U.S. REAL PRODUCT AND REAL FACTOR INPUT, 1929–1967

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The objective of this paper is to provide a conceptual basis for separating social product and social factor input accounts into price and quantity components. Despite the essential similarity between concepts of real product and real factor input, the measurement of social factor outlay in constant prices is not well established in social accounting practice.

Production accounts are constructed for the United States in current and constant prices, including social product and social factor outlay, for the period 1929–1967. The resulting estimates are applied to the measurement of total factor productivity and the study of the responsiveness of product and factor intensities to price changes.

1. INTRODUCTION

Within the framework of social accounting the production account includes an allocation of the total social product among final uses such as private and public consumption, capital formation, and net exports. The factor outlay account includes a similar allocation of factor outlay among productive factors—labor services and various types of capital services. As an accounting identity the value of the social product is equal to the value of outlays on factor services required for production. The objective of this paper is to provide a conceptual basis for separating social product and social factor input into price and quantity components.[1]

The measurement of social product in current and constant prices is well established in accounting practice. For most countries with production accounts a separation of the social product into price and quantity components is available. Each delivery of social product to final demand involves a commodity or service flow that may be separated into price and quantity components. Quantities and prices of individual commodities and services are combined into indexes of real product and its price or implicit deflator.

An analysis of the sources of economic growth requires the measurement of social factor outlay in current and constant prices. The conceptual basis for separation of factor outlay into price and quantity components is identical to that for social product. Each outlay on factor services must be separated into price and quantity components. Price and quantities of the individual factor services are combined into indexes of real factor input and its price. As an illustration, the value of labor services may be divided between wage rate and quantity of labor time. The product of the two is the outlay on labor services or labor compensation.

[1]The measurement of social factor input in constant prices was proposed by Copeland [6] and has been discussed from the viewpoint of social accounting by Stone [33], Kendrick [24], and Jorgenson and Griliches [23]. Social factor input in constant prices is not included in the United Nations system of standard national accounts [37] or in the United States national income and product accounts [28, 29, 30].

Despite the essential similarity between concepts of real product and real factor input, the measurement of social factor outlay in constant prices is not well established in social accounting practice. The chief remaining problem is the measurement of capital input in real terms. We have attempted to provide a conceptual basis for measuring real capital input in a previous paper.[2] An accounting imputation is required for separation of outlay on capital services into price and quantity components. Our method for imputation is based on the correspondence between asset prices and service prices implied by the equality between the value of an asset and the discounted value of its services. This method for imputation requires the same data as the perpetual inventory method for measurement of capital stock, together with data on property compensation by legal form of organization.

In this paper we present production accounts for the United States in current and constant prices, including social product and social factor outlay, for the period 1929–1967. Deconsolidation by commodities or by industrial sectors may be carried out along conventional lines, resulting in product and factor outlay accounts for each sector and incorporating inter-industry transactions in current and constant prices. Income, expenditure, and capital finance accounts may also be separated into price and quantity components. The uses of capital finance correspond to changes in the quantity of national wealth, while revaluations correspond to changes in its price.[3] In this paper we discuss price and quantity measurement only for the production account.

The principal applications of measures of real product and real factor input are to the study of production. We apply our estimates to the measurement of total factor productivity in the United States. We also measure the elasticity of substitution between labor and capital input and the elasticity of transformation between investment and consumption goods output. Our study of total factor productivity extends that of Jorgenson and Griliches [23], providing measurements for a considerably longer period of time and analyzing the growth of real factor input in more detail. Our estimates of the elasticities of substitution and transformation provide an alternative characterization of production possibilities to that given by Arrow, Chenery, Minhas, and Solow [2].

2. The Production Account in Current Prices

The fundamental accounting identity for the production account is that the value of output is equal to the value of input. Letting q_i represent the price of the ith output and Y_i its quantity and letting p_j represent the price of the jth input and X_j its quantity, this accounting identity may be written:

$$q_1 Y_1 + q_2 Y_2 + \ldots + q_m Y_m = p_1 X_1 + p_2 X_2 + \ldots + p_n X_n.$$

The first accounting problem is to define appropriate concepts of output and input. We define the value of output as gross value added from the point of view of the producer. For each sector we measure revenue as net proceeds to

[2]Christensen and Jorgenson [5].

