NOTES AND BOOK REVIEWS

# DIVISIA INDEX NUMBERS AND PRODUCTIVITY MEASUREMENT

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## INTRODUCTION

A recent note by Merrilees [15] concludes that Divisia index numbers as employed in our studies of productivity change [11, 14] represent a new perspective on the analysis of productivity. Merrilees concludes further that Divisia index numbers are appropriate for the explanation of productivity change, but that they should not be used for measurement. In this note we examine and reject both conclusions.

## DIVISIA INDEX NUMBERS

A Divisia index number, like any other, is based on price and quantity data.[1] A Divisia quantity index has a rate of growth equal to a weighted average of rates of growth of its component quantities. Similarly, a Divisia price index has a rate of growth equal to a weighted average of rates of growth of its component prices. The weights in either case are the relative value shares of each component in total value.

The first published paper employing Divisia index numbers in productivity measurement is that of Solow [17].[2] Solow's paper embodies two important innovations: (1) A Divisia index of total factor input, obtained by weighting rates of growth of capital and labor to obtain the rate of growth of total factor input. (2) An interpretation of the resulting Divisia index of total factor productivity in terms of shifts in an aggregate production function.

Subsequently, Denison employed Divisia indexes for (1) measuring growth in U.S. total factor productivity [4] and (2) international comparisons of productivity growth [5]. Denison used a Divisia index of total factor input, weighting rates of growth of capital and labor to obtain a rate of growth of total factor input. Unlike Solow he also weighted rates of growth of individual components of labor and capital to obtain Divisia indexes of labor and capital input.

Our study of total factor productivity did not pioneer in the use of Divisia indexes. We related a Divisia index of total factor productivity to shifts in a production possibility frontier with many outputs and many inputs. For this purpose we introduced a Divisia index of total output as well as total factor input. Divisia indexes of this type were analyzed in greater detail by Richter [16] and, recently, by Hulten [13]. We also introduced methods of measuring capital input that utilize the basic duality between prices of capital services and quantities of capital stock analyzed by Arrow [1] and Hall [12]. Finally, we introduced new measurements of investment goods prices and utilization of capital.

## MEASUREMENT AND EXPLANATION

Accurate measurement does not necessarily provide an explanation of growth in total factor productivity. The point of our original paper is that much measured growth in total factor productivity is a result of inaccurate measurements. Errors of

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[1]These index numbers were first proposed by Divisia in 1925 [6, 7, 8]. Somewhat more accessible discussions of Divisia's work may be found in Frisch's survey article on the theory of index numbers [10] and in Wold's book on demand analysis [18].

[2]Application of Divisia index numbers was also suggested by Divisia in 1952 [9, pp. 53–54].

aggregation can be eliminated by applying Divisia aggregation as suggested by Solow. Our innovation was to apply these methods consistently not only to the weighting of growth rates of capital and labor but also to the weighting of growth rates of components within the capital and labor aggregates and to the weighting of components of total output.

Merrilees argues that Divisia indexes differ from ordinary arithmetic indexes of real product and that Divisia indexes are, therefore, suspect. This argument is entirely mistaken. Divisia indexes are defined for continuous time. In any practical application a discrete approximation to these indexes must be made. Alternative approximations correspond to alternative index number formulas.

As an example, suppose we consider the rate of growth of a conventional Laspeyres index of real product:

$$\frac{Q_1^L - Q_0^L}{Q_0^L} = \frac{\sum p_{0i}q_{1i}}{\sum p_{0i}q_{0i}} - \frac{\sum p_{0i}q_{0i}}{\sum p_{0i}q_{0i}},$$
$$= \frac{\sum p_{0i}q_{1i}}{\sum p_{0i}q_{0i}} - 1.$$

In these formulas $Q^L$ is the Laspeyres index of real product, $\{p_{0i}\}$ are base period prices, $\{q_{0i}\}$ are base period quantities, and $\{q_{1i}\}$ are the quantities in the succeeding period.

Next we consider a "Laspeyres" approximation to the Divisia index of real product:

$$\frac{Q_1^D - Q_0^D}{Q_0^D} = \sum \frac{p_{0i}q_{0i}}{\sum p_{0i}q_{0i}} \cdot \frac{q_{1i} - q_{0i}}{q_{0i}}$$
$$= \frac{\sum p_{0i}q_{1i}}{\sum p_{0i}q_{0i}} - 1.$$

We conclude that the rate of growth of this approximation to the Divisia index is identical with the rate of growth of the conventional arithmetic Laspeyres index of real product.

If the rate of growth of the Laspeyres approximation to the Divisia index is identical to the rate of growth of a conventional arithmetic Laspeyres index, how can the use of Divisia index numbers affect the measurement of real product, real factor input, or total factor productivity? The answer to this is that Divisia indexes are "chain-linked"; for each year the current prices are used as a base in estimating the rate of growth to the following year. The process is followed for each year in succession and the year-to-year rates of growth are linked into a chain index.

The main advantage of a chain index is in the reduction of errors of approximation as the economy moves from one production configuration to another. If weights could be changed continuously, errors of this type would be eliminated. This property of Divisia indexes, called "invariance" by Richter, characterizes no other index number. Discrete chain-linked index numbers reduce errors of approximation to a minimum. For this reason chain indexes rather than a single base period should be used in real product accounting and productivity measurement.

The Laspeyres approximation to the Divisia index of total factor productivity was employed in our original study of productivity change [14]. In more recent work Christensen and Jorgenson [2, 3] have employed an alternative approximation to the Divisia index that is symmetric in prices and quantities, satisfying the factor-reversal test of index numbers. This approximation also satisfies the time-reversal test; that is, the data of the two time periods enter the formulas symmetrically.

## Conclusion

Merrilees' criticisms of Divisia index numbers are entirely misdirected. The Divisia index number formula has been standard in productivity measurement since

Solow's pioneering 1957 paper. The use of these index numbers has been gradually extended until it encompasses not only productivity measurement, but also measurement of sub-aggregates such as capital and labor and measurement of real product.

Chain-linked indexes provide discrete approximations to the growth rate of real product and real factor input with minimum error. Alternative discrete approximations correspond to conventional index numbers such as the Laspeyres formula analyzed above. We conclude that chain-linked indexes provide the most satisfactory basis for index numbers of real product, real factor input, and total factor productivity. Index numbers of this type should be made standard in national accounting measurements of real product just as they have been made standard in the study of total factor productivity.

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