# AN ANALYSIS OF THE DISTRIBUTION OF WAGES IN COPENHAGEN IN THE SECOND QUARTER OF 1951*

BY KJELD BJERKE

*Danish Statistical Office*

This article presents the results of a study of distributions of wage rates in approximately 250 trades, comprising 225,000 workers, in Copenhagen in the second quarter of 1951. It examines particularly the effects of heterogeneity within trades and aggregation upon the resulting distributions, both for individual trades and for all trades combined. Separate distributions are studied for men and women, for skilled and unskilled, and for three types of institutional wage payment systems.

Examination of the Danish income distribution shows that it is neither normal nor log-normal. It is distinctly positively skewed and leptokurtic. A breakdown by sex, district categories, and trade or occupation in certain cases leads to distributions which are less skewed; but almost all the distributions examined probably deviate significantly from normal or log-normal distributions. The distributions seem to be too heterogeneous to fulfill the conditions for normality or log-normality.

However, it seems evident that if one could obtain data solely on wage earners and their wages, there would be a better prospect of obtaining simple distributions which might turn out to be normal or log-normal. This could, in fact, be expected if the data in question could be broken down sufficiently by sex, age, trade and occupation, and wage system. In this way it would be possible to verify some of the views advanced in the first part of this study, which appeared in the preceding issue of this *Review*. Whether the work within a trade can be said to be homogeneous is of course difficult to decide, and only experts in the fields in question can reasonably be expected to be able to do so.

Through a classification of this type it will also be possible to demonstrate how the aggregation problem by itself can explain why normal or log-normal distributions do not remain normal or log-normal when they are aggregated, since the conditions for their doing so are very stringent: for aggregated distributions to be log-normal, the individual distributions must be log-normal, their standard deviations must be the same, and the averages of the individual distributions should be log-normally distributed, cf. [1] p. 110. If these conditions are not fulfilled, the aggregated distributions usually are not log-normal. As demonstrated by Hill [2], aggregation of normal distributions with different standard deviations but the same averages will lead to leptokurtic distributions.

In Denmark excellent data are available which can be used for a detailed analysis of these problems. These data have kindly been placed at my disposal. This paper will discuss their analysis.

*Based on chapters III and IV of the author's book, *Indkomst-og Lønfordelinger*, Copenhagen, 1965.

For the second quarter of 1951[1] the Danish Employers Federation undertook a study of the distribution of wages for a total of 225,000 wage earners, or almost 90 per cent of all the wage earners who were employed by the members of the Federation at the end of June. The study of wage distribution was limited to production workers proper, and certain groups of workers with special wage systems were excluded. The study was undertaken separately for each trade, and within each trade individual occupational groups were distinguished. A distinction was made between piece work and time work. To avoid duplication in those cases (about $\frac{1}{3}$) where in the course of the quarter a worker had had both piece work hours and time work hours, corrections were made so that each worker was included in the number of piece-workers and the number of time-workers, with weights determined by the relative amount of time work and piece work [3].

The data are subdivided by sex and by skill level. The latter distinction is not entirely meaningful because unskilled workers contain groups of wage earners with widely different qualifications. However, I have retained it because a further breakdown was not possible, nor did it seem essential for this study.

In the survey a distinction was made between three main types of wages: piece wages, standard wages, and minimum wages [4, 5, 6]. Piece wage rates are fixed in accordance with performance. A distinction is made between quantitative rates, where wages are fixed in accordance with output, and time rates, where standard times are fixed for the performance of the job and payment for the work corresponds to standard time multiplied by a certain hourly wage. According as the work is performed quickly or slowly, the hourly wage will vary. Certain schedules of wages are fixed by collective bargaining either for the whole country or for regions, and these will normally be in force as long as the collective agreement remains in force. Such agreements on piece work rates will most frequently be met with among artisans. In manufacturing industries the usual thing is for workshop schedules to be fixed within individual enterprises between the workers and the employer. These workshop schedules are not part of the collective agreements and can be changed whenever the parties agree to do so. In Denmark, wages include the cost of living allowance. Since in 1951 this allowance was chiefly paid as a fixed amount per hour, an element of time wage was thus introduced into the piece work rates.

In the case of time wages a distinction is made between the standard wage system and the minimum wage system. Standard wages are normally the same for all workers in the same trade irrespective of qualifications, age, etc. The wage is agreed centrally, and the rates remain in force for the full period of the collective agreement. However, certain personal allowances may be paid and there may be a certain graduation by length of service. In the minimum wage system rates are fixed by individual agreements between the worker and his employer. The centrally-fixed minimum wage rate forms a floor. Usually the wages actually paid will be considerably above the minimum wage rate. Within the period of the collective agreement wages may be altered for the individual worker; on the other hand, workers cannot make collective demands for increased wages, and employers cannot collectively reduce wages.

[1]There is also a study for the fourth quarter of 1956 and the second quarter of 1962.

The data include only the earnings from normal work. Thus overtime pay and payment for night work are not included, nor is dirt money or holiday allowance.

It falls outside the scope of this survey to go into the general theories of wage fixing in present-day Danish society since it is not the wage level as such which is of interest in this connection, but wage differences among the individual workers. I shall therefore also leave out of account the general problems connected with wage drift.

On the other hand, there may be reason to consider what the special Danish problem (*viz.*, that wages are determined in three ways: through collective agreements, through adjustment by cost of living index, and by wage drift) means to the wage distributions, especially if, as in this case, the data have been broken down by wage system. What is of special importance is the extent to which it may be expected that wage drift will re-establish an agreement between wage and performance for the two wage systems where this is most likely, namely piece work rate and minimum rate. The tendency to self-regulation through wage drift is clear. This fact is important in the examination of the tendency for wage distributions to be normal or log-normal. It does therefore seem justifiable to consider total wages inclusive of the cost of living allowance. If the offset effect could be assumed to be of no importance, the analysis would have been more rational if total wages less (at least) the regular cost of living allowance had been used. It should be underlined that with the correction of the general theory effected by wage drift, the approach advocated by Tinbergen [7] will be relevant. The individual differences in attributes will, of course, be able to manifest themselves in wages through wage drift.

*Trades with more than 90 observations: main results*

It has been mentioned above that a decision as to how homogeneous the work is within the individual trades is needed to demonstrate that homogeneous work can lead to log-normal or normal distributions. By way of introduction, I shall therefore discuss the work which the Danish Employers Federation has kindly undertaken in order to illustrate this question. For all the nearly 250 trades represented the Employers Federation has assessed the heterogeneity of the work and subdivided the trades into three groups:

A = slightly heterogeneous
B = moderately heterogeneous
C = strongly heterogeneous.

The Employers Federation has pointed out to me that such a breakdown is naturally extremely difficult to make and is subjective. The fact that there are 250 trades in this survey is due to the fact that most trades are split into two parts according to wage system. The actual number of trades will therefore be approximately half of the 250. If we compare this breakdown with a decision as to whether the trades deviate significantly or not from normal or log-normal distributions, we shall be able to determine whether the degree of homogeneity affects normality.

The trades for which information is available include a widely varying number of workers, some trades seeming to be substantially larger than others. All other things being equal, it must be justifiable to expect that trades with few workers can to a greater degree be homogeneous than trades with many workers. That this is so is confirmed by Table 1. The table also shows, however, that there does not seem to be much difference between groups A and B, and in the following analysis these two groups will be considered together.