[3]The compilation of national accounts in constant prices has been discussed by Stone [33] and more recently by Broderick [3], Burge [4], Courbis [7], Fabricant [12], and Geary [15].

the sector and outlay as gross expenses of the sector. Our concept of gross value added is intermediate between gross product at market prices and at factor cost, as these terms are conventionally employed. The value of output is net of taxes on output while the value of input is gross of taxes on input. The justification for this definition is that the main analytical use of the production account is in the study of producer behaviour. Revenue and outlay must be measured from the producer's point of view.

In implementing the production account for the United States we confine our attention to the private domestic economy. We exclude government since government product is equal to labor compensation in the government sector by definition. The services of capital in the government sector are ignored, so that production accounts for private and government sectors are not comparable. Our concept of private domestic output treats direct taxes in the same way as in the U.S. national income and product accounts. However, rather than include or exclude all indirect taxes from the value of output, we exclude indirect business taxes charged against revenue, such as excise or sale taxes, and include indirect business taxes charged to the producer as part of outlay on productive factors, such as property taxes. Taxes on output reduce the net proceeds of the business sector and subsidies increase these proceeds; accordingly, we add production subsidies in arriving at the value of output from the producer's point of view.[4]

In measuring gross private domestic product for the United States our treatment of excise and sales taxes, business nontax payments, and customs duties is symmetric in that each is excluded from the value of output. Excise and sales taxes and nontaxes[5] are deducted from revenue in arriving at net proceeds to the producer. Customs duties are part of the outlay on imports of commodities and services of the foreign sector and must be excluded from value added in the private domestic sector.

In the U.S. national income and product accounts the services of owner-occupied housing and structures utilized by non-profit institutions are included in the product of the private sector. The flows of capital services resulting from investment in housing by owner-occupiers and investment in structures by non-profit institutions are not recorded in market transactions. The value of the service flow must be imputed from data on rental values. The treatment of capital services from consumers' durables and producers' durables used by non-profit institutions is not symmetrical with that of housing and structures. Purchases of consumers' durables are treated as part of personal consumption expenditures and purchases of producers' durables by non-profit institutions are treated as part of private investment, but the service flow from these durables is not included in private product.

We treat the services of owner-utilized consumers' durables and producers' durables utilized by non-profit institutions symmetrically with the services of owner-occupied housing and the structures of non-profit institutions. Purchases

[4]The evaluation of output from the producer's point of view is equivalent to incorporating indirect taxes included in outlay on productive factors in factor cost. As Stone [33] points out, output must be evaluated at market prices in order for value added to be equal to deliveries to final demand.

[5]See [29] for a description of nontax payments included in the U.S. national accounts.

of new consumers' durables and purchases of producers' durables by institutions are included in private investment. This change from the conventions of the U.S. national income and product accounts leaves the value of the total product unaltered. We then impute the value of services of consumers' durables and producers' durables owned by institutions from rental values implied by the imputed service flow for owner-occupied housing and institutional structures. We add the resulting service flow to the product of the private sector. This change increases the value of the total product and requires data for the imputation of the rental value of these capital services.

Given our definitions of output and input, we may describe more explicitly the measurement of gross private domestic product and gross private domestic factor outlay. The value of gross product is defined as private gross national product less rest of the world product,[6] less income orginating in government enterprises,[7] plus the value of the services of consumers' durables and producers' durables utilized by institutions,[8] less federal indirect business tax and nontax accruals, except for capital stock tax,[9] less state and local indirect business tax and nontax accruals, except for motor vehicle licences, property taxes, and other taxes,[10] plus subsidies and less current surplus of federal and state and local government enterprises.[11] The resulting value of gross private domestic product for the year 1958 is presented in Table 1.

The value of gross private domestic factor outlay is equal to the value of gross private domestic product by definition. The value of factor outlay is the sum of income originating in private enterprises and in private households and institutions,[12] plus the imputed value of the services of consumers' durables and durables utilized byinstitutions,[13] plus indirect business taxes charged to the producer as part of factor outlay, as described in the definition of gross product. The value of factor outlay also includes capital consumption allowances, business transfer payments, and the statistical discrepancy[14] arising from differences between the product side and the factor outlay side of the production account. Capital consumption allowances are part of the outlay on capital services and are included in the rental value of capital services. Business transfer payments and the statistical discrepancy are taken as part of income from capital. The resulting value of gross private domestic factor outlay for the year 1958 is given in Table 1.

