INCOME AND WAGE DISTRIBUTIONS
PART I: A SURVEY OF THE LITERATURE

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This paper, the first of a two-part series, surveys the literature in the field of income and wage distributions. The author divides work in this area into two schools: the theoretic-statistical school, and the socio-logical school. Within each of these groups he reviews leading contributions. He then examines the work of Tinbergen, which, the author feels, fits into neither of the older classifications; rather, Tinbergen approaches the distribution of income as a problem in analyzing the supply of and demand for various attributes, such as intelligence, physical strength, ability to get along with people, etc. In conclusion, the author points out areas which he feels need further work. The paper is based upon the author's book in Danish, *Indkomst-og lønfordelinger*.

Many economists have been fascinated by skewed income and wage distributions. Several subtle theories have been advanced to explain this skewness, and the apparent stability and regularity of these distributions have led to attempts at expressing the income distribution in terms of certain distribution laws, such as the Pareto distribution, the log-normal distribution, or the Gram-Charlier type A distribution. According to background, knowledge, inclination, and temperament, writers have more or less one-sidedly emphasized different factors in their explanations of the skewness. They can roughly be divided into two main schools. The first comprises statisticians and economists, who regard income formation as the product of certain stochastic processes. I shall refer to this group as the theoretic-statistical school. The other school has to a greater degree considered the problem from a sociological point of view, pointing out that differences in income level for different social groups may lead to income distributions characterized by positive skewness. I shall call this the sociological school. Finally, Tinbergen has tried to explain the formation of the income distribution on the basis of a modern theory of supply and demand. Naturally, there are also some authors who have tried to bridge the gap between these two schools, and these attempts are particularly interesting.

In the following sections I have tried to describe the considerations of the various statisticians and economists concerning the causes of the shape of the income and wage distributions, and, particularly in the closing remarks, attempted some assessments of the theories which have been advanced. Since the theoretic-statistical school simplifies the problems in many ways, I shall start by discussing this school.

THE THEORETIC-STATISTICAL SCHOOL

This school is based, to a very considerable extent, on a special formulation of the central limit theorem as expressed in the law of proportionate effect.

The law of proportionate effect states that if the change in a variate y at any step of a process of change may be expected to be proportional to the preceding variate x, the distribution becomes log-normal, e.g., $\log x_n$ will be distributed normally.

Aitchison and Brown (1, Chapter 10) discuss various examples where the law of proportionate effect seems to have been fulfilled, including certain biological processes and certain breakage processes. More to the point, it is interesting that certain output distributions—e.g., the quantity of goods produced per unit of time—seem to be approximately log-normally distributed. If this is the case, it may be assumed that certain wage distributions could show the same form. It may therefore be asked what could cause output distributions to approximate log-normal distributions.

Roy [2] emphasizes that the primary condition for obtaining log-normal distributions must be that attributes interact *multiplicatively*, but adds that other conditions must also be fulfilled, including the following: (1) attributes of different kinds must not be correlated; (2) attributes must follow a binomial distribution, and (3) the standard deviation must be presumed to be fairly constant within the individual distributions of attributes. Given all these conditions, the quantity of goods produced by workers per unit of time will follow approximately a log-normal distribution. This will apply also to wages if they are positively correlated with output.[1]

To obtain log-normal distributions it is, in Roy's opinion, necessary that the quantities (the different attributes) should be uncorrelated; but as Haldane [4] has demonstrated for three variables, log-normal distributions are produced even if the quantities are correlated, if the coefficients of variation are constant. The correlation coefficients may be different as long as they are positive. Roy points out that the assumptions of symmetric distributions may not hold, so that we should not expect full agreement with log-normality for the output distributions. By applying tests for skewness and humpedness Roy has studied concretely 12 output distributions. The types of work involved are of a rather primitive character, i.e., rather homogeneous. Some of the distributions seem to have a tendency towards negative skewness, but on the whole the distributions seem to be rather more log-normal than normal.

Roy discusses in some detail why agreement with the log-normal distribution is not quite reached and mentions that in the case of output per hour such factors as number of hours worked, illness, etc., are not considered. So even if output per hour does not come very close to the log-normal distribution, it is quite possible that a distribution of output per year would give better agreement. He does not try to give any concrete, measurable distributions of talents; but assumes that the conditions for a binomial distribution are to some extent fulfilled, and assumes without submitting any proof that attributes should act multiplicatively on each other, not additively. He mentions (p. 493) that it may be considered probable that when workers become indisposed,

[1]Boissevain [3] has also shown that normal distributions of attributes interacting multiplicatively result in income distributions with positive skewness. However, he emphasizes that for certain occupations in which the scarcity factor exercises a strong influence, it may not be possible to trace this influence.

for example, their output will be reduced in a constant ratio rather than by a constant absolute amount. He also thinks that women's output will tend to be a certain fraction of the output of men of similar skill, age, education and so on. To this may be added that it is a general experience that slow manual workers are also often poor workers, which seems to indicate a multiplicative adverse effect. Roy advances no proof of his assertions, but his views seem reasonable. Of course, we cannot know whether the interaction of attributes which takes place is multiplicative. It can assume a multitude of other forms.