TABLE 1

NUMBER OF TRADES BY DEGREE OF HETEROGENEITY

| Number of workers | Number of Trades | | | Percentage Distribution of Number of Trades | | | |
|---|---|---|---|---|---|---|---|
| | A | B | C | A | B | C | Total |
| 0– 39 | 17 | 10 | 1 | 60.7 | 35.7 | 3.6 | 100.0 |
| 40– 89 | 32 | 30 | 9 | 45.1 | 42.2 | 12.7 | 100.0 |
| 90–119 | 11 | 13 | 1 | 44.0 | 52.0 | 4.0 | 100.0 |
| 120–159 | 11 | 14 | 3 | 39.3 | 50.0 | 10.7 | 100.0 |
| 160–499 | 23 | 29 | 4 | 41.1 | 51.8 | 7.1 | 100.0 |
| 500 and over | 10 | 9 | 20 | 25.6 | 23.1 | 51.3 | 100.0 |
| TOTAL | 104 | 105 | 38 | 42.1 | 42.5 | 15.4 | 100.0 |

I shall now proceed to an analysis of the data. By way of preliminary guidance the following information is given about the eight main groups:

| | | |
|---|---|---|
| Men | Skilled | Piece work rate |
| | | Minimum wages |
| | | Standard wages |
| | | (Other) |
| | Unskilled | Piece work rate |
| | | Minimum wages |
| | | Standard wages |
| | | (Other) |
| Women | | Piece work rate |
| | | Minimum wages |
| | | Standard wages |
| | | (Other) |

"Other" has not been dealt with. The group numbers only 16 trades, of which only one has more than 90 observations. Within the other groups, for the individual trades with at least 90 observations, a graphical description (by means of fractile diagrams[2]) and a description by means of the moments of the distribution curves have been made.

All distributions are presented both on the ordinary scale and on the logarithmic scale. As will be discussed later, there are more trades that are log-normal than normal, and therefore the survey has primarily aimed at an analysis of the distributions in relation to the log-normal distribution. In addition to the mean and the standard deviation, two additional characteristics have been

[2]The fractile diagrams are available in manuscript.

examined: the $\sqrt{\beta_1}$ test and the $a$-test. $\sqrt{\beta_1}$, as will be known, gives a measure of skewness [8]. As a measure of whether the distributions are leptokurtic or platykurtic the $a$-test developed by R. C. Geary has been used [8, 9, 10]; $a$ is defined as follows.

$$a = \frac{\sum_i |x_i - \bar{x}|}{\sqrt{n \sum_i (x_i - \bar{x})^2}}$$

In a normal distribution this ratio has the value 0.7979. For platykurtic distributions the ratio will be higher, and lower for leptokurtic distributions.

I have not used the usual $\beta_2$ test, which is useful only when the sample includes at least 200 observations. As the intervals in the wage distributions are very small, I have not used Sheppard's correction and similar corrections for the higher moments. Besides the tests mentions, finally, I have used a $\chi^2$ test, and in this connection I have given a comparison between the expected distribution and the actual distribution as well as the individual $u$-values. A later section describes trades with between 40 and 90 observations. The limit of 40 was chosen because the $a$-test is not applicable below that limit. The tests which have been used thus contribute to illustrating essential and different aspects of the distributions so that they can be described quite exhaustively. As will appear later from the analysis, some of the distributions are leptokurtic and positively skewed. This means that there must be a tendency to large $\chi^2$s because the $\chi^2$ is not independent of the other two tests.

It has been mentioned that aggregation may lead to leptokurtic distributions, but aggregation may, of course, also result in the distributions being skewed—possibly negatively, but particularly positively. Also, it is possible that a ceiling or a floor to wages may lead to a significant $\chi^2$ without skewed distributions.

For the above mentioned main groups, the wage levels are given in Table 2. The means are given both for all trades and for those trades in which there are at least 90 observations, since it is these latter trades which are studied in this section.

It will be seen that the wage level is almost the same for all trades combined and for the selected trades, which comprise approximately 93 per cent of all observations. Naturally, there is considerable variation among categories, but the table does show what we already know:

1. That piece work wages are higher than wages based on other systems, and that standard wages are lowest.

2. That women's average hourly wage is about $\frac{2}{3}$ of that of male workers.

3. That skilled workers have a higher average hourly wage than unskilled workers.

4. That the difference between piecework rates and minimum wage rates is greatest for unskilled workers.

TABLE 2

NUMBER OF OBSERVATIONS AND AVERAGE WAGES IN COPENHAGEN, 2ND QUARTER 1951

| | Number of Observations | | | | Average Hourly Wage in Øre | | | |
|---|---|---|---|---|---|---|---|---|
| | Piece-Wages | Minimum Wages | Standard Wages | Total | Piece-Wages | Minimum Wages | Standard Wages | Total |
| *All trades* | | | | | | | | |
| Men: | | | | | | | | |
| Skilled | 17,456 | 15,710 | 1,065 | 34,231 | 460 | 416 | 392 | 437 |
| Unskilled | 13,307 | 9,196 | 10,978 | 33,481 | 412 | 334 | 329 | 363 |
| Total | 30,763 | 24,906 | 12,043 | 67,712 | — | — | — | 401 |
| Women | 14,285 | 4,768 | 6,544 | 25,597 | 275 | 235 | 235 | 257 |
| TOTAL | 45,048 | 29,674 | 18,587 | 93,309 | — | — | — | 361 |
| *Trades with at least 90 observations* | | | | | | | | |
| Men: | | | | | | | | |
| Skilled | 16,033 | 14,802 | 523 | 31,358 | 460 | 417 | 389 | 438 |
| Unskilled | 12,219 | 8,924 | 10,169 | 31,312 | 410 | 333 | 328 | 361 |
| Total | 28,252 | 23,726 | 10,692 | 62,670 | — | — | — | 400 |
| Women | 13,441 | 4,433 | 6,061 | 23,935 | 274 | 233 | 235 | 257 |
| TOTAL | 41,693 | 28,159 | 16,753 | 86,605 | — | — | — | 360 |

If we consider the number in all trades, almost $\frac{3}{4}$ are men. There are considerable differences from one wage system to another. The greater part of male workers are paid either on the piecework or the minimum wage system. Most unskilled workers are paid by the piecework system, but there are a considerable number who are paid minimum or standard wages. The greater part of women are paid piecework rates, but considerable numbers are paid on the basis of minimum or standard wages.

The aggregate distributions were tested for log-normality by using all three tests. The tests show that the distributions are not log-normal. In the fractile diagrams the distributions take on an S-shape.

To determine whether the wage system has any influence on the aggregated distributions, Table 3a, 3b, and 3c compare the actual frequency distributions with normal distributions. The frequency distributions for both piece wages and minimum wages are leptokurtic, and in the case of piece wages clearly positively skewed. It is strange that the frequencies for piece wages in the low wage brackets are considerably smaller in the actual distributions than in the normal distributions. However, this is not the case for minimum wages for skilled men. Here the frequencies are greater. For the other minimum wage distributions it is interesting that there seems to be a floor in the downward direction. The tables show that the institutional factor (the wage system) is of decisive importance.