In separating the values of gross product and gross factor outlay into price and quantity components, we find it useful to divide total product between consumption and investment goods and total factor outlay between capital and

[6]All references to data from the U.S. national income and product accounts will be to *The National Income and Product Accounts of the United States*,1929–1965, *Statistical Tables, A Supplement to the Survey of Current Business*, August 1966, henceforward *NIP* [28], and subsequent national income issues of the *Survey of Current Business*, unless otherwise indicated. *NIP* [28], Table 1.7.

[7]*NIP* [28], Table 1.13.

[8]These values are imputed by methods discussed in detail in our previous paper, [5], Section 5.

[9]*NIP* [28], Table 3.1.

[10]*NIP* [28], Table 3.3.

[11]*NIP* [28], Tables 3.1 and 3.3.

[12]*NIP* [28], Table 1.13.

[13]See footnote 8, above.

[14]*NIP* [28], Table 1.9.

TABLE 1

PRODUCTION ACCOUNT, GROSS PRIVATE DOMESTIC PRODUCT AND FACTOR OUTLAY, UNITED STATES, 1958 (CURRENT PRICES)[a]

	PRODUCT	
1.	Private gross national product (Table 1.7)	405.2
2.	− Income originating in government enterprises (Table 1.13)	4.8
3.	− Rest of the world gross national product (Table 1.7)	2.0
4.	+ Services of consumers' durables (our imputation)	39.8
5.	+ Services of durables held by institutions (our imputation)	.3
6.	− Federal indirect business tax and nontax accruals (Table 3.1)	11.5
7.	+ Capital stock tax (Table 3.1, footnote 2)	—
8.	− State and local indirect business tax and nontax accruals (Table 3.3)	27.0
9.	+ Business motor vehicle licences (Table 3.3)	.8
10.	+ Business property taxes (Table 3.3)	13.8
11.	+ Business other taxes (Table 3.3)	2.9
12.	+ Subsidies less current surplus of federal government enterprises (Table 3.1)	2.7
13.	− Current surplus of state and local government enterprises (Table 3.3)	1.8
14.	= Gross private domestic product	418.4
	FACTOR OUTLAY	
1.	Capital consumption allowances (Table 1.9)	38.9
2.	+ Business transfer payments (Table 1.9)	1.6
3.	+ Statistical discrepancy (Table 1.9)	1.6
4.	+ Services of consumers' durables (our imputation)	39.8
5.	+ Services of durables held by institutions (our imputation)	.3
6.	+ Certain indirect business taxes (product account above, 7 + 9 + 10 + 11)	17.4
7.	+ Income originating in business (Table 1.13)	312.2
8.	− Income originating in government enterprises (Table 1.13)	4.8
9.	+ Income originating in households and institutions (Table 1.13)	11.4
10.	= Gross private domestic factor outlay	418.4

[a]All table references are to *The National Income and Product Accounts of the United States, 1929–1965, Statistical Tables, A Supplement to the Survey of Current Business*, August, 1966.

labor services. In the U.S. national income and product accounts total output is divided among durables and structures output (which we denote investment goods output) and nondurables and services output (which we denote consumption goods output).[15] Our definition of durables output includes consumers' durables, as in the U.S. national accounts. Our definition of services output includes the services of consumers' durables and institutional durables along with services output included in the U.S. accounts. The output of the foreign and government sectors consists entirely of services, so that we define the output of services by the private sector as services included in gross national product,[16] less the product of foreign and government sectors (including government enterprises),[17] plus the services of consumers' durables and durables utilized by non-profit institutions.

The value of factor outlay in the private domestic sector includes the labor compensation of employees in private enterprises and in private households and non-profit institutions,[18] plus the labor compensation of self-employed persons.[19]

[15]*NIP* [28], Table 1.3.

[16]*NIP* [28], Table 1.5.

[17]*NIP* [28], Tables 1.7, 1.13.

[18]*NIP* [28], Table 1.13.