Lydall [5], dealing solely with wage distributions, thinks that inborn intelligence seems normally distributed, but the different social groups one belongs to and the different education one has the chance to obtain influence the opportunity for expansion of ability.

From a psychological point of view the idea that talents may tend to act multiplicatively is not unacceptable, but there does not seem to be any literature on the subject. Whether or not talents act multiplicatively or additively, or both, or whether there is a different combination, it will of course be interesting to see if it is possible to find log-normal or normal distribution laws, if the condition that the work performed is relatively homogeneous is met. If this is possible, we should have a basis for positing that the distribution of talents contributes to explaining the shape of the personal income distribution. This seems particularly clear if the homogeneous work mainly leads to log-normal distributions since in that case the positive skewness already exists, but positively skewed aggregate distributions may also result even if the homogeneous groups should be normally distributed since such a positive skewness can be produced through aggregation. As will be mentioned later, a Gram-Charlier A-distribution may also result; such a distribution may be produced under quite simplified assumptions, but it may also arise through aggregation. Of course, the whole theory may be somewhat tenuous, because there is no guarantee that earnings are proportionally or linearly connected with talents. Other forms are quite possible.

Thus, the conditions for obtaining a log-normal distribution are not too stringent, so it is quite possible that the necessary assumptions will be fulfilled. The real question arising from Roy's studies is therefore the following: Is it possible that different attributes act multiplicatively together and is not this fact—if it is a fact—the sufficient and real explanation of the tendency towards log-normal output distributions and possibly also certain wage distributions. Only a really thorough analysis of homogeneous output distributions or wage distributions can throw more light on this question.

There are other researchers who have discussed the question of the formation of the income distribution on a more general basis, if from very special assumptions. The experience which has been gained in the course of time, and which Pareto [6] formulated for the higher income brackets, has caused various researchers to ask: Can we set up certain simplified assumptions which are of such a nature that they will produce the Pareto distribution or other specified distributions?

The authors I shall discuss in this section all take a special theoretic-statistical view of the formation of the income distribution. Given an arbitrary

initial distribution, a shock system is set in motion, which in turn will trigger a stochastic process. These authors then show, using different assumptions, that any initial income distribution will converge towards certain distributions: a Pareto distribution, a log-normal distribution, or a Gram-Charlier A-distribution.

The necessary condition for such a convergence is that the transition probabilities, i.e., the probability of moving from a given income interval to the next, are independent of the original shape of the distribution (the frequencies) and only dependent on the ratio between the income before and that after the transition. In short, the authors assume the law of proportionate effect is operating—i.e., that the matrix of transition probabilities must be constant through time, based on a certain form of Markov process (see [7] and [8]). I shall now discuss some of these authors' ideas to show the influence of the assumption about the size of intervals in the income distribution on the ultimate frequency distribution which is produced.

Champernowne [9] assumes that his model is produced through a stochastic process (a shock process), and that the stochastic process is constant through time. He operates with certain special simplified assumptions, although he points out that even if the simplified assumptions were abandoned the general conclusions would not be altered. Among the conditions he sets up are:

1. The income intervals in the income distribution are assumed to follow a geometric progression.

2. No person is assumed to move more than one interval upwards, whereas he can very well move down several intervals.

3. Income is assumed not to increase indefinitely.

4. The law of proportionate effect applies to the transition probability.

Finally, he assumes that for every income-receiver who disappears there is an heir to his income in the following year, which implies that the number of incomes is constant while those who receive the income change. This assumption does not seem very realistic. After a sufficiently long time this model will lead to a Pareto distribution.

Champernowne next abandons some of the simplifying assumptions and, for example, allows for the influence of age and occupation, showing that this does not change the general conclusions that can be drawn. He also relaxes the assumptions limiting upward shifts of income to one range in a year, and limits the basic assumption that the prospects of various amounts of percentage change of income are independent of initial income so that it applies to higher incomes only. About his simplified assumptions and reality Champernowne remarks (p. 319): "The forces determining the distribution of incomes in any community are so varied and complex, and interact and fluctuate so continuously, that any theoretical model must either be unrealistically simplified or hopelessly complicated. We shall choose the former alternative but then give indications that the introduction of some of the more obvious complications of the real world does not seem to disturb the general trend of our conclusions." Beginning one's analysis under strictly simplified assumptions is often a necessary procedure.

Many analyses remain in this rather unrealistic position. It is therefore attractive that Champernowne tries to set up additional, more realistic assumptions in his model in order to bring it closer to reality.