It has been mentioned that skewness in income and wage distributions can partly be ascribed to different income levels and partly to the fact that the individual distributions are generally positively skewed. That wage levels are different in the individual trades is not surprising since the trades include both

TABLE 3A

COMPARISON BETWEEN ACTUAL AND NORMAL DISTRIBUTIONS BY WAGE SYSTEM
SKILLED WORKERS, 90 OBSERVATIONS OR MORE

| Hourly Wage in Øre | Piece Wages | | | Minimum Wages | | | Standard Wages | | |
|---|---|---|---|---|---|---|---|---|---|
| | I | II | I–II | I | II | I–II | I | II | I–II |
| 239.5–249.5 | 2 | 49.0 | – 47.0 | | | | | | |
| 249.5–259.5 | 5 | 23.3 | – 18.3 | | | | | | |
| 259.5–269.5 | 9 | 32.7 | – 23.7 | | | | | | |
| 269.5–279.5 | 32 | 45.2 | – 13.2 | 68 | 8.9 | 59.1 | | | |
| 279.5–289.5 | 32 | 61.4 | – 29.4 | | | | | | |
| 289.5–299.5 | 61 | 82.0 | – 21.0 | 42 | 12.2 | 29.8 | 9 | 9.4 | – 0.4 |
| 299.5–309.5 | 87 | 107.6 | – 20.6 | 38 | 26.0 | 12.0 | | | |
| 309.5–319.5 | 160 | 138.9 | 21.1 | 81 | 51.9 | 29.1 | 3 | 8.0 | – 5.0 |
| 319.5–329.5 | 135 | 176.3 | – 41.3 | 118 | 97.0 | 21.0 | 8 | 12.9 | – 4.9 |
| 329.5–339.5 | 250 | 219.9 | 30.1 | 194 | 170.1 | 23.9 | 9 | 19.5 | –10.5 |
| 339.5–349.5 | 200 | 269.7 | – 69.7 | 261 | 279.7 | – 18.7 | 15 | 27.5 | –12.5 |
| 349.5–359.5 | 227 | 325.2 | – 98.2 | 310 | 431.2 | –121.2 | 99 | 36.2 | 62.8 |
| 359.5–369.5 | 266 | 385.6 | –119.6 | 368 | 623.3 | –255.3 | 32 | 44.4 | –12.4 |
| 369.5–379.5 | 360 | 449.4 | – 89.4 | 595 | 844.8 | –249.8 | 59 | 50.9 | 8.1 |
| 379.5–389.5 | 430 | 515.1 | – 85.1 | 879 | 1,073.5 | –194.5 | 38 | 54.4 | –16.4 |
| 389.5–399.5 | 454 | 580.3 | –126.3 | 1,149 | 1,279.1 | –130.1 | 51 | 54.3 | – 3.3 |
| 399.5–409.5 | 538 | 642.9 | –104.9 | 1,468 | 1,428.9 | 39.1 | 42 | 50.5 | – 8.5 |
| 409.5–419.5 | 707 | 700.2 | 6.8 | 1,996 | 1,496.6 | 499.4 | 53 | 43.9 | 9.1 |
| 419.5–429.5 | 963 | 749.7 | 213.3 | 1,862 | 1,469.7 | 392.3 | 38 | 35.6 | 2.4 |
| 429.5–439.5 | 1,209 | 789.3 | 419.7 | 1,928 | 1,353.3 | 574.7 | 25 | 27.0 | – 2.0 |
| 439.5–449.5 | 1,445 | 817.0 | 628.0 | 1,139 | 1,168.3 | – 29.3 | 8 | 19.1 | –11.1 |
| 449.5–459.5 | 1,337 | 831.4 | 505.6 | 888 | 945.7 | – 57.7 | 22 | 12.6 | 9.4 |
| 459.5–469.5 | 1,228 | 831.8 | 396.2 | 484 | 717.7 | –233.7 | 3 | 7.7 | – 4.7 |
| 469.5–479.5 | 1,078 | 818.2 | 259.8 | 298 | 510.7 | –212.7 | | | |
| 479.5–489.5 | 824 | 791.3 | 32.7 | 215 | 340.7 | –125.7 | | | |
| 489.5–499.5 | 600 | 752.3 | –152.3 | 115 | 213.1 | – 98.1 | | | |
| 499.5–509.5 | 528 | 703.3 | –175.3 | 92 | 125.0 | – 33.0 | | | |
| 509.5–519.5 | 407 | 646.4 | –239.4 | 72 | 68.7 | 3.3 | | | |
| 519.5–529.5 | 342 | 584.1 | –242.1 | 48 | 35.4 | 12.6 | | | |
| 529.5–539.5 | 280 | 518.9 | –238.9 | 36 | 17.1 | 18.9 | | | |
| 539.5–549.5 | 204 | 453.3 | –249.3 | 17 | 7.8 | 9.2 | | | |
| 549.5–559.5 | 191 | 389.2 | –198.2 | | | | | | |
| 559.5–569.5 | 159 | 328.6 | –169.6 | | | | | | |
| 569.5–579.5 | 182 | 272.8 | – 90.8 | | | | | | |
| 579.5–589.5 | 156 | 222.6 | – 66.6 | | | | | | |
| 589.5–599.5 | 132 | 178.6 | – 46.6 | | | | 9 | 9.1 | – 0.1 |
| 599.5–609.5 | 111 | 140.9 | – 29.9 | | | | | | |
| 609.5–619.5 | 116 | 109.3 | 6.7 | | | | | | |
| 619.5–629.5 | 76 | 83.4 | – 7.4 | | | | | | |
| 629.5–639.5 | 74 | 62.5 | 11.5 | 41 | 5.6 | 35.4 | | | |
| 639.5–649.5 | 54 | 46.1 | 7.9 | | | | | | |
| 649.5–659.5 | 48 | 33.4 | 14.6 | | | | | | |
| 659.5–669.5 | 42 | 23.8 | 18.2 | | | | | | |
| 669.5–679.5 | 34 | 16.7 | 17.3 | | | | | | |
| 679.5–689.5 | 30 | 11.5 | 18.5 | | | | | | |
| 689.5–699.5 | 22 | 7.8 | 14.2 | | | | | | |
| 699.5–709.5 | 28 | 5.2 | 22.8 | | | | | | |
| 709.5–869.5 | 178 | 8.9 | 169.1 | | | | | | |
| TOTAL | 16,033 | 16,033 | 0.0 | 14,802 | 14,802 | 0.0 | 523 | 523 | 0.0 |

NOTE: I = Actual distribution. II = Normal distribution.