[19]Self-employed persons include proprietors and unpaid family workers. Alternative methods for imputation of the labor compensation of the self-employed are reviewed by Kravis [27].

We estimate labor compensation of the self-employed by multiplying the compensation of employees by the ratio of proprietors and unpaid family workers to full-time equivalent employees in each sector. Our estimates of non-farm proprietors and employees are those of the Office of Business Economics. Our estimates of non-farm unpaid family workers are those of Kendrick, allocated among sectors in proportion to the number of proprietors in each sector, as Kendrick suggests. Our estimates of persons engaged in the farm sector are from Kendrick.[20] In effect we assume that for each sector the average labor compensation of proprietors and unpaid family workers is equal to average labour compensation of full-time equivalent employees in the same sector. The sectors utilized in carrying out this imputation are: (1) The farm sector—agriculture, forestry, and fisheries, (2) mining, (3) contract construction, (4) nondurable manufacturing, (5) durable manufacturing, (6) transportation, (7) communication, (8) electric, gas, and sanitary services, (9) wholesale and retail trade, (10) finance, insurance, and real estate, (11) services. This method of imputation is only one of many that have been proposed; Denison has suggested that the results are likely to be biased in the direction of allocating too large a portion of proprietors' income to labor compensation.[21]

All factor outlay not allocated to labor is allocated to capital.[22] Specifically, the value of outlay on capital services includes the following: property income of self-employed persons, the portion of proprietor's income not allocated to labor compensation; profits, rentals, and interest; capital consumption allowances; business transfer payments; the statistical discrepancy; indirect business taxes that are part of the outlay on productive factors, such as motor vehicle licenses, property taxes, and other taxes; and the imputed value of the services of consumers' durables and producers' durables utilized by institutions.[23] Gross private domestic product and factor outlay in current prices for 1929–1967 are given in Table 2. Total product is divided between gross private domestic investment and gross private domestic consumption. Total factor outlay is divided between labor compensation and property compensation.

3. Price and Quantity Index Numbers

To separate flows of product and factor outlay into prices and quantities, we introduce price and quantity index numbers. As an example, we consider the value of output, say qY, introduced in the production accounts. Suppose that there are m components to the value of output,

$$qY = q_1 Y_1 + q_2 Y_2 + \ldots + q_m Y_m.$$

[20]These data have been compiled for John W. Kendrick's forthcoming study, *Postwar Productivity Trends in the United States*, for the National Bureau of Economic Research [25]. We are indebted to Kendrick for providing us with these data in advance of publication. The conceptual basis for compilation of the data is the same as in Kendrick's *Productivity Trends in the United States* [26]. The Office of Business Economics data on non-farm proprietors and employees are from *NIP* [28], Tables 6.4 and 6.6.

[21]Denison [9], page 4.

[22]This is a consequence of the accounting identity between the value of output and the value of input.

[23]Of these components of gross factor outlay, the statistical discrepancy is the only component that might be partly assigned to labor compensation. We assume that any discrepancy reflects errors in reporting property income rather than labor income.

TABLE 2

GROSS PRIVATE DOMESTIC PRODUCT AND FACTOR OUTLAY, 1929–1967 (CURRENT PRICE)

