A NOTE ON MEASURING SECTORAL INPUT PRODUCTIVITY*

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This note attempts to shed some light on the relationship between the total factor productivity derived from national income accounts and the total input productivity based upon input-output accounts, especially on a sectoral basis. Since there has been no positive evidence to support a constancy between changes in net and gross output in individual industries, the formulation of a measure of sectoral input productivity change by using the formula of the Divisia index based on input-output accounts may be valuable in examining possible biases which are associated with a common notion of the total factor productivity. An operational definition of sectoral input productivity change and its relation to sectoral total factor productivity are discussed in the present note, in addition to its empirical application to the Japanese data.

I. INTRODUCTION

During the last decade a considerable number of countries have started to publish a series of real domestic product or real value added by industry within their national income framework. In addition to this, a compilation of input-output tables at some intervals, say every five years, has become a regular function of the national income authority in some countries. Under these circumstances, a disaggregation of the measurement of total factor productivity change of a national economy[1] into the level of individual industries will become one of the more fashionable things to do. The general approach used in analyses of this kind is to compute the portion of the rate of growth which is not accounted for by the growth rate of measured inputs. This residual is attributed to technical progress or to the growth of input efficiency. A common notion, which has been implicitly or explicitly assumed thus far, is a production function for real net output[2] whose arguments are the inputs of labor and capital or the primary inputs; in other words, there has been no room for considering the contribution due to possible shifts of purchased intermediate inputs.

In principle, some part of the measured changes in the net output function can be generated by actual movement along the overall production function, and the conventionally measured rate of primary factor productivity growth therefore may well be biased. This may be especially true in estimating sectoral factor productivity changes, since there has been no positive evidence to support a constancy between changes in net and gross output in individual industries.

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[1]The latest and the most extensive study on this problem can be found in Jorgenson and Griliches [1].

[2]In what follows, the terms net and gross will be used in relation to intermediate inputs, instead of depreciation.

Although the rate of growth of primary input productivity is not necessarily the same as that of total input efficiency, it may be possible to derive an operational relation between these two kinds of input productivity changes with properly designed national accounts.

The present paper attempts to provide a measure of sectoral input productivity change by using the formula of the Divisia index based on input-output accounts. Section II will present a formulation of sectoral total input productivity change based on input-output accounts and will discuss its nature by comparing it with a conventionally defined measure of sectoral primary input productivity change. In Section III, empirical results in terms of indices of sectoral input productivity changes, both total and primary, will be shown using Japanese data.

II. Sectoral Input Productivity Change: Formulations and Its Nature

An accounting identity which can be derived from the standard input-output table can be written as

$$p_j X_j = \sum_{i=1}^{N} p_i X_{ij} + \sum_{k=1}^{K} p_k V_{kj} \qquad (j = 1, \ldots, N)$$

where

x_j: physical output in the j-th sector

X_{ij}: physical intermediate flow from the i-th sector to the j-th sector

V_{kj}: the k-th primary input in the j-th sector

p_j: price of the j-th output

p_k: price of the k-th primary input.

Also, the following identity will be defined as the national demand-supply relation,

$$p_i X_i = \sum_{j=1}^{N} p_i X_{ij} + \sum_{l=1}^{L} p_i Y_{li}$$

where Y_{li}: the l-th component of final demand in the i-th sector. National income identity is defined as

$$\sum_{j=1}^{N} \sum_{k=1}^{K} p_k V_{kj} = \sum_{i=1}^{N} \sum_{l=1}^{L} p_i Y_{li}$$

Within this accounting framework, the Divisia index of total input productivity in the j-th sector can be derived as

$$\sum_i a_{ij} x_{ij} + \sum_k f_{kj} v_{kj}$$

where

$$a_{ij} = p_i X_{ij}/p_j X_j, \qquad f_{kj} = p_k V_{kj}/p_j X_j$$

$$x_{ij} = \dot{X}_{ij}/X_{ij} \qquad v_{kj} = \dot{V}_{kj}/V_{kj}$$

and a dot over a variable indicates a derivative with respect to time. The total input productivity change in the j-th sector can be defined as

$$e_j = x_j - \sum_i a_{ij}x_{ij} - \sum_k f_{kj}v_{kj}$$

where $x_j = \dot{X}_j / X_j$. In other words, the total input productivity change defined here corresponds to the difference between the growth rate of output (gross) and the growth rate of total inputs weighted by the value term coefficients of the input-output table.[3] A national index of total productivity change, therefore, can be defined as,

$$e = \sum_j w_j e_j$$

where

$$w_j = p_j X_j / \sum_j p_j X_j.^4$$

Now let us consider the corresponding definition of sectoral primary input productivity change which will be obtained from the net output (or real value added) function. Assuming that the data on gross output, intermediate inputs and primary inputs valued at constant prices are available, we can compute net output (or value added in real terms) by using the double deflation scheme.[5] The change in the real value added between the periods 0 and t will be as follows,

$$Z_j^* = Z_j^t / Z_j^0 = \frac{p_j^0 X_j^t - \sum_i p_i^0 X_{ij}^t}{p_j^0 X_j^0 - \sum_i p_i^0 X_{ij}^0}$$