TABLE 3B

COMPARISON BETWEEN ACTUAL AND NORMAL DISTRIBUTIONS BY WAGE SYSTEM
UNSKILLED WORKERS, 90 OBSERVATIONS OR MORE

| Hourly Wage in Øre | Piece Wages | | | Minimum Wages | | | Standard Wages | | |
|---|---|---|---|---|---|---|---|---|---|
| | I | II | I–II | I | II | I–II | I | II | I–II |
| 239.5–249.5 | 2 | 249.2 | −247.2 | | | | 1 | 72.8 | − 71.8 |
| 249.5–259.5 | 115 | 87.1 | 27.9 | | | | 1 | 93.1 | − 92.1 |
| 259.5–269.5 | 26 | 111.1 | − 85.1 | 0 | 351.9 | −351.9 | 1 | 180.4 | −179.4 |
| 269.5–279.5 | 47 | 139.5 | − 92.5 | 263 | 268.4 | − 5.4 | 6 | 317.5 | −311.5 |
| 279.5–289.5 | 84 | 172.3 | − 88.3 | 396 | 403.8 | − 7.8 | 318 | 507.2 | −189.2 |
| 289.5–299.5 | 74 | 209.4 | −135.4 | 679 | 562.6 | 116.4 | 1,336 | 735.5 | 600.5 |
| 299.5–309.5 | 145 | 250.5 | −105.5 | 904 | 726.1 | 177.9 | 1,706 | 968.1 | 737.9 |
| 309.5–319.5 | 234 | 294.8 | − 60.8 | 1,397 | 867.8 | 529.2 | 1,020 | 1,156.9 | −136.9 |
| 319.5–329.5 | 309 | 341.4 | − 32.4 | 1,151 | 960.6 | 190.4 | 579 | 1,254.9 | −675.9 |
| 329.5–339.5 | 461 | 389.1 | 71.9 | 1,178 | 984.9 | 193.1 | 3,225 | 1,235.7 | 1,989.3 |
| 339.5–349.5 | 546 | 436.2 | 109.8 | 613 | 935.2 | −322.2 | 536 | 1,104.6 | −568.6 |
| 349.5–359.5 | 656 | 481.3 | 174.7 | 564 | 822.4 | −258.4 | 501 | 896.3 | −395.3 |
| 359.5–369.5 | 1,049 | 522.5 | 526.5 | 496 | 669.8 | −173.8 | 181 | 660.2 | −479.2 |
| 369.5–379.5 | 1,115 | 558.2 | 556.8 | 365 | 505.3 | −140.3 | 123 | 441.4 | −318.4 |
| 379.5–389.5 | 990 | 586.7 | 403.3 | 373 | 353.0 | 20.0 | 75 | 267.9 | −192.9 |
| 389.5–399.5 | 868 | 606.9 | 261.1 | 152 | 228.4 | − 76.4 | 109 | 147.6 | − 38.6 |
| 399.5–409.5 | 799 | 617.6 | 181.4 | 104 | 136.9 | − 32.9 | 100 | 73.8 | 26.2 |
| 409.5–419.5 | 549 | 618.5 | − 69.5 | 53 | 76.0 | − 23.0 | 74 | 33.5 | 40.5 |
| 419.5–429.5 | 466 | 609.5 | −143.5 | 36 | 39.0 | − 3.0 | 78 | 13.8 | 64.2 |
| 429.5–439.5 | 446 | 591.0 | −145.0 | 77 | 18.6 | 58.4 | | | |
| 439.5–449.5 | 323 | 563.9 | −240.9 | 35 | 8.2 | 26.8 | | | |
| 449.5–459.5 | 348 | 529.4 | −181.4 | | | | | | |
| 459.5–469.5 | 298 | 489.0 | −191.0 | | | | | | |
| 469.5–479.5 | 329 | 444.5 | −115.5 | | | | | | |
| 479.5–489.5 | 310 | 397.6 | − 87.6 | | | | | | |
| 489.5–499.5 | 234 | 349.9 | −115.9 | | | | | | |
| 499.5–509.5 | 187 | 303.1 | −116.1 | | | | | | |
| 509.5–519.5 | 154 | 258.2 | −104.2 | | | | 199 | 7.8 | 191.2 |
| 519.5–529.5 | 137 | 216.5 | − 79.5 | | | | | | |
| 529.5–539.5 | 143 | 178.7 | − 35.7 | | | | | | |
| 539.5–549.5 | 87 | 145.0 | − 58.0 | | | | | | |
| 549.5–559.5 | 85 | 115.9 | − 30.9 | 88 | 5.1 | 82.9 | | | |
| 559.5–569.5 | 80 | 91.1 | − 11.1 | | | | | | |
| 569.5–579.5 | 52 | 70.5 | − 18.5 | | | | | | |
| 579.5–589.5 | 69 | 53.6 | 15.4 | | | | | | |
| 589.5–599.5 | 51 | 40.2 | 10.8 | | | | | | |
| 599.5–609.5 | 47 | 29.6 | 17.4 | | | | | | |
| 609.5–619.5 | 44 | 21.5 | 22.5 | | | | | | |
| 619.5–629.5 | 28 | 15.3 | 12.7 | | | | | | |
| 629.5–639.5 | 33 | 10.7 | 22.3 | | | | | | |
| 639.5–649.5 | 28 | 7.4 | 20.6 | | | | | | |
| 649.5–659.5 | 23 | 5.0 | 18.0 | | | | | | |
| 659.5–869.5 | 148 | 9.6 | 138.4 | | | | | | |
| TOTAL | 12,219 | 12,219 | 0.0 | 8,924 | 8,924 | 0.0 | 10,169 | 10,169 | 0.0 |

NOTE: I = Actual distribution. II = Normal distribution.

TABLE 3C

COMPARISON BETWEEN ACTUAL AND NORMAL DISTRIBUTIONS BY WAGE SYSTEM

WOMEN, 90 OBSERVATIONS OR MORE

| Hourly Wage in Øre | Piece Wages | | | Minimum Wages | | | Standard Wage | | |
|---|---|---|---|---|---|---|---|---|---|
| | I | II | I–II | I | II | I–II | I | II | I–II |
| 159.5–169.5 | 3 | 160.6 | −157.6 | | | | | | |
| 169.5–179.5 | 9 | 114.9 | −105.9 | 4 | 111.1 | −107.1 | 2 | 239.8 | −237.8 |
| 179.5–189.5 | 46 | 178.1 | −132.1 | 135 | 134.4 | 0.6 | 276 | 213.6 | 62.4 |
| 189.5–199.5 | 80 | 263.6 | −183.6 | 226 | 238.8 | − 12.8 | 339 | 334.9 | 4.1 |
| 199.5–209.5 | 133 | 372.5 | −239.5 | 687 | 372.0 | 315.0 | 671 | 476.1 | 194.9 |
| 209.5–219.5 | 281 | 502.6 | −221.6 | 438 | 508.3 | − 70.3 | 723 | 613.3 | 109.7 |
| 219.5–229.5 | 503 | 647.7 | −144.7 | 533 | 609.1 | − 76.1 | 742 | 716.1 | 25.9 |
| 229.5–239.5 | 1,431 | 797.0 | 634.0 | 671 | 640.0 | 31.0 | 832 | 757.9 | 74.1 |
| 239.5–249.5 | 1,525 | 936.5 | 588.5 | 507 | 589.9 | − 82.9 | 805 | 727.0 | 78.0 |
| 249.5–259.5 | 1,758 | 1,050.8 | 707.2 | 515 | 476.8 | 38.2 | 553 | 632.1 | − 79.1 |
| 259.5–269.5 | 1,720 | 1,125.8 | 594.2 | 372 | 338.0 | 34.0 | 509 | 498.1 | 10.9 |
| 269.5–279.5 | 1,227 | 1,151.9 | 75.1 | 127 | 210.1 | − 83.1 | 174 | 355.8 | −181.8 |
| 279.5–289.5 | 1,055 | 1,125.3 | − 70.3 | 108 | 114.6 | − 6.6 | 131 | 230.3 | − 99.3 |
| 289.5–299.5 | 797 | 1,049.9 | −252.9 | 41 | 54.8 | − 13.8 | 63 | 135.1 | − 72.1 |
| 299.5–309.5 | 654 | 935.3 | −281.3 | 27 | 23.0 | 4.0 | 69 | 71.9 | − 2.9 |
| 309.5–319.5 | 510 | 795.6 | −285.6 | | | | 54 | 34.6 | 19.4 |
| 319.5–329.5 | 359 | 646.3 | −287.3 | | | | 45 | 15.1 | 29.9 |
| 329.5–339.5 | 299 | 501.3 | −202.3 | | | | | | |
| 339.5–349.5 | 270 | 371.3 | −101.3 | | | | | | |
| 349.5–359.5 | 189 | 262.6 | − 73.6 | | | | | | |
| 359.5–369.5 | 127 | 177.4 | − 50.4 | | | | | | |
| 369.5–379.5 | 112 | 114.4 | − 2.4 | 42 | 12.1 | 29.9 | | | |
| 379.5–389.5 | 85 | 70.5 | 14.5 | | | | 73 | 9.3 | 63.7 |
| 389.5–399.5 | 61 | 41.4 | 19.6 | | | | | | |
| 399.5–409.5 | 33 | 23.3 | 9.7 | | | | | | |
| 409.5–419.5 | 26 | 12.5 | 13.5 | | | | | | |
| 419.5–429.5 | 32 | 6.4 | 25.6 | | | | | | |
| 429.5–789.5 | 116 | 5.5 | 110.5 | | | | | | |
| TOTAL | 13,441 | 13,441 | 0.0 | 4,433 | 4,433 | 0.0 | 6,061 | 6,061 | 0.0 |