Year	1. Gross Private Domestic Product	2. Investment Goods Product	3. Consumption Goods Product	4. Labor Compensation	5. Property Compensation
1929	103.0	28.4	74.5	56.2	46.8
1930	89.8	20.2	69.5	51.4	38.4
1931	77.0	14.1	62.9	43.2	33.8
1932	57.9	7.1	50.7	33.4	24.5
1933	55.5	7.5	48.0	31.0	24.4
1934	60.0	10.4	49.6	35.2	24.9
1935	69.1	12.7	56.4	38.3	30.8
1936	76.4	17.0	59.3	42.9	33.5
1937	84.9	19.7	65.2	49.1	35.8
1938	77.4	15.3	62.1	45.4	32.0
1939	84.9	19.3	65.6	48.9	36.0
1940	93.4	23.8	69.5	52.9	40.5
1941	115.7	37.0	78.7	64.9	50.8
1942	143.2	47.6	95.6	81.5	61.7
1943	168.7	60.6	108.2	96.5	72.2
1944	177.2	61.3	116.0	103.1	74.1
1945	175.2	52.6	122.5	103.3	71.8
1946	190.5	49.9	140.6	115.2	75.3
1947	218.2	64.2	154.0	132.9	85.3
1948	239.6	72.7	166.9	145.9	93.7
1949	236.1	72.2	164.0	143.5	92.6
1950	269.1	91.2	177.9	156.3	112.8
1951	307.3	106.2	201.0	177.4	129.9
1952	323.1	108.2	214.9	188.9	134.2
1953	340.2	115.3	225.1	202.7	137.5
1954	343.1	110.9	232.1	200.8	142.2
1955	374.7	128.6	246.1	216.5	158.2
1956	396.4	135.3	261.1	234.0	162.5
1957	415.1	140.0	275.1	245.9	169.2
1958	418.4	130.4	288.0	245.1	173.3
1959	453.4	146.8	306.6	265.5	187.8
1960	472.5	148.8	323.7	278.7	193.8
1961	487.2	147.4	339.7	284.7	202.5
1962	423.5	163.5	360.0	302.6	220.9
1963	550.9	173.2	377.7	316.8	234.1
1964	588.5	186.7	410.8	338.4	250.1
1965	640.7	204.7	436.0	362.7	278.0
1966	700.8	223.6	477.2	397.1	303.7
1967	732.0	226.9	505.0	423.1	308.9

We must introduce index numbers for the price of output q and the quantity of output Y, defined in terms of the prices $\{q_i\}$ and quantities $\{Y_i\}$ of the m components. Differentiating totally with respect to time and dividing both sides by the corresponding total value of output, we obtain:

$$\frac{\dot{q}}{q} + \frac{\dot{Y}}{Y} = \Sigma w_i \left[\frac{\dot{q}_i}{q_i} + \frac{\dot{Y}_i}{Y_i}\right],$$

with weights $\{w_i\}$ given by the relative shares of the value of the ith output in

the value of total output:

$$w_i = \frac{q_i Y_i}{\Sigma q_i Y_i}.$$

We define the price and quantity indexes of output in terms of rates of growth of the prices and quantities of individual components; the rates of growth of the price index q and the quantity index Y are

$$\frac{\dot{q}}{q} = \Sigma w_i \frac{\dot{q}_i}{q_i}, \qquad \frac{\dot{Y}}{Y} = \Sigma w_i \frac{\dot{Y}_i}{Y_i},$$

respectively. These index numbers are Divisia price and quantity indexes.[24] Rates of growth of the Divisia indexes of prices and quantities add up to the rate of growth of the value of output (factor reversal test) and are symmetric in different directions of time (time reversal test). They also have the reproductive property that a Divisia index of Divisia indexes is a Divisia index of the components.

For application to data for discrete points of time an approximation to the Divisia indexes for continuous time is required. Price and quantity index numbers originally discussed by Fisher [13] may be used for this purpose:

$$\log q_t - \log q_{t-1} = \Sigma \bar{w}_{it}[\log q_{it} - \log q_{i,t-1}],$$
$$\log Y_t - \log Y_{t-1} = \Sigma \bar{w}_{it}[\log Y_{it} - \log Y_{i,t-1}],$$

where the weights w_{it} are arithmetic averages of the relative shares in the two periods,

$$\bar{w}_{it} = \tfrac{1}{2} w_{it} + \tfrac{1}{2} w_{i,t-1}.$$

These index numbers have been suggested as a discrete approximation to the Divisia index by Tornquist [36]. Obviously, the discrete and continuous index numbers are equal if relative shares are constant. If shares are not constant, the discrete approximation involves an error that depends on the variability of the relative shares and the length of the time period.

Divisia index numbers for discrete time are symmetric in data of different time periods (time reversal). They have the basic reproductive property that a discrete Divisia index of discrete Divisia indexes is a discrete Divisia index of the components. Theil [34] has demonstrated that the sum of changes in logarithms of discrete Divisia indexes of price and quantity is approximately equal to the change in the logarithm of the value (factor reversal). The factor reversal test is satisfied exactly if relative shares are constant; the accuracy of the approximation depends on the change in relative shares. As a practical matter this approximation is extremely accurate for annual time series of national accounting aggregates such as consumption; Theil shows that the error averages only 0.01 per cent of the annual rate of growth in the value of consumption in The Netherlands for the period 1921–1963.