Champernowne demonstrates that the key assumptions leading to the Pareto distribution are those of proportionate effect and of geometric progression. He says (p. 346): "The above examples are probably sufficient to illustrate the theory that the approximate observance of Pareto's law which has so often been remarked upon is not an illusion or coincidence, but has its explanation in a similarity at different high-income levels of the prospects of given proportionate changes of income. They can do little more than illustrate the theory, since they are built on the artificial simplifying assumption that these prospects of change remain constant through time at each income level. It will be readily appreciated that any model catering for prospects which are not constant through time is much more complicated and the results obtainable are far less clear."

Aitchison and Brown [10] investigated the effect of altering the assumption of geometric progression. They consider first income intervals of the same size, and later a model with infinitesimal income intervals. They operate with the usual assumptions for the law of proportionate effect.

In discussing the model with infinitesimal income intervals, the authors set up the problem as follows (page 94): "Let us denote by $F_t(x_t)$ the distribution function of income at time t, that is to say the probability of an income being not greater than x_t at time t is $F_t(x_t)$. Here we must define the transition probabilities in continuous terms and this is done by specifying the probability that a person with income in the interval $(x_t, x_t + dx_t)$ at time t will have income in the interval $(x_{t+1}, x_{t+1} + dx_{t+1})$ by time $t + 1$. This we shall denote by $dG_t(x_{t+1}, x_t)$ and the basic postulate now asserts that $dG_t(x_{t+1}, x_t)$ depends only on t and the ratio x_{t+1}/x_t. Thus we may write $dG_t(x_{t+1}, x_t) = dH_t(x_{t+1}/x_t)$. These assumptions must lead to a log-normal distribution.

Aitchison and Brown point out that the assumption of the law of proportionate effect requires homogeneous groups. A possibility would be a subdivision by trades: if the income distributions within individual trades are log-normal, the aggregated distribution may also be log-normal, if the variance for individual distributions is constant, and the average incomes of the individual trades are log-normally distributed. Aitchison and Brown discuss one example with 109 trades which seems to fulfill the conditions. A good approximation is obtained despite the stringency of the required assumptions.

The authors mentioned so far have assumed that the distribution of income in any year is a product of the distributions of the preceding years through a system of random shocks. Varying with the assumptions chosen, the law of proportionate effect must lead to log-normal distributions or Pareto distributions. Rutherford [11] points out that the empirical distributions of income in a number of countries do not seem to fulfill these conditions. To obtain a better fit, he introduces a birth and death process. A constant annual inflow of newcomers to the income distribution, of the same age, is assumed. The income distribution of the newcomers is log-normally distributed. The incomes of the newcomers are dependent on their own education and abilities and the

social status and income of their parents. These attributes are also assumed to be log-normally distributed. The average income and the variance for these newcomers are assumed to be constant. Normal mortality in accordance with the known death rate data is postulated, and mortality is independent of income. The model subjects the newcomers to a series of normally-distributed independent stochastic shocks in successive years. Like the authors mentioned above, Rutherford operates with the law of proportionate effect. If the shock system has a mean of zero and constant variance, he shows that the aggregated income distribution will be the Gram-Charlier Type A, i.e., an S-shaped distribution which to some extent—but not fully—seems to approximate the actual distributions he has studied.

Rutherford then abandons some of the stringent assumptions. The annual inflow is not assumed to be constant, but to increase exponentially. The shock system is allowed to have a mean different from zero and is assumed to be lepto-kurtic. On the other hand, he maintains the assumption that the shock system is independent of the preceding income distribution, i.e., the probability of a given change in income is not dependent on the existing income distribution. The assumption that the mean value of the shock stytem differs from zero will lead to skewness in the aggregated distribution. The resulting model is applied to data from various countries, and seems to achieve fairly good agreement.

Rutherford's analysis—and those of the above-mentioned authors—requires that the process goes on over a long period so that the population consists exclusively of newcomers. On this assumption Rutherford's model is interesting, and his criticism of Champernowne's assumptions of constant inflow and outflow seem justified.

I have dwelt upon Rutherford's elegant theories because it seems possible to explain the S-shaped distributions he has found by means of his theoretic model. But such S-shaped distributions (lepto-kurtic distributions) may also result from the aggregation of normal or log-normal distributions, even if the distributions have the same average, if the standard deviations are different, cf. [28]. It thus cannot be ruled out that the tendency for the income distributions to become S-shaped may be due to aggregation alone.

One cannot help wondering whether the changes in the income distributions in the course of time, which may be due to many different influences, are subject to such a relatively constant probability matrix. If this is the case, it would be expected that as societies reach a more advanced stage of development, the income distributions for the different countries would approach the same shape. According to the stage of development of the countries their distributions would be close to or far from that shape. A characteristic feature of economic development has been increasing equality of the income distribution caused by the rise of wage-earners and salaried employees. The interesting question is whether we are approaching a state of equilibrium where the shape of the income distribution will be relatively permanent.