NOTE: I = Actual distribution. II = Normal distribution.

men and women, and both skilled and unskilled. This difference in wages is illustrated in Table 4.

It will be seen that the distributions, and consequently also the wage levels for the different categories, are staggered. On the whole, the wage levels are highest for skilled workers on piece wages, lowest for women on standard wages. A comparison with normal distributions with a given deviation and mean can be utilized to get a rough impression of what the skewness in the individual distributions means to the skewness in the aggregated distributions—and since the normal distributions for the aggregated distributions have also been calculated, it will also be possible to say something about the effect of the difference in wage level on the shape of the wage distribution.

The difference between the actual distribution and the sum of the normal distributions for the individual trades should thus illustrate the effect of skewness in the individual distributions, since the difference in wage level has not been

TABLE 4

DISTRIBUTION OF INDIVIDUAL TRADES BY AVERAGE WAGES (90 OBSERVATIONS OR MORE)

| Average hourly age in Øre | Skilled | | | Unskilled | | | Women | | | Total | | | In all |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | P | M | S | P | M | S | P | M | S | P | M | S | |
| 0–189 | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 190–199 | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 200–209 | – | – | – | – | – | – | – | 1 | 2 | – | 1 | 2 | 3 |
| 210–219 | – | – | – | – | – | – | – | 1 | 1 | – | 1 | 1 | 2 |
| 220–229 | – | – | – | – | – | – | – | 2 | – | – | 2 | – | 2 |
| 230–239 | – | – | – | – | – | – | 2 | 2 | 2 | 2 | 2 | 2 | 6 |
| 240–249 | – | – | – | – | – | – | 1 | 2 | 1 | 1 | 2 | 1 | 4 |
| 250–259 | – | – | – | – | – | – | 5 | 2 | 4 | 5 | 2 | 4 | 11 |
| 260–269 | – | – | – | – | – | – | 2 | – | 1 | 2 | – | 1 | 3 |
| 270–279 | – | – | – | – | – | – | 3 | – | 1 | 3 | – | 1 | 4 |
| 280–289 | – | – | – | – | – | – | 1 | – | – | 1 | – | – | 1 |
| 290–299 | – | – | – | – | – | – | 3 | – | – | 3 | – | – | 3 |
| 300–309 | – | – | – | – | – | 3 | 1 | – | – | 1 | – | 3 | 4 |
| 310–319 | – | – | – | – | 1 | 7 | – | – | – | – | 1 | 7 | 8 |
| 320–329 | – | – | – | – | 1 | 1 | – | – | – | – | 1 | 1 | 2 |
| 330–339 | – | – | – | – | – | 3 | 1 | – | – | 1 | – | 3 | 4 |
| 340–349 | – | – | – | – | 1 | 2 | – | – | – | – | 1 | 2 | 3 |
| 350–359 | 1 | 1 | 1 | – | 2 | – | – | – | – | 1 | 3 | 1 | 5 |
| 360–369 | – | 1 | – | 1 | 2 | – | – | – | – | 1 | 3 | – | 4 |
| 370–379 | – | 1 | – | – | 2 | – | – | – | – | – | 3 | – | 3 |
| 380–389 | – | 2 | 1 | 4 | 1 | 1 | – | – | – | 4 | 3 | 2 | 9 |
| 390–399 | – | 4 | 1 | 2 | – | – | – | – | – | 2 | 4 | 1 | 7 |
| 400–409 | – | 1 | – | 1 | – | – | – | – | – | 1 | 1 | – | 2 |
| 410–419 | 1 | 6 | 2 | 1 | – | 1 | – | – | – | 2 | 6 | 3 | 11 |
| 420–429 | 1 | 6 | – | – | – | 1 | – | – | – | 1 | 6 | 1 | 8 |
| 430–439 | 2 | 2 | – | – | – | – | – | – | – | 2 | 2 | – | 4 |
| 440–449 | 4 | 3 | – | – | – | – | – | – | – | 4 | 3 | – | 7 |
| 450–459 | 2 | 1 | – | 1 | – | – | – | – | – | 3 | 1 | – | 4 |
| 460–469 | 5 | – | – | 1 | – | – | – | – | – | 6 | – | – | 6 |
| 470–479 | 4 | – | – | 3 | – | – | – | – | – | 7 | – | – | 7 |
| 480–489 | 2 | – | – | – | – | – | 1 | – | – | 3 | – | – | 3 |
| 490–499 | 2 | – | – | 1 | – | – | – | – | – | 3 | – | – | 3 |
| 500–509 | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 510–519 | – | – | – | 1 | – | – | – | – | – | 1 | – | – | 1 |
| 520–529 | – | – | – | 1 | – | – | – | – | – | 1 | – | – | 1 |
| 530–539 | 1 | – | – | – | – | – | – | – | – | 1 | – | – | 1 |
| 540–549 | 1 | – | – | – | – | – | – | – | – | 1 | – | – | 1 |
| 550–559 | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 560–569 | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 570–579 | 1 | – | – | – | – | – | – | – | – | 1 | – | – | 1 |
| 580–589 | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 660–669 | – | – | – | – | – | – | – | – | – | – | – | – | – |
| | 27 | 28 | 5 | 17 | 10 | 19 | 20 | 10 | 12 | 64 | 48 | 36 | 148 |

NOTE: P = Piece rates. M = Minimum wage system. S = Standard wage system.

eliminated in this comparison. The difference between the sum of the normal distributions of the individual trades and the normal distribution of all trades as a whole should, on the other hand, give an impression of the importance which should be attached to differences in wage levels. The estimates we get in this way are rather rough ones; nevertheless they do give a certain impression of the