It is convenient to have the product of price and quantity indexes equal to the value of transactions so that standard accounting identities hold for variables

[24]The economic interpretation of Divisia indexes of total factor productivity has been discussed by Solow [32], Richter [31], and Jorgenson and Griliches [23].

defined as price and quantity index numbers. Accordingly, we construct discrete Divisia price indexes as the value in current prices divided by the discrete Divisia quantity index. The resulting price indexes are approximately equal to the Divisia price indexes.

In defining the price and quantity of output we distinguish between the price representing proceeds to the producer and the price paid by the ultimate consumer. The difference between the two prices includes excise and sales taxes. Just as price and quantity index numbers may be defined in terms of the prices and quantities of the components of output, we may define a tax index, incorporating the effective tax rate, in terms of prices, quantities, and tax rates of the components of output. Let the market price of output q^+ equal the product of the producers' price q and unity plus the effective tax rate $1+t$. The value of output at market prices is

$$q^+ Y = (1 + t)q Y.$$

We now define the value of output at market prices in terms of prices, quantities and tax rates of the components of output:

$$\begin{aligned} q^+ Y &= \Sigma q_i^+ Y_i, \\ &= \Sigma (1 + t_i) q_i Y_i. \end{aligned}$$

Proceeding as before, we differentiate totally with respect to time, obtaining:

$$\frac{(1 \dot{+} t)}{1 + t} + \frac{\dot{q}}{q} + \frac{\dot{Y}}{Y} = \Sigma w_i \left[\frac{(1 \dot{+} t_i)}{1 + t_i} + \frac{\dot{q}_i}{q_i} + \frac{\dot{Y}_i}{Y_i} \right].$$

The rate of growth of the tax index $1+t$ is

$$\frac{(1 \dot{+} t)}{1 + t} = \Sigma w_i \frac{(1 \dot{+} t_i)}{1 + t_i};$$

rates of growth of the price and quantity indexes are the same as before. The effective tax rate is the tax index less unity.

Again, it is convenient to preserve equality between the product of price, quantity, and tax indexes and the value of transactions. Accordingly, we construct an index of taxes $1+t$ by dividing the value of transactions at market prices by the value of transactions at producers' prices. The resulting tax index is approximately equal to the Divisia tax index defined for discrete points of time.

4. Total Product in Constant Prices

We now turn to separation of gross product and gross factor outlay from the production account into price and quantity indexes of product and factor input. Product is allocated between consumption and investment goods and factor input is allocated between capital and labor services. Consumption goods include nondurable goods and services; investment goods include durable goods and structures. We construct quantity index numbers of output and of final sales for these two types of output from data for the corresponding components of gross national product in constant prices.[25] Change in business inventories

[25] *NIP* [28], Tables 1.5 and 1.6, except for structures—see Section 5 below.

in constant prices is defined as the difference between index numbers of output and of final sales in constant prices. The product of the rest of the world and government sectors is composed entirely of services. The price index for the product of each of these sectors is assumed to be the same as for services as a whole. Quantity index numbers for the services of consumers' durables and institutional durables are constructed as part of our imputation of the value of these services and will be described below.

The value of output from the point of view of the producing sector excludes certain indirect business taxes less subsidies. The price of output is implicit in the value of output and the quantity index of output described above. The market price of final sales is the price index implicit in the quantity index of final sales described above and the value of final sales at market prices as calculated from the U.S. national accounts. The tax index is implicit in the value of final sales from the point of view of the producing sector and the value of final sales at market prices. Price and quantity index numbers for gross private domestic product and final sales from the point of view of the producing sector are given for 1929–1967 in Table 3.

We require a division of output between consumption and investment goods. Sales and excise taxes must be allocated between these two categories of output. If taxes were assessed only on the basis of deliveries to final demand, we could allocate them directly between investment and consumption goods deliveries. In fact a substantial portion of sales and excise taxes falls on deliveries to intermediate demand; examples would include taxes on airline tickets, automobiles, gasoline, telephone services, and business machines. A completely satisfactory allocation of these taxes would require a detailed input-output analysis. The data required to carry out this analysis on an annual basis are unavailable. We have allocated the taxes in proportion to the value of consumption and investment goods output in the value of final sales. This is equivalent to assuming that the effective tax rate is the same for consumption and investment goods. Price and quantity index numbers for consumption and investment goods output are given in Table 3, together with the relative share of investment goods output in the value of total output.