Kravis [12] has discussed the reasons why income distributions seem to be more equal in the West European countries and North America than in the developing countries, and he concludes (p. 416):

"We conclude that the explanation of the greater income equality that is found in the developed countries lies in the social and economic conditions that distinguish them from the underdeveloped countries. The developed countries have proceeded farther in the integration of all segments of their populations into their social and economic life. In the underdeveloped countries, on the other hand, barriers to equal educational opportunity and direct economic discrimination on the basis of race, nationality, language, dress, or other characteristics tend to a greater degree than in the developed countries to exclude some groups from competition on equal terms with others in the population. While early changes in economic structure bring greater economic differentiation and thus may produce more inequality than prevailed in the pre-industrial society, further economic growth, accompanied by the spread of education, the rise of insurance, the growing importance of the corporation, and the tendency for the labor share in income to rise relative to the property share, produces a movement toward the more equal distribution of income."

Soltow [13] has analysed the importance of changes in education, age, and occupation on the levelling of income. He comes to the following conclusion for the U.S.A. (p. 453):

"Age has had some effect in increasing inequality because of the relative shift from younger to older people. Its continuing effect in the next twenty years will be negligible. Historically, shifts in the importance of the various occupations to those having less income dispersion has reduced inequality. Education shifts, although not as important historically as occupation shifts, will continue to be a strong factor in decreasing inequality of income as the average educational attainment level is raised."

Lydall [5] has examined the distributions of wages in different countries; he has shown that rich countries normally have wage distributions with a smaller dispersion than poor countries and that the dispersion seems smallest in the communist countries. He attributes the differences to education and to the importance of agriculture in the national economy.

Benoit Mandelbrot [14, 15, and 16] also considers the Pareto distribution or, as he calls his special adaptation of the law, the Pareto-Lévy law. Like other authors he stresses that the law is valid only for high incomes, and in contrast with Champernowne, he demonstrates that it is not necessary to assume that the law of proportionate effect is valid. Mandelbrot also offers a discussion of the connection between abilities and the Paretian distribution of income. It is assumed that each individual chooses the occupation that offers him most. Mandelbrot develops the concept of "weights" for various occupations which are equal to the number of factors (abilities) that must be large in order for an individual to receive and accept a large offer in a given occupation. He shows that if the total income distribution is Paretian with exponent α, the offers accepted from each occupation taken separately will also be Paretian, but with an exponent of the form $w(n)\alpha$, where $w(n)\alpha$ is the weight. If the weight is large, the law of Pareto becomes practically useless and should be replaced by the log-normal distribution.

The quantity $w(n)$ has special interest. Mandelbrot shows that the quantity (weight) will be 1 when only one of the ability factors is dominating. If there are two factors dominating then the weight will be 2, and the weight will increase when more and more ability factors are necessary for the job. The larger the number of factors, the fewer the number of people deriving high income from that occupation, and conversely, most very highly paid people will be highly specialized.

Partly attached to this school is also Friedman [17], who emphasizes that differing propensities to undertake risks influence the distribution of income and wealth. He points out that for certain occupations (those where there are considerable opportunities for profit and loss) there will be a skew distribution of income and wealth resulting from the risk factor. Many people within the industry do not obtain the special remuneration. Only a chosen few will get it, and this fact gives two distributions which when added together give a distribution with positive skewness.

Rutherford's observations had the advantage that they seemed to lead to distributions corresponding to the actual distributions. By introducing a birth and death process he showed that the ultimate shape towards which the income distribution had to converge under certain simplified assumptions was a Gram-Charlier A distribution, and that a modified Gram-Charlier distribution could—with quite good approximation—be applied to the actual income distributions for different countries. We should, therefore, perhaps go a little more deeply into the more general requirements for obtaining the Gram-Charlier distribution. Cramér [18, 19, 21] has discussed these more general requirements and demonstrated that a modified Gram-Charlier distribution (the P distribution) is to be preferred because this has the attractive characteristic that the serial development becomes asymptotic. A limitation of Cramér's analysis, however, is that he assumes only an additive effect. As demonstrated by Roy and others there seems to be a case for assuming that the interaction of the different stochastic variables in income and wage distributions may be multiplicative.

The Sociological School

The sociological school takes a general view which differs widely from stochastic shock theory, that the shape of the income distribution depends—apart from the distribution of talents—on a number of institutional factors which influence the income level in the different trades. This difference in income level—in conjunction with the distribution of talents—is a contributory factor in producing a positively skewed income distribution. There are a great number of these institutional factors: sex, age, occupation, education, trade, geographical differences, the distribution of wealth, etc. If changes occur in these factors and if these changes affect income levels, the shape of the income distribution will be changed. The views of the two schools thus differ widely. The theoretic-statistical school assumes a process leading to convergence towards a definite type of distribution, whereas the sociological school says that the shape of the income distribution at any given time is the result of a historical process, and

the shape may therefore be changed when the relative importance of the different institutional factors is changed. The importance of the historical process is stressed in the influence of the distribution of wealth on the income distribution by way of the interest yield.