TABLE 5A

THE IMPORTANCE OF AGGREGATION TO WAGE DISTRIBUTIONS. SKILLED MEN—MINIMUM WAGES

| Hourly Wage in øre | Actual Distribution $a^{I}$ | Aggregated Distribution on the Basis of Normal Distributions for the Individual Trades $b^{II}$ | Normal Distribution for all Trades $c^{III}$ | I–II (Skewness in the Individual Distributions) d | II–III (Difference in Wage Level) e | I–III |
|---|---|---|---|---|---|---|
| 259.5–269.5 | 68 | 14.1 | 8.9 | 30.8 | 28.3 | 59.1 |
| 269.5–279.5 | | 9.0 | | | | |
| 279.5–289.5 | | 14.1 | | | | |
| 289.5–299.5 | 42 | 24.6 | 12.2 | 17.4 | 12.4 | 29.8 |
| 299.5–309.5 | 38 | 41.0 | 26.0 | − 3.0 | 15.0 | 12.0 |
| 309.5–319.5 | 81 | 68.9 | 51.9 | 12.1 | 17.0 | 29.1 |
| 319.5–329.5 | 118 | 111.6 | 97.0 | 6.4 | 14.6 | 21.0 |
| 329.5–339.5 | 194 | 172.7 | 170.1 | 21.3 | 2.6 | 23.9 |
| 339.5–349.5 | 261 | 256.1 | 279.7 | 4.9 | − 23.6 | − 18.7 |
| 349.5–359.5 | 310 | 365.4 | 431.2 | − 55.4 | − 65.8 | −121.2 |
| 359.5–369.5 | 368 | 514.6 | 623.3 | −146.6 | −108.7 | −255.3 |
| 369.5–379.5 | 595 | 718.5 | 844.8 | −123.5 | −126.3 | −249.8 |
| 379.5–389.5 | 879 | 973.8 | 1,073.5 | − 94.8 | − 99.7 | −194.5 |
| 389.5–399.5 | 1,149 | 1,252.8 | 1,279.1 | −103.8 | − 26.3 | −130.1 |
| 399.5–409.5 | 1,468 | 1,499.2 | 1,428.9 | − 31.2 | 70.3 | 39.1 |
| 409.5–419.5 | 1,996 | 1,648.7 | 1,496.6 | 347.3 | 152.1 | 499.4 |
| 419.5–429.5 | 1,862 | 1,655.6 | 1,469.7 | 206.4 | 185.9 | 392.3 |
| 429.5–439.5 | 1,928 | 1,512.4 | 1,353.3 | 415.6 | 159.1 | 574.7 |
| 439.5–449.5 | 1,139 | 1,255.7 | 1,168.3 | −116.7 | 87.4 | − 29.3 |
| 449.5–459.5 | 888 | 947.0 | 945.7 | − 59.0 | 1.3 | − 57.7 |
| 459.5–469.5 | 484 | 652.3 | 717.7 | −168.3 | − 65.4 | −233.7 |
| 469.5–479.5 | 298 | 416.4 | 510.7 | −118.4 | − 94.3 | −212.7 |
| 479.5–489.5 | 215 | 253.7 | 340.7 | − 38.7 | − 87.0 | −125.7 |
| 489.5–499.5 | 115 | 154.6 | 213.1 | − 39.6 | − 58.5 | − 98.1 |
| 499.5–509.5 | 92 | 90.6 | 125.0 | 1.4 | − 34.4 | − 33.0 |
| 509.5–519.5 | 72 | 58.4 | 68.7 | 13.6 | − 10.3 | 3.3 |
| 519.5–529.5 | 48 | 39.2 | 35.4 | 8.8 | 3.8 | 12.6 |
| 529.5–539.5 | 36 | 26.7 | 17.1 | 9.3 | 9.6 | 18.9 |
| 539.5–549.5 | 17 | 18.2 | 7.8 | − 1.2 | 10.4 | 9.2 |
| 549.5– | 41 | 36.1 | 5.6 | 4.9 | 30.5 | 35.4 |
| TOTAL | 14,802 | 14,802 | 14,802 | 0 | 0 | 0 |

mechanics of aggregation. As the calculations are quite time consuming, they have only been done for skilled workers on minimum wages and for women on piece wages. The results are shown in Tables 5A and 5B.

If we consider men on minimum wages, the comparison between the actual distribution and the normal distribution for all trades as a whole shows that the actual distribution is leptokurtic and positively skewed. This derives both from the shape of the individual distributions (the deviation) and from the difference in wage level. The tendency to positive skewness also seems to be attributable to the influence of both factors.

For women on piece wages, Table 5B also shows that leptokurtosis and positive skewness can both be ascribed to the shape of the individual distributions

TABLE 5B

THE IMPORTANCE OF AGGREGATION TO WAGE DISTRIBUTIONS. WOMEN—PIECE-WAGES

| Hourly Wage in øre | Actual Distribution $a^{I}$ | Aggregated Distribution on the Basis of Normal Distributions for the Individual Trades $b^{II}$ | Normal Distribution for all Trades $c^{III}$ | I−II (Skewness in the Individual Distributions) d | II−III (Difference in Wage Level) e | I−III |
|---|---|---|---|---|---|---|
| 159.5–169.5 | 3 | 58.6 | 160.6 | − 55.6 | −102.0 | −157.6 |
| 169.5–179.5 | 9 | 42.3 | 114.9 | − 33.3 | − 72.6 | −105.9 |
| 179.5–189.5 | 46 | 75.3 | 178.1 | − 29.3 | −102.8 | −132.1 |
| 189.5–199.5 | 80 | 128.5 | 263.6 | − 48.5 | −135.1 | −183.6 |
| 199.5–209.5 | 133 | 224.7 | 372.5 | − 91.7 | −147.8 | −239.5 |
| 209.5–219.5 | 281 | 383.8 | 502.6 | −102.8 | −118.8 | −221.6 |
| 219.5–229.5 | 503 | 632.1 | 647.7 | −129.1 | − 15.6 | −144.7 |
| 229.5–239.5 | 1,431 | 964.6 | 797.0 | 466.4 | 167.6 | 634.0 |
| 239.5–249.5 | 1,525 | 1,285.1 | 936.5 | 239.9 | 348.6 | 588.5 |
| 249.5–259.5 | 1,758 | 1,492.9 | 1,050.8 | 265.1 | 442.1 | 707.2 |
| 259.5–269.5 | 1,720 | 1,556.9 | 1,125.8 | 163.1 | 431.1 | 594.2 |
| 269.5–279.5 | 1,227 | 1,445.2 | 1,151.9 | −218.2 | 293.3 | 75.1 |
| 279.5–289.5 | 1,055 | 1,195.6 | 1,125.3 | −140.6 | 70.3 | − 70.3 |
| 289.5–299.5 | 797 | 921.6 | 1,049.9 | −124.6 | −128.3 | −252.9 |
| 299.5–309.5 | 654 | 703.4 | 935.3 | − 49.4 | −231.9 | −281.3 |
| 309.5–319.5 | 510 | 547.4 | 795.6 | − 37.4 | −248.2 | −285.6 |
| 319.5–329.5 | 359 | 432.7 | 646.3 | − 73.7 | −213.6 | −287.3 |
| 329.5–339.5 | 299 | 345.2 | 501.3 | − 46.2 | −156.1 | −202.3 |
| 339.5–349.5 | 270 | 265.5 | 371.3 | 4.5 | −105.8 | −101.3 |
| 349.5–359.5 | 189 | 203.3 | 262.6 | − 14.3 | − 59.3 | − 73.6 |
| 359.5–369.5 | 127 | 150.7 | 177.4 | − 23.7 | − 26.7 | − 50.4 |
| 369.5–379.5 | 112 | 107.4 | 114.4 | 4.6 | − 7.0 | − 2.4 |
| 379.5–389.5 | 85 | 74.9 | 70.5 | 10.1 | 4.4 | 14.5 |
| 389.5–399.5 | 61 | 51.0 | 41.4 | 10.0 | 9.6 | 19.6 |
| 399.5–409.5 | 33 | 34.6 | 23.3 | − 1.6 | 11.3 | 9.7 |
| 409.5–419.5 | 26 | 22.5 | 12.5 | 3.5 | 10.0 | 13.5 |
| 419.5–429.5 | 32 | 16.0 | 6.4 | 16.0 | 6.6 | 25.6 |
| 429.5– | 116 | 79.2 | 5.5 | 36.8 | 73.7 | 110.5 |
| TOTAL | 13,441 | 13,441.0 | 13,441.0 | 0.0 | 0.0 | 0.0 |

and the difference in wage levels. Thus the wage system seems to be very important for the shape of the distribution function. This fact seems, if anything, to support the general views of the sociological school. On the other hand, Aitchison and Brown, among others, have pointed out that the assumptions for using the shock considerations must be that the trades concerned are homogeneous.