5. Total Factor Input in Constant Prices

The input of the producing sector is divided between labor and capital services. We present quantity indexes for input of each type. The construction of a quantity index of labor input begins with private domestic persons engaged; our estimates of persons engaged are described above.[26] Our estimates for the non-farm sector are identical to those of the Office of Business Economics for full-time equivalent employees and proprietors; we add Kendrick's estimates of unpaid family workers to obtain total persons engaged. For the farm sector we employ Kendrick's estimates.[27] Persons engaged is essentially the stock of labor and must be adjusted for hours utilized per person to obtain a measure of

[26]Persons engaged includes full-time equivalent employees, proprietors, and unpaid family workers.

[27]See footnote 20, above.

the quantity of labor input. Man-hours are also estimated by Kendrick and we employ his estimates for the private domestic sector.[28]

The assumption that effective labor services are proportional to the stock of labor is obviously incorrect. On the other hand the assumption that effective labor services can be measured directly from data on man-hours is equally incorrect, as Denison [8] has pointed out. The intensity of effort varies with the number of hours worked per week, so that effective labor input can be measured accurately only if data on man-hours are corrected for the effects of variations in the number of hours per man on effective labor input. Denison [10] suggests that the stock of labor provides an upper bound for effective labor services while the number of man-hours provides a lower bound. He estimates effective labor input by correcting man-hours for variations in labor intensity. We employ Denison's correction for intensity, but we apply this correction to actual hours per man rather than potential hours per man.

It is desirable to distinguish among outputs of different types and to deflate each type of output separately; similarly, it would be desirable to distinguish among different categories of labor, classified by sex, race, number of years of schooling, occupation, age, and so on. Labor input is defined as a quantity index of labor inputs of each type; corresponding to the quantity index of labor input there is a price index for labor representing the aggregate wage rate. Denoting the quantity index by L and the price index by p^L the value of labor input is the sum of the values of labor inputs:

$$p^L L = \Sigma\, p_j{}^L L_j,$$

where labor input of each type is measured in effective man-hours and the prices are corresponding hourly wage rates. Proceeding as before, we obtain indexes of the wage rate and quantity of labor,

$$\frac{\dot{p}^L}{p^L} = \Sigma\, v_j \frac{\dot{p}_j{}^L}{p_j{}^L}, \qquad \frac{\dot{L}}{L} = \Sigma\, v_j \frac{\dot{L}_j}{L_j},$$

where the weights $\{v_j\}$ are the relative shares of each type of labor in the value of total labor input.

For each category of labor, man-hours are the product of persons engaged, say n_j, and effective hours per person, say h_j. The index of total labor input may be rewritten:

$$\frac{\dot{L}}{L} = \Sigma\, v_j \frac{\dot{n}_j}{n_j} + \Sigma\, v_j \frac{\dot{h}_j}{h_j}.$$

Where N is person engaged and H is effective hours per man, the index may be finally rewritten in the form:

$$\frac{\dot{L}}{L} = \Sigma\, v_j \left(\frac{\dot{n}_j}{n_j} - \frac{\dot{N}}{N}\right) + \Sigma\, v_j \left(\frac{\dot{h}_j}{h_j} - \frac{\dot{H}}{H}\right) + \frac{\dot{N}}{N} + \frac{\dot{H}}{H};$$

the first term is the change in labor input per person due to shifts in the composition of the labor force, the second is the change in labor input per hour due to

[28]See footnote 20, above.

