains with real factor input change rather than productivity increase.

1. THE JORGENSON–GRILICHES TECHNIQUE

The usual social accounting identity is retained, namely that for each accounting period the value of output is equal to the value of input:

$$q_1 Y_1 + q_2 Y_2 + \ldots + q_m Y_m = p_1 X_1 + p_2 X_2 + \ldots + p_n X_n \tag{1}$$

*Date received: October 30, 1970.

[1]The basic technique of Jorgenson and Griliches has been employed in at least five or six articles. Only the two major works will be referred to.

[2]Data and time periods not strictly comparable.

TABLE 1

SOURCES OF PRODUCTIVITY ESTIMATION ERRORS, U.S. 1945–1965

	%p.a.
Previously accepted rate of productivity estimate	1.6
New estimate	0.10
Upward bias in old estimate	1.50
Comprised of six components:	
(1) Errors of aggregation, given the broad inputs of capital and labour	0.11
(2) Errors of aggregation within the broad capital input category	0.36
(3) Errors of aggregation within the broad labor input category	0.48
(4) Errors in investment goods price index	0.08
(5) Errors in assumption that the utilization of capital services is proportional to capital stock	0.60
(6) Errors in assumption that the utilization of labor services is proportional to labor stock	−0.15

Source: Jorgenson and Griliches (1967) passim.

where

Y_i = quantity of ith output,
X_j = quantity of jth input,
q_i = price of ith output,
p_j = price of jth input.

After totally differentiating equation (1) with respect to time and then manipulating, the authors finally define the rates of real output and real input as:

$$\frac{\dot{Y}}{Y}=\sum W_i\frac{\dot{Y}_i}{Y_i} \quad \text{and} \quad \frac{\dot{X}}{X}=\sum V_j\frac{\dot{X}_j}{X_j}$$

where the dots have the usual time change meaning; W_i is the relative share of the ith output in the value of total output, and V_j is the relative share of the jth input in the value of the total input.

The rate of total productivity is defined as:[3]

$$\frac{\dot{Y}}{Y}-\frac{\dot{X}}{X}=\sum W_i\frac{\dot{Y}_i}{Y_i}-\sum V_j\frac{\dot{X}_j}{X_j} \qquad (2$$

2. INTERPRETATION OF THE DIVISIA TOTAL PRODUCTIVITY INDEX

The Divisia index denoting equation (2) may be interpreted as "a line integral, so that its value normally depends on the path of integration; even if the path returns to its initial value the index of productivity may increase or decrease."[4] The authors go on to declare that even for a production function characterized by constant returns to scale with all factors paid the value of their marginal products, the rate of growth of real output may exceed or fall short of the rate of growth of real input. These two propositions are stressed by the authors to free themselves from accusations that their (1967) conclusion of a near zero productivity increase is a tautology.

[3]In the Christensen–Jorgenson paper discrete analogues are used in lieu of the continuous time variables.

[4]Jorgenson and Griliches (1967), p. 253.

I do not dispute either of the two propositions, although there seems a danger that their true significance may not be fully grasped by the cursory reader. The first proposition is well known to students who understand the (analogous) principles of least squares regression and its relation to fitting a trend to time series data. To some extent the second proposition follows from the first; that is, as long as the data traces a slightly irregular path there would be a tendency for the Divisia index of productivity to differ from its base level (i.e. the rate of change is different from zero), notwithstanding the absence of a discernible secular trend. What happens if we add the condition that the time series variables follow regular (no cycles whatever) trends? Does the second proposition still hold?

The positive answer to this question facilitates distinction of the Divisia index approach from the usual social accounting solution, where the answer is an unequivocal no. The difference arises from the fact that in the conventional social accounting framework everything proceeds with arithmetic mean weights, whereas now geometric weights are proposed. To illustrate, take a one (natural) input, two (consumer) good model, in which the two industries differ in relative sizes. In terms of the model outlined in section 1 above, we can let $W_1 > W_2$ and $V = 1.0$ because there is only one input. Given these assumptions, only in the special case in which productivity increase is the same in both industries will the arithmetic base system yield the same answer as the geometric base system. Thus by adopting the Divisia index schema it is possible to reach the paradoxical situation of knowing that aggregate (arithmetic weighted) outputs grew by say 5 percent per year and that aggregate (arithmetic weighted) inputs also grew by 5 percent per year, but still maintaining that the (Divisia) productivity rate of change is different from zero, at say 1 percent per year.

From these simplified examples it is clear that the Divisia index number approach represents a powerful tool and is obviously a new perspective on the analysis of productivity. But is it a better perspective? It seems to the present writer that Divisia index numbers can be a useful aid in explaining the rate of productivity but not to the measurement of such. The rationale underlying the index is that it eliminates errors of aggregation which befog conventional aggregate indexes. This means that the Divisia real output index can be conceived as forming the end product of a production function process which conforms to Theil's condition for perfect aggregation; i.e. output is dependent on the values of the aggregate inputs only and is independent of the distribution of inputs. In contrast in a conventional system a unique (i.e. single valued) function cannot be guaranteed. But is this a limitation on measurement as opposed to explanation?