The aggregated wage distributions deviate significantly from log-normal distributions. The aggregated distributions are leptokurtic and generally positively skewed. Rutherford has pointed out that under certain assumptions about the birth and death process it is possible to arrive at distributions of Gram–Charlier Type A, which resemble the actual income distributions. Cramér argues that his P-distribution, which he has not used as a model for income distributions, should be preferred to the Gram–Charlier distribution, which it

resembles, because the P-distribution has asymptotic properties. Hill has demonstrated that an aggregation gives leptokurtic distributions if the distributions have the same average but different deviations.

One may be a little skeptical as regards the theories of the shock process because a simple aggregation may seem to lead to leptokurtic distributions. Nevertheless one is tempted to examine whether some of the aggregated distributions which are dealt with here should be P-distributed. As the calculation of Cramér's P-distribution is quite time consuming, it has only been undertaken for skilled men on piece wages, using the logs of wages as the variable. I have chosen a distribution of piece wages on the assumption that here wages might probably be fixed on the basis of an interaction of different attributes. The procedure adopted in the calculations has been described in detail by Cramér [11]. The sums of squares arrived at in the calculation of the $\chi^2$ test if the log-normal and the P-distribution, respectively, are used as models are shown in Table 6. This table shows that the $\chi^2$ sums are too great, that the actual distribution accordingly also deviates significantly from the P-distribution. The P-distribution is not nearly as leptokurtic as the actual distribution. In the P-distribution, however, the $\chi^2$ sum is far smaller than the corresponding $\chi^2$ sum in the log-normal distribution.

TABLE 6

COMPARISON BETWEEN LOG-NORMAL DISTRIBUTION, CRAMÉR'S P-DISTRIBUTION AND THE ACTUAL DISTRIBUTION, ALL SKILLED MEN ON PIECE WAGES

| | Log-normal Distribution $\frac{(a_j - u\hat{p}_j)^2}{u\hat{p}_j}$ | Cramér's P-distribution $\frac{(a_j - u\hat{p}_j)^2}{u\hat{p}_j}$ | |
|---|---|---|---|
| 2.37931–2.43056 | 4.03762 | 24.08 | 2.375–2.425 |
| 2.43056–2.44638 | 35.65830 | 1.23 | 2.425–2.450 |
| 2.44638–2.47640 | 20.42488 | 0.05 | 2.450–2.475 |
| 2.47640–2.50447 | 46.07642 | 65.46 | 2.475–2.500 |
| 2.50447–2.53084 | 3.65704 | 85.45 | 2.500–2.525 |
| 2.53084–2.55570 | 61.58706 | 11.25 | 2.525–2.550 |
| 2.55570–2.57921 | 110.27981 | 0.31 | 2.550–2.575 |
| 2.57921–2.60152 | 134.29133 | 157.92 | 2.575–2.600 |
| 2.60152–2.62273 | 61.23745 | 176.02 | 2.600–2.625 |
| 2.62273–2.65273 | 398.23730 | 101.66 | 2.625–2.650 |
| 2.65273–2.67164 | 446.38924 | 210.29 | 2.650–2.675 |
| 2.67164–2.69854 | 21.43245 | 0.00 | 2.675–2.700 |
| 2.69854–2.72387 | 120.73203 | 65.09 | 2.700–2.725 |
| 2.72387–2.74780 | 196.98490 | 40.02 | 2.725–2.750 |
| 2.74780–2.77779 | 83.95073 | 2.43 | 2.750–2.775 |
| 2.77779–2.79900 | 3.90973 | 27.79 | 2.775–2.800 |
| 2.79900–2.82575 | 0.49194 | 0.69 | 2.800–2.825 |
| 2.82575–2.85095 | 15.25356 | 3.00 | 2.825–2.850 |
| 2.85095–2.87477 | 36.78113 | 9.51 | 2.850–2.875 |
| TOTAL $\Sigma x^2$ | 1,801.41292 | 982.25 | |

*Analysis of the Individual Wage Distributions*

Next we shall consider the distributions of the individual trades in relation to the $\chi^2$ test, the $\sqrt{\beta_1}$ test, and the *a*-test. We will test how many of the 148

trades are log-normally distributed, because the tendency to log-normal distributions seems, as mentioned, greater than that to normal distributions.

The main result may be seen in Table 7. This table shows the relation between homogeneity, number of workers in the individual trades, and log-normality. Other things being equal, it is probably to be expected that wage distributions with many observations will be aggregated to a greater extent than trades with few observations.

The table shows that it is wage distributions with few observations which are likely to be at the same time less heterogeneous (groups A and B) and non-significant (column is). However, as there are also many less heterogeneous trades with a large number of observations, this may reflect the difficulty of taking the aggregation problem sufficiently into account. The tests give more stringent limits of heterogeneity than the subdivision into the A, B, and C groups. If relatively normal distributions are chiefly produced in trades with few observations, it must be borne in mind that the tests are less sensitive when the number of observations is small than when it is great. I think that the $\sqrt{\beta_1}$ test is the most relevant one to look at. From the table it may be seen that out of 28 trades in the C group only one is non-significant by the $\sqrt{\beta_1}$ test, while of 120 A and B trades 34 are non-significant.

In the following section I have tried to demonstrate the extent to which log-normal distributions occur, and whether such distributions are due to the work being homogeneous. This seems to be the case to a certain extent. This is, of course, of interest; but it is also of importance to see what the other distributions look like. Solely because of the difference in wages one would expect different frequency distributions, and moreover the degree of heterogeneity will undoubtedly also affect the shape of the distribution. If the groups are more or less non-competing, this fact by itself may result in positively skewed distributions within (aggregated) trades. The shape of the distributions is also influenced by their standard deviations. This is seen most clearly in a comparison of trades with piece wages and minimum wages on the one hand, and trades with standard wages on the other hand.

The following table summarizes all of the groups discussed. By wage system and degree of heterogeneity the trades break down as follows:

| | Degree of Heterogeneity | | | |
|---|---|---|---|---|
| | Least A | Medium B | Greatest C | Total |
| Piece wages | 25 | 26 | 13 | 64 |
| Minimum wages | 16 | 23 | 9 | 48 |
| Standard wages | 14 | 16 | 6 | 36 |
| TOTAL | 55 | 65 | 28 | 148 |

It will be seen that there are relatively many piece wage trades in the A and B groups, and relatively many minimum wage trades in the B group.

There are considerably more positively skewed distributions than negatively skewed, and it is even more characteristic that the distributions are leptokurtic.