The sociological school has a long history. Taussig [22] discussed the inequality of income distribution and its causes, and went into differences in remuneration between the different classes. He mentioned that differences in wages or salaries may be classified in two groups. The first group comprises the factors which act toward equalization considering the nature of the work performed, and the second group comprises the factors which lead to differences in pay in spite of the nature of the work performed (real wage differences). If there were a perfectly free choice among different trades, there would be only levelling wage differences caused by the first group of factors. Under these assumptions dirty and unpleasant work would be expected to attract better wages than pleasant work. Seasonal work would be better paid than other work, owing to the greater risk of fluctuating employment. Finally, better education should lead to better pay for more exacting work. But there is not free competition; on the contrary, there are various barriers among trades. These barriers Taussig attributed (p. 136) to "expense of education and training; the subtle influence of environment; and finally differences in inborn gifts." In view of these factors, he thought that society could be divided into five non-competing groups: (1) day laborers, (2) those not needing specialized skill, (3) the aristocracy of the manual laboring class, (4) the well-to-do, and (5) the highest class of the well-to-do. Cutting across this class system, he mentioned particularly the position of women, giving various reasons why women are paid lower wages. He mentioned that women are less productive than men, and that many, particularly young women, live with their parents and are unmarried. They can be underpaid since there is a great supply of them; and since they have no family to maintain, the demand for high wages is not so insistent.

Pigou [23] held similar views. In a chapter on Pareto's law he discussed the reasons why the income distribution is skewed. He thought that if the income distribution were to depend solely on personal characteristics, we should have a normal distribution. He thus considered the attributes additively. But the income distribution also depends on the distribution of wealth, and the latter is skewed.

Spengler has stressed [24] the influence upon the income distribution of (1) occupation, (2) regional differences, (3) education, (4) age, (5) sex, and (6) race. He assumes a skewed distribution for gifts, and mentions the multiplicative interaction of gifts, where he refers to Roy and Boissevain.

Thomas Mayer [25] also emphasizes that occupations requiring higher education receive better pay, leading to positively skewed distributions even if the distribution of gifts should be normally distributed.

In this connection Jacob Mincer's observations [26] must also be mentioned. He too is critical of the theoretic-statistical school, saying (p. 283): "With few exceptions, the sole purpose of the models is to rationalize a presumed mathematical form of the aggregate." As a partial explanation of the fact that

income distributions are positively skewed, Mincer points out that in occupations requiring a higher education higher wages are received. Moreover, the differences in wages do not differ according to an additive constant but by a multiplicative factor. This would imply a tendency to positively skewed distributions. Furthermore, the difference in wages by age is widest for occupations requiring a higher education. Workers with a good education have thus the greatest possibilities of perfecting their knowledge or skill. This should lead to further skewness in the distributions. These circumstances, Mincer underlines, give only a partial explanation of the origin of the skewness. For American conditions Mincer thinks that he can demonstrate that about one-third of the standard deviation and skewness can be attributed to training factors. Skewness is increased considerably if the age factor is included.

Lydall [5] treats the shape of the wage distributions in detail. He stresses four factors: (1) inborn differences in abilities, (2) the influence of social status (environment), (3) differences in education, and (4) the hierarchical structure. While the first three of these factors would lead to log-normal lepto-kurtic distributions, the last would lead to a Pareto distribution in the upper range. It is with respect to the last point that Lydall's theory is new. People with the highest salaries are in his view not paid according to their abilities, but according to the post they maintain in a hierarchical system. This means that the wage level will be determined according to their responsibility as leaders. To account for the Pareto distributions, Lydall set up some simplified assumptions: (1) the leader in a certain class has responsibility for a fixed number of persons in the class below his own; and (2) the salary the leader gets constitutes a fixed proportion of the whole salary in the class below his own. These simplified assumptions must lead to a Pareto distribution.

Gerard Adams [27], among others, has carried out an interesting attempt at evaluation of the different factors. Adams explains the skew income distribution in the usual way: differences in average income by occupation, education, age and geographical location, and the number of people with low incomes, as well as skewness within individual homogeneous groups. These considerations are to some extent tautological. Adams attempts to measure the importance of the socio-economic factors, basing his study on the Michigan Survey of Consumer Finances. This material includes only white males; Adams introduces neither sex nor race in his analysis. He shows—both by means of an analysis of variance and a regression analysis—that income depends, as expected, on a number of socio-economic factors (occupation, education, age, geographical location, etc.). The socio-economic variables can explain forty to fifty per cent of the variations in income, and these factors have a significant influence. Adams then examines the distribution of the residual. He comes to the interesting result that for the higher social groups there is a tendency towards a skew distribution of the residual, while the distribution for the lower groups tends to be normal. It therefore seems as if the residual may be dependent on the size of previous income, which is the condition for a tendency towards log-normal distribution.

Hill [28] has undertaken for the U.K. an investigation similar to the one Adams has done for the U.S.A., as has the present author [29] for Denmark.