Then the primary input productivity change in the j-th sector can be defined as

$$e_j^* = z_j^* - \sum_k f_{kj}^* v_{kj}$$

where

$$f_{kj}^* = p_k^0 V_{kj}^0 / p_j^0 X_j^0 - \sum_i p_i^0 X_{ij}^0$$

The next problem we have to examine here concerns the relationship between total input productivity change and primary input productivity change in individual sectors, namely the relationship between e_j and e_j^*. It can be shown that total input productivity change is generally less than primary input productivity change. More specifically, the following relation holds between e_j and e_j^*, namely

$$e_j = e_j^*(1 - \delta_j^0)$$

[3]Here the value term coefficients include the value share of primary inputs in the total value of output, in addition to those coefficients used in the usual input-output table.

[4]Alternatively, $e^* = \Sigma\, w_j^* e_j$, where $w_j^* = p_j X_j / \Sigma_t\, \Sigma_i\, p_i Y_{ii}$, may be defined. However, e^* is not the weighted average of e_j's since $\Sigma\, w_j^* \neq 1$.

[5]See United Nations Report [2].

where $1 - \delta_j^0 = (p_j^0 X_j^0 - \Sigma_i p_i^0 X_{ij}^0)/p_j^0 X_j^0$ and $0 < 1 - \delta_j^0 < 1$.[6] In other words, total input productivity change is the product of primary input productivity change and the value added ratio, which is generally less than unity. This relation can also be extended in the national index, i.e., $e = e^*(1 - \delta^0)$ where $1 - \delta^0 = \Sigma_i \Sigma_l p_i^0 \cdot Y_{li}^0 / \Sigma_j p_j^0 X_j^0$.[7]

In summing up, it has been shown that a measure of sectoral total input productivity change based upon the gross output function can be defined by applying the formula of the Divisia index to the input-output account, and also that it can be easily transformed into sectoral primary input productivity change by multiplication by the corresponding value added ratio. In addition to this, since the value added ratios in individual sectors are, in general, significantly different from unity, it may not be appropriate to use the primary input productivity change as an indicator of technological progress or input efficiency.

III. Empirical Results

A computation of sectoral total input productivity change, defined in the previous section, requires the following statistical information: (i) consistently compiled input-output tables which must be valued at constant prices,[8] and (ii) capital and labor inputs in the corresponding sectoral classification. In case of the Japanese data, three consistently compiled input-output tables, for 1955, 1960 and 1965, all with 1960 prices, and series of capital stock (1960 prices) and labor inputs in the same industry classification are now available.[9] By using these data, total input productivity change, e_j, and primary input produc-

[6]Since, under the constant price valuation scheme

$$e_j = \frac{p_j^0 X_j^t}{p_j^0 X_j^0} - \sum_i \frac{p_i^0 X_{ij}^0}{p_j^0 X_j^0}\left(\frac{p_i^0 X_{ij}^t}{p_i^0 X_{ij}^0}\right) - \sum_i \frac{p_j^0 V_{kj}^0}{p_j^0 X_j^0}\left(\frac{p_k^0 V_{kj}^t}{p_k^0 V_{kj}^0}\right)$$

$$= \frac{1}{p_j^0 X_j^0}\left\{p_j^0 X_j^t - \sum_i p_i^0 X_{ij}^t - \Sigma p_k^0 V_{kj}^t\right\}$$

while,

$$e_j^* = \frac{1}{p_j^0 X_j^0 - \sum_i p_i^0 X_{ij}^0}\left\{p_j^0 X_j^t - \sum_i p_i^0 X_{ij}^t - \sum_k p_k^0 V_{kj}^t\right\}.$$

[7]The definition of e^* can be found in the previous footnote. By using this definition,

$$(1 - \delta)e^* = (1 - \delta)\Sigma w_j^* e_j = (1 - \delta)\sum_i \frac{p_j X_j}{\sum_i \sum_l Y_{li} p_i} e_j$$

$$= (1 - \delta)\frac{\sum_i p_j X_j}{\Sigma \Sigma Y_{li} p_i} \Sigma \frac{p_j X_j}{\sum_i p_j X_j} e_j = \Sigma w_j e_j = e.$$

[8]"Consistently" implies that concepts and definitions such as classifications, valuation procedures, treatments of by-products, imports, etc., which are usually needed to compile input-output accounts, are the same among input-output tables.