First, as the above examples also illustrated, it seems contrary to common sense to maintain that an actual increase in G.N.P. of say 5 percent is "really" only, say, 4 percent because the latter figure is what results when geometric mean weights are used in order to ensure a unique production function.

Second, effective use can be made of the aggregation bias to explain productivity change. Massell (1961) found that about a third of productivity change could be explained by inter-industry shifts; the latter being synonomous with aggregation bias. Thus the impact of dynamic resource reallocation on productivity change is highlighted —a feature which would be easily slurred over if economists reclined to the habit of measuring productivity advance at near zero *ab initio*. If shifts between goods and factors were frozen our material welfare would abate considerably, so it is useful to have this specific relative contribution made as explicit as possible.

Third, with the new approach there seems a danger that measurement and explanation become confounded at the expense of an unscientific basis to the latter. For example, questions dealing with the nature and magnitude of the effect of education on the quality of labor are not adequately dealt with—instead they are assumed. The amount of schooling and related variables are prime candidates for explaining the growth of the quality of the labor force; but they are only hypotheses not facts. The relationship between education and labor quality change is apt to be quite complex, as Nelson and Phelps (1966) interestingly point out. Indeed, since Christensen and Jorgenson (1970,

p. 48) recognize that further compositional changes need to be considered on the basis of race, sex, age, occupation, etc., there is a danger that the rate of change in quality will always be understated by their method. This problem is tentatively confirmed. The estimates calculated by Christensen and Jorgenson (Table 7, p. 39) give rates of growth of labor productivity quality in the order of 0.6 and 0.7 percent per year. A more direct and general estimate is provided by the David and deKlundert (1965) model[5] which calculates the rate at about 2.2 percent per year.

3. Concluding Remarks

At the end of their 1967 paper Jorgenson and Griliches felt undisturbed by their conclusion that the rate of productivity in the postwar U.S. economy was near zero, given their framework. In their words: "Our conclusion is not that advances in knowledge are negligible, but that the accumulation of knowledge is governed by the same economic laws as any other process of capital accumulation. Costs must be incurred if benefits are to be achieved" (p. 274).

This conclusion can be compared to an earlier statement by the authors which recognized that their definition of change in total factor productivity was the same as that suggested by Abramovitz, namely "the effect of 'costless' advances in applied technology, managerial efficiency, and industrial organization" (p. 250).

The contradiction between these two quotations is standard; there cannot be many compilers of conventional productivity indexes who fail to appreciate that "costless" advances are not really such and somewhere in the system inputs are needed for knowledge to advance. There can be few proponents of traditional productivity indexes who believe the opportunity cost of productivity increase is zero. While it is encouraging to see economists explicitly stating that productivity advancement has a positive opportunity cost, this hardly justifies abandonment of the traditional productivity concept. The latter concept documents the proximate cause of rising standards of living and also provokes investigation into the deeper determinants of productivity increase over time.

Although the aforementioned contradiction perturbs neither Jorgenson and Griliches (1967) nor compilers of traditional productivity indexes, it does leave the Christensen and Jorgenson (1970) results in an embarrassing position. Committed to balancing real inputs with real outputs (in the context of estimation of productivity change), their new results reveal a fairly large gap between input and output unexplained. They have failed to account for all the real inputs. This unexplained gap pinpoints the serious pitfall in their analysis which we noted before in comparing their labor quality change estimate with that of David and deKlundert.

At this point my case for orthodoxy rests. The problems of measurements and explanation of total productivity change need to be kept separate. For the three reasons expounded above, Divisia index numbers have no role to play in measurement, but should and do provide a powerful tool in the explanation of productivity change. In passing, nothing said above detracts from the individual comments and procedures in the Jorgenson–Griliches and Christensen–Jorgenson articles which without doubt contain a wealth of insight and invaluable material. Rather, the issue in debate is the question of at what stage should Divisia index numbers be introduced—within the social accounting schema proper or in the explanatory stage following?

Selected References

L. Christensen and D. Jorgenson, "U.S. Real Product and Real Factor Input, 1929–1967," *Review of Income and Wealth*, March 1970.

P. David and T. deKlundert, "Biased Efficiency Growth and Capital–Labor Substitution in the U.S., 1899–1960," *American Economic Review*, June 1965.

[5]They utilize a factor augmenting model, $Q = [(E_L L)^{-p} + (E_K K)^{-p}] - 1/P$ to estimate $\dot{E}_L/E_L$ and $\dot{E}_K/E_K$, where E_L, E_K represent efficiency levels of labor and capital respectively. Time periods not strictly comparable.

E. Denison, Comment, *American Economic Review*, May 1966.
D. Jorgenson and Z. Griliches, "The Explanation of Productivity Change," *Review of Economic Studies*, July 1967.
B. Massell, "A Disaggregated View of Technical Change," *Journal of Political Economy*, December 1961.
R. Nelsen and E. Phelps, "Investment in Humans, Technological Diffusion, and Economic Growth," *American Economic Review*, May 1966.
R. and N. Ruggles, "Concepts of Real Capital Stocks and Services," in N.B.E.R. (1961), *Output, Input and Productivity Measurement*.

Editor's note:

Jorgenson and Griliches respond to this comment in "Divisia Index Numbers and Productivity Measurement," *Review of Income and Wealth*, Vol. 17, No. 2.