The authors discussed so far have viewed the determinants of the distribution of income as the result of certain stochastic processes or as being institutionally determined. Tinbergen [30] considers income distribution as a result of the supply of and demand for attributes. As he says, there is no general theory which can explain income distribution (p. 156): "The fairly satisfactory state of affairs with respect to the statistical description of income distribution contrasts with an unsatisfactory state in the area of economic interpretation. No generally accepted interpretation of the statistical regularities seems to exist and most economic textbooks do not even deal with such an interpretation. This is the more remarkable since the inequality in income distribution is at the bottom of some of the most important problems of economic policy." What is lacking is "a precise description of the mechanism of income formation in terms of the usual instruments of economic analysis."

Tinbergen starts by saying that he is going to adopt the usual procedure in economic analysis by asking how supply and demand determine the shape of the income distribution (the parameters). He emphasizes that his analysis applies not only to labour, but also to capital and land. He assumes the market to be split up into homogeneous groups (occupations), with homogeneous "talent groups." He next assumes that the price of labour, i.e., wages, can be described as a single function of a set of variables, describing his variables (p. 157) as "the degree to which certain attributes and qualities are needed for contributions to be made." It is thus the evaluation of different attributes he is interested in, and these attributes are "intelligence, ability to work under unfavourable physical conditions, ability to deal with other people and so on." He calls the evaluation of the attributes which are demanded s and those supplied t. Both the "demanders" and the "suppliers" must be assumed to have certain ideas of the shape of the frequency distributions for the attributes; if these distributions were identical, i.e., if the number of units of each category required and the number available at a given wage were the same, production would be organized in such a way that every job would be performed by the person who was qualified to perform it. Now, we cannot expect this to be the case; therefore certain tensions arise between the attributes that are available and those that are required, and it is these tensions that affect the shape of the income distribution.

To make the approach clear, Tinbergen only includes two variables from the supply and demand side, respectively (t_1, t_2) and (s_1, s_2), and he assumes that they are normally distributed, but with a different average and a different standard deviation. Further, he assumes no correlation between s_1 and s_2 or between t_1 and t_2. As a decisive simplification he begins by assuming that demand is inelastic, i.e., only the adjustment of supply is considered. Then Tinbergen assumes that the individual supplier will maximize his utility function. Utility is assumed to be measurable, and equal percentage increases in income are assumed to cause equal additions to utility, and equal increases in "tension" between s_1 and t_1—whether positive or negative—to cause parabolic increases in disutility. With a given income distribution the individual will choose his job so as to maximize his utility function.

The process of income formation now aims at merging the correlation surfaces for the t distribution and the s distribution, at the same time maximizing the utility function. Since the demand frequency distribution is assumed to be inelastic this means that the t-surface has to be deformed so as to coincide with the s-surface, otherwise there will not be equilibrium in all compartments of the market.

Given all of these assumptions, and in particular the assumption that different attributes are uncorrelated, Tinbergen arrives at the result that (p. 168):

> "an income scale would be possible where log I is a quadratic function of the 'degrees' s_1 and s_2 but without a 'mixed term'. This means that, generally, income has to depend on the degrees of the attributes of the contributions. The size of the coefficients, which are indicative of the extra remuneration wanted if a higher degree is required, depends on certain 'tensions' again, namely, for the linear terms . . . on the tensions between average required and average available degrees; and for the quadratic terms on the tensions between the dispersion of the required and the available degrees. If the dispersion in the required degrees is larger than that in the degrees available, there will be a positive quadratic term, meaning that successive equal increases in required degrees will have to be remunerated by increasing additional percentage income increases. If the dispersion in the required degrees is smaller than that in the degrees available, successive equal increases in required degrees can be remunerated with decreasing additional percentage income increases. These scales do not depend on the requirements made for the other degree."

The "mixed term" vanishes since uncorrelated attributes are assumed. The income parameters, whose size is dependent on "certain tensions" between the supply and demand parameters, must produce a skewed distribution function for income. As Tinbergen points out, if tension can be reduced, it will also be possible to reduce the inequality of the income distribution. If the assumptions are tightened further and it is assumed that the standard deviations in the s and t distributions are identical, Tinbergen points out that log-normal distributions will be produced. Tinbergen assumes that homogeneous groups are considered and accordingly mentions that an aggregation under the above-mentioned assumptions will lead to log-normal distributions. Aitchison and Brown have, as previously mentioned [1] formulated certain conditions for getting aggregated log-normal distributions.

In conclusion, Tinbergen relaxes certain of his assumptions. Assumed values of attributes may of course be substituted for real values. Demand as well as supply may be allowed to vary. The consequences would be that the demand for qualified workers will rise at the expense of unqualified workers. It is in reality the wages drift problem which is coming in here. An adjustment from both the supply and the demand side will increase the possibilities of adjustment. Like Pigou, Tinbergen draws attention to the influence of the distribution of wealth on the income distribution and vice-versa. Through these generalizations realistic assumptions are obtained.