[9]The number of classifications is 20, including one sector of "not elsewhere classified." All data are prepared by the Economic Planning Agency, Government of Japan.

TABLE 1
(1955–1960)

	$1 - \delta_j$ ('60)	e_j	e_j/x_j	$[e_j^*(1 - \delta_j)]$	$[e_j^*/Z_j^*]$	$[e_j^*]$
1. Agriculture	0.2308	0.6	5.4	0.4	16.5	2.1
2. Mining	0.3900	9.7	39.5	6.8	60.5	17.5
3. Food	0.8557	8.6	37.0	15.7	50.5	18.4
4. Textiles	0.8851	12.2	30.5	21.2	60.2	23.9
5. Paper Products	0.6188	12.5	28.4	17.4	42.3	28.2
6. Chemicals	0.6321	12.5	28.1	16.6	38.7	26.2
7. Metals	0.5494	8.8	17.2	8.2	28.4	15.0
8. Metal Products	0.6051	11.0	18.2	8.4	21.7	13.9
9. Machinery	0.5952	14.4	22.3	13.9	34.9	23.4
10. Electrical Machinery	0.6420	14.0	19.1	9.1	18.9	14.3
11. Transport Machinery	0.6430	14.4	22.5	16.5	38.5	25.7
12. Miscellaneous Manufacturing	0.6369	7.6	17.6	8.9	32.1	14.1
13. Public Utilities	0.4980	17.2	35.7	9.6	39.4	19.4
14. Trade	0.3014	4.8	19.3	2.2	27.6	7.5
15. Transport and Communications	0.3294	14.9	34.6	5.5	34.3	16.7
16. Financial	0.2654	24.4	47.8	6.0	44.7	22.8
17. Services	0.2675	4.7	19.2	1.6	27.4	6.0

Note: All data, except $(1 - \delta_j)$, are in percentage units. Value shares are taken from 1960 figures.

TABLE 2
(1960–1965)

	$1 - \delta_j$('60)	e_j	e_j/x_j	$[e_j^*(1 - \delta_j)]$	$[e_j^*/Z_j^*]$	$[e_j^*]$
1. Agriculture	0.2308	−18.4	—	−6.3	—	27.5
2. Mining	0.3900	9.1	42.0	4.0	134.9	10.4
3. Food	0.8557	10.2	26.2	10.7	17.3	12.5
4. Textiles	0.8851	3.4	13.5	1.4	7.5	1.6
5. Paper Products	0.6188	25.8	31.3	29.0	45.8	47.0
6. Chemicals	0.6331	40.1	36.4	49.2	50.4	77.8
7. Metals	0.5494	15.5	18.3	15.1	27.3	27.5
8. Metal Products	0.6051	45.6	32.7	54.6	49.8	90.3
9. Machinery	0.5952	6.0	8.4	6.6	10.9	11.1
10. Electrical Machinery	0.6420	25.0	27.8	38.1	42.2	59.3
11. Transport Machinery	0.6430	68.7	50.5	90.7	63.5	141.0
12. Miscellaneous Manufacturing	0.6369	21.6	23.8	26.5	40.1	41.6
13. Public Utilities	0.4980	27.4	38.0	11.4	32.3	22.9
14. Trade	0.3014	40.9	43.3	14.5	56.0	48.4
15. Transport and Communications	0.3294	8.4	12.8	5.3	24.7	16.2
16. Financial	0.2654	32.4	35.6	7.9	31.1	29.7
17. Services	0.2675	16.8	29.7	4.3	37.6	16.3

Note: All data, except $(1 - \delta_j)$, are in percentage units. Value shares are taken from 1960 figures.

tivity change, e_j^*, are shown in Tables 1 and 2, where Construction and Real Estate sectors are omitted.[10]

Five years average rates of total input productivity change, i.e., e_j for 1955–1960 and 1960–1965, are generally lower than those of primary input productivity changes, i.e., e_j^*, as has been expected from the previous section. Furthermore, adjusted primary input productivity changes, i.e., $e_j^*(1 - \delta_j)$, show to a considerable extent similar orders of magnitude with total input productivity changes, as shown in columns 3 and 5 in the tables. The existence of those industries which give relatively large differences between e_j and $e_j^*(1 - \delta_j)$, for example Food, Textiles, Transportation and Communication, and Finance in 1955–1960, may indicate imperfections in compilation of input-output accounts including their transformation into constant price valuations.

REFERENCES

[1] Jorgenson, D. W.. and Z. Griliches, "The Explanation of Productivity Change," *Review of Economic Studies*, vol. 34 (1967).
[2] United Nations, "A System of National Accounts," 1968.

[10]Since the data on capital input is capital stock (not transformed into capital service), including nonresidential and residential buildings, the growth rates of capital stock are much larger than those of output (net and gross) even after adjusting by the corresponding value shares.