TABLE 7

HETEROGENEITY AND SIGNIFICANCE BY NUMBER OF OBSERVATIONS

| Number of observations | $\chi^2$-test | | | | | | | | $\sqrt{\beta_1}$-test | | | | | | | | $a$-test | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | | B | | A+B | | C | | A | | B | | A+B | | C | | A | | B | | A+B | | C | |
| | is | s | is | s | is | s | is | s | is | s | is | s | is | s | is | s | is | s | is | s | is | s | is | s |
| 90–199 | 6 | 19 | 8 | 21 | 14 | 40 | 2 | 4 | 8 | 17 | 11 | 18 | 19 | 35 | 1 | 5 | 7 | 18 | 13 | 16 | 20 | 34 | 4 | 2 |
| 200–399 | 3 | 12 | 4 | 16 | 7 | 28 | – | 2 | 3 | 12 | 5 | 15 | 8 | 27 | – | 2 | 2 | 13 | 8 | 12 | 10 | 25 | – | 2 |
| 400 and over | 2 | 13 | 2 | 14 | 4 | 27 | – | 20 | 3 | 12 | 4 | 12 | 7 | 24 | – | 20 | 5 | 10 | 5 | 11 | 10 | 21 | 3 | 17 |
| TOTAL | 11 | 44 | 14 | 51 | 25 | 95 | 2 | 26 | 14 | 41 | 20 | 45 | 34 | 86 | 1 | 27 | 14 | 41 | 26 | 39 | 40 | 80 | 7 | 21 |

NOTE: is = insignificant, s = significant.

TABLE 8

HETEROGENEITY AND SIGNIFICANCE BY NUMBER OF OBSERVATIONS

| Number of observations | $\chi^2$-test | | | | | | | | $\sqrt{\beta_1}$ -test | | | | | | | | $a$-test | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | | B | | A+B | | C | | A | | B | | A+B | | C[1] | | A | | B | | A+B | | C[1] | |
| | is | s | is | s | is | s | is | s | is | s | is | s | is | s | is | s | is | s | is | s | is | s | is | s |
| 1. 40–89 | 16 | 16 | 15 | 15 | 31 | 31 | (4) | (5) | 14 | 18 | 16 | 14 | 30 | 32 | (3) | (6) | 19 | 13 | 14 | 16 | 33 | 39 | (6) | (3) |
| 2. 90 and over | 11 | 44 | 14 | 51 | 25 | 95 | 2 | 26 | 14 | 41 | 20 | 45 | 34 | 86 | 1 | 27 | 14 | 41 | 26 | 39 | 40 | 80 | 7 | 21 |
| 3. TOTAL | 27 | 60 | 29 | 66 | 56 | 126 | 6 | 31 | 28 | 59 | 36 | 59 | 64 | 118 | 4 | 33 | 3 | 54 | 40 | 55 | 73 | 109 | 13 | 24 |

[1]Reliability is questionable because of small numbers.

NOTE: is = insignificant, s = significant.

Especially in the C group there are, as is to be expected, relatively many positively skewed and leptokurtic distributions. On the other hand, there is only one distribution which is platykurtic, while as expected there are relatively many within the A and B groups, a total of 17. Normal distributions are, as expected, to be found primarily within the A and B groups.

*Trades with less than 90 Observations*

This section considers trades with 40–89 observations. Like the distributions for larger trades discussed in the preceding section, the aggregated distributions seem not to be log-normal. The calculations of the different tests have been carried through in the same way as explained above. To show the importance of aggregation, a comparison was made between homogeneity and number of workers for all trades. This comparison is shown in Table 8, together with the results given above for larger trades.

Looking first at the $\sqrt{\beta_1}$ test, it will be seen that log-normality depends on homogeneity and the tension of the aggregation.

If we look at the three dimensional tests for trades with 40–89 observations, Table 9 shows that there is a greater tendency to log-normality than in the trades with 90 or more observations, because the aggregation effect is not as great.

TABLE 9

INTERRELATION BETWEEN THE THREE TESTS WITH RESPECT TO SIGNIFICANCE, INCLUDING ILLUSTRATION OF HETEROGENEITY
TRADES WITH BETWEEN 40 AND 89 OBSERVATIONS

| | $\sqrt{\beta_1}$-test | $\chi^2$-test | $a$-test | Number of Observations | Distribution with Respect to Heterogeneity | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | A | % | B | % | A+B | % | C | % |
| 1 | is | is | is | 19 | 10 | 31.3 | 8 | 26.7 | 18 | 29.0 | 1 | 11.1 |
| 2 | is | is | (s) | 5 | 2 | 6.2 | 3 | 10.0 | 5 | 8.1 | — | — |
| 3 | is | (s) | is | 4 | 2 | 6.2 | 2 | 6.7 | 4 | 6.5 | — | — |
| 4 | (s) | is | is | 7 | 2 | 6.2 | 3 | 10.0 | 5 | 8.1 | 2 | 22.2 |
| 5 | is | (s) | (s) | 5 | — | — | 3 | 10.0 | 3 | 4.8 | 2 | 22.2 |
| 6 | (s) | is | (s) | 4 | 2 | 6.2 | 1 | 3.3 | 3 | 4.8 | 1 | 11.1 |
| 7 | (s) | (s) | is | 9 | 5 | 15.7 | 1 | 3.3 | 6 | 9.6 | 3 | 33.4 |
| | | 1–7 | | 53 | 23 | 71.8 | 21 | 70.0 | 44 | 70.9 | 9 | 100.0 |
| 8 | (s) | (s) | (s) | 18 | 9 | 28.2 | 9 | 30.0 | 18 | 29.1 | — | — |
| | | 1–8 | | 71 | 32 | 100.0 | 30 | 100.0 | 62 | 100.0 | 9 | 100.0 |

NOTE: is = insignificant; s = significant.

The test procedure seems to show nearly the same result for groups A and B. If we look at the $\sqrt{\beta_1}$ test, which for several reasons I prefer, for these two groups, 48 per cent are insignificant, whereas only 28 per cent were insignificant for trades with 90 or more observations.

For the other tests we get similar results.

What conclusion can we draw from the data concerning the problem of the tendency toward log-normality when the groups are homogeneous and the numbers of observations are small (no aggregation)? It seems to me that it would be relevant to look at the test under two conditions: (a) all three tests are insignificant, and (b) the $\sqrt{\beta_1}$ test is insignificant. For the first test we get the following result.

| | *Number of observations* | |
|---|---|---|
| | *40–89* | *90 and over* |
| Groups A and B combined: | | |
| Number significant | 44 | 110 |
| Number insignificant | 18 | 10 |
| Insignificant as % of total | 29 | 8 |
| Group C: | | |
| Number significant | 8 | 27 |
| Number insignificant | (1) | (1) |
| Insignificant as % of total | (11) | (4) |

It is obvious that the degree of heterogeneity has an effect on the probability of obtaining log-normal distributions and that the number in the trades—the probability of aggregation—also has an effect.

The second test, with only the $\sqrt{\beta_1}$ test insignificant, gives the following result:

| | | |
|---|---|---|
| Groups A and B combined: | | |
| Number significant | 32 | 86 |
| Number insignificant | 30 | 34 |
| Insignificant as % of total | 48 | 28 |
| Group C:[3] | | |
| Number significant | (6) | 27 |
| Number insignificant | (3) | (1) |
| Insignificant as % of total | (33) | (4) |

The figures seem to show that homogeneous trades with small numbers will have the greatest tendency to be log-normal.

[3]The numbers are small for groups up to 400 observations. I do not think the figures for smaller groups are reliable.

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