Concluding Remarks

This review of the different theories of the determination of income and wage distributions has shown how differently the problems can be viewed. The discussion concerns both the total income distribution, i.e., the aggregated distribution, and the individual distributions. Man's proclivity towards generalizations and the inclination of economists towards simplifying problems in their working hypotheses are both well-known phenomena. However, this simplification is balanced by some research workers' urge to show the complexities of life and to underline, accordingly, that the end result is produced by the intereaction of many factors. The sociological school is an example of an explanation being sought through a discussion of the great number of factors which influence the aggregated income distribution. The importance of these factors is not the same from time to time, nor from one country to another—and the shape of the income distribution is therefore influenced by the relative strength of the different factors. Such a demonstration is of great interest, particularly if a time analysis of the influence of the factors is undertaken at the same time. Also the quantitative evaluation of the influence of the factors at a given time undertaken by Adams and Hill (as well as myself) must be said to be very valuable, particularly if the income of the individual person is divided into the part that can be explained through the influence of the different socio-economic factors and a remainder which may have been purged so much that it becomes a stochastic variable.

Some of the sociological views of the importance of the institutional factors nevertheless do not give any proper explanation because these institutional factors are themselves a result of certain processes. It may therefore be asked what are the underlying forces which have produced the sociological factors which influence income distributions. A dynamization of the conceptual apparatus of the sociological school seems to be lacking.

The question remains whether the theoretic-statistical school has something fundamental when they show, to be sure under extremely simplified assumptions, that a given initial income distribution, under certain specified assumptions, will converge, through a shock system, towards a specific shape (Pareto distribution, log-normal distribution, Gram-Charlier distribution). In an interesting article [31] Kendall says the following about this, pp. 14–15:

> "Champernowne (1953) was, I think, the first to propose an explanation in stochastic terms, individuals moving from one income group to another with certain transition probabilities. We can now write down models to explain the Pareto distribution which, to put it no higher, have considerable plausibility. Hart and Prais (1956) have applied similar ideas in studies of business concentration. Work of this type is in a phase of such rapid development that it is difficult to say where it will lead us; but the prospects are very encouraging."

It probably cannot be ruled out that the many factors affecting the income earning of individuals can have the mentioned effect. If this is not necessarily reflected in a specific ultimate distribution shape, this may be due to the fact that the simplified assumptions about the transition probabilities cannot be

maintained, e.g., because of shifts in the social structure. On the other hand, it is curious to see that the shape of the income distribution for wage earners in Denmark, which follows neither a Pareto distribution nor a log-normal distribution, seems to be the same for different subgroups, and this applies to conditions both before and after the war. The difference in the distributions seems to manifest itself solely in a different standard deviation.[2] This peculiarity is in harmony with Rutherford's considerations because his assumptions lead to distributions which correspond to the distributions which can actually be ascertained. The problem therefore remains whether the shape of the aggregated income distribution is determined by a country's phase of development, or whether there is something stable behind it in the sense that the influence of *many* elements on the incomes of individual persons tends to produce a specific shape of the income distribution. This is the general approach of the two schools; deviating from this, of great importance, is Tinbergen's analysis. Tinbergen justly emphasizes that so far no real attempt has been made to give a precise description, by means of the usual concepts of economic analysis, of the mechanism contributing to the formation of the income distribution. He considers the income distribution and its shape as a result of supply of and demand for attributes. Since the supply of and demand for these attributes differ, certain tensions will arise between the frequency distributions. He demonstrates that the "tensions" between the supply and demand parameters (average assessments of the attributes and the standard deviations) must result in the distribution function for the income distribution becoming skewed, the skewness being determined by the extent of the tensions. The conclusion may therefore be drawn that if a reduction of tensions were possible, the consequence would be a tendency towards a specific shape of the income distribution.

Taking Tinbergen's considerations as a point of departure, one's thoughts might pursue a different course. Behind the shock theory lies the idea that the general trend may lead towards a specific ultimate distribution. It may be more fruitful to assume that behind the historically determined distributions there is a structural distribution which has some basic shape, which can be expressed through some general formulation.

Now in Denmark there is a tradition dating back as far as Westergaard's days that when we deal with aggregated quantities, we shall try to split them up since if we get down to homogeneous groups we may possibly expect normal distributions. This view I have accepted in studying the individual wage distribution statistics for the second quarter of 1951 compiled by the Danish Employers' Association.

The analysis of the wage distributions becomes all the more interesting in the light of Roy's study of output distributions in which he tries to show that attributes tend to act multiplicatively, and that this leads to log-normal distributions. If, then, there is agreement between output and wages, the result should be log-normal wage distributions—i.e., positively skewed distributions. This positive skewness in the individual distribution caused by the

[2]The data available from the Danish Employers' Association, on the other hand, do not seem to fulfill the mentioned conditions. The shape is too different for the individual trades, which may be due to the more detailed breakdown.

multiplicative effect should explain the tendency towards positive skewness of the income and wage distributions.

The second explanation of the positive skewness is of a socio-economic nature and has to do with the difference in income level which exists between different occupations, and the fact that many people have low incomes. Explaining these, in turn, is the influence of the environment in which one grows up and of the education one has received.

When we get down to individual trades, possibly in certain cases homogeneous trades, the question arises what the result would be if we were to go the opposite way, i.e., aggregate. Hill has emphasized that even if normal distributions have the same average, the aggregation will give a lepto-kurtic distribution if the standard deviations in the distributions are different.

In [31] Kendall has some interesting views of aggregation. He says on pp. 13–14:

> "To construct anything approaching a realistic model of a social situation is a formidable undertaking. The first difficulty is one of aggregations.
>
> "Sometimes we may be reasonably certain that the aggregative process is not distorting constituent series; for example, the sum of a number of Pareto distribution has, as a rule, the shape of a Pareto distribution, so that aggregation may blur the edges but will not obscure the main outline. Economic enquiries are particularly vulnerable to this difficulty and it is not always to be resolved by subdivision of the data—in one sense we have to aggregate to some extent to get a pattern at all. We must clearly take care not to aggregate too much. But how we know what is 'too much' is a question I cannot answer."

Aitchison and Brown have emphasized that the conditions for getting aggregated log-normal distributions are exceedingly stringent as the individual distributions must have the same standard deviation and the averages of the distributions must be log-normal. The Danish aggregated wage distributions, for instance, are not, indeed, log-normal. Thus, there is quite a case for lepto-kurtosis in the distributions—and that is precisely what characterizes the distributions Rutherford has been working on. Accordingly, we have on the one hand a theory derived from certain simplified assumptions, and on the other hand, the more commonplace thing that an aggregation seems to give certain tendencies in the same direction. It seems hard to find a way; but perhaps the problems may be seen from the point of view that the postulated shock processes are operating—but the simplified assumptions are not fulfilled. Therefore attention will naturally be drawn to the concrete situation, and one will explain the income distribution on this basis. Tinbergen's tensions may be varying and may thus explain changes in the shape of the distribution. It may be asked whether the equalization of incomes which has taken place will not lead to a more stable income distribution, so that to a greater extent than previously we are approaching the theoretically ideal situation; but who can tell whether this situation is going to last.

A study of the literature about the personal income distribution does not seem to give a unique explanation of the process of income formation

and the personal income distribution at any given time; but on the basis of the views of the different authors I am inclined to hold the conventional opinion that any observed income distribution is a result both of a shock process and of sociological factors. The problem that has not been solved is a real dynamization of the explanations of both the shock theory and the sociological school. Behind wage determination there are factors of supply and demand, as mentioned by Tinbergen and Mandelbrot; but nobody seems to have taken up the problem of how the special wage-fixing which takes place between organizations might influence the income distribution.

Now it must not be forgotten that wage-fixing dictated by the relative positions of power of the parties will primarily influence the level of the wage. Wages are to a very great extent fixed individually, and changes in wages take place through wage drift, where the qualifications of the individual are undoubtedly of great importance in fixing wages; it must not be forgotten, however, that qualifications do not attain their full scope at standard wages—a fact that must be included in the explanation of the actual wage distribution. Bearing in mind a phenomenon like wage drift, one should not underestimate the importance of Tinbergen's theory of tensions nor the importance of the theories touching the problem of the way in which attributes interact.

In this connection it should be stressed that Tinbergen's theory of tensions naturally is important especially in connection with the remuneration of employees. The theory of tensions will not apply directly to the income distribution for proprietors since residual income is determined by output prices, the amount of goods sold, input prices, and the consumption of raw materials, but also by the level of wages—and this is how the theory of tensions will exercise influence.

In conclusion, I think that it will probably be possible to develop a system embodying Rutherford's, Mandelbrot's, and Tinbergen's thought, taking into account certain assumptions about the institutional factors. Rutherford's somewhat simplified assumptions could be incorporated in Tinbergen's economic theory, but in such a way that the model is applied to a society with certain given institutions. But we shall still be faced with the multiplicity of reality, even an elaborate system can only give an idea of the dynamics of forces which have been operating and are still operating in the formation of the personal income distribution.

Presently-available theories of the determinants of the income distribution are probably not adequate. It has already been mentioned that a dynamization of the views of the sociological school must be considered imperative. The shock theory suffers from the defect that a once-only shock is assumed. Actually a dynamic analysis would require that the effect of numerous small shocks at small time intervals on income formation should be investigated.

It might be interesting to test, through studies for many countries, whether certain standard shapes for the income distribution can be expected, considering the level of development of the country—and whether the deviations from the standard among the countries classified by strata might in themselves reflect differences in standard deviation.

Difference in standard deviation would then probably express something about the more special deviations in the institutional factors, while the fact

that there should be standards at all would say something about the effects of a shock system. According to what has been stated above, we should not expect the effects of this system to give the same effect within strata because the institutional factors differ widely. Similar considerations may be advanced in connection with the historical development. These would be possible ways of seeking knowledge about the factors operating in the formation of the unequal income distribution.

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