TABLE 3

GROSS PRIVATE DOMESTIC PRODUCT AND FINAL SALES, 1929–1967 (CONSTANT PRICES OF 1958)

Year	1. Gross Private Domestic Product, Quantity Index	2. Gross Private Domestic Product, Price Index	3. Gross Private Domestic Final Sales, Quantity Index	4. Gross Private Domestic Final Sales, Price Index	5. Effective Tax Rate, Final Sales	6. Consumption Goods Product, Quantity Index	7. Consumption Goods Product, Price Index	8. Investment Goods Product, Quantity Index	9. Investment Goods Product, Price Index	10. Relative Share of Investment Goods Product
1929	189.8	0.543	185.9	0.544	0.018	136.275	0.547	55.781	0.509	0.276
1930	172.1	0.522	172.2	0.523	0.019	132.291	0.525	41.253	0.489	0.225
1931	159.1	0.484	160.9	0.486	0.021	128.840	0.488	31.097	0.453	0.183
1932	135.6	0.427	141.2	0.428	0.027	118.260	0.429	17.642	0.405	0.123
1933	132.0	0.420	135.7	0.420	0.040	113.791	0.422	18.548	0.403	0.135
1934	141.8	0.423	143.7	0.422	0.050	117.234	0.432	25.064	0.414	0.173
1935	153.9	0.449	150.9	0.450	0.047	124.285	0.454	30.325	0.418	0.184
1936	171.5	0.445	167.8	0.447	0.046	131.804	0.450	41.077	0.415	0.223
1937	183.0	0.464	176.9	0.465	0.045	139.840	0.467	44.620	0.442	0.232
1938	173.2	0.447	174.7	0.448	0.046	140.153	0.443	34.272	0.447	0.198
1939	188.5	0.450	186.5	0.453	0.044	146.147	0.449	43.755	0.442	0.228
1940	205.5	0.454	199.6	0.457	0.046	153.778	0.452	53.265	0.447	0.255
1941	236.0	0.491	225.4	0.494	0.046	164.364	0.479	73.076	0.506	0.319
1942	257.8	0.555	253.1	0.551	0.039	178.567	0.535	80.802	0.589	0.332
1643	277.5	0.608	277.4	0.610	0.037	180.380	0.600	98.066	0.618	0.359
1944	291.1	0.609	292.5	0.610	0.042	188.830	0.614	103.207	0.594	0.346
1945	284.5	0.616	286.9	0.614	0.049	192.278	0.637	92.600	0.568	0.300
1946	274.0	0.695	264.8	0.695	0.054	195.802	0.718	77.297	0.646	0.262

1947	279.9	0.780	280.3	0.780	0.049	193.836	0.794	85.665	0.749	0.294
1948	297.6	0.805	293.2	0.801	0.049	203.862	0.819	93.524	0.777	0.303
1949	297.7	0.793	301.8	0.793	0.050	206.087	0.796	91.290	0.791	0.306
1950	328.9	0.818	321.0	0.817	0.050	214.858	0.828	113.906	0.801	0.339
1951	351.4	0.874	341.0	0.871	0.048	228.406	0.880	122.928	0.864	0.346
1952	360.4	0.896	357.4	0.896	0.048	237.323	0.905	122.964	0.880	0.335
1953	378.9	0.898	378.1	0.899	0.049	247.628	0.909	131.165	0.879	0.339
1954	375.8	0.913	377.6	0.913	0.045	250.337	0.927	125.156	0.886	0.323
1955	406.7	0.921	400.6	0.920	0.047	262.884	0.936	143.864	0.894	0.343
1956	416.3	0.952	411.8	0.951	0.047	272.994	0.956	143.264	0.945	0.341
1957	422.8	0.982	421.7	0.981	0.046	281.133	0.978	141.574	0.989	0.337
1958	418.4	1.000	419.9	1.000	0.045	287.953	1.000	130.421	1.000	0.312
1959	445.7	1.017	441.1	1.017	0.047	300.725	1.020	144.979	1.013	0.324
1960	457.3	1.033	454.1	1.032	0.049	310.005	1.044	147.263	1.010	0.315
1961	466.3	1.045	464.4	1.045	0.047	320.353	1.061	145.736	1.012	0.303
1962	495.3	1.057	489.8	1.057	0.048	334.981	1.075	160.431	1.019	0.312
1963	515.5	1.069	510.0	1.069	0.048	346.273	1.091	169.410	1.022	0.314
1964	544.1	1.082	538.5	1.082	0.048	363.320	1.106	181.165	1.030	0.317
1965	579.2	1.106	570.9	1.106	0.047	383.562	1.137	196.323	1.043	0.320
1966	615.6	1.138	603.0	1.138	0.044	406.587	1.174	209.890	1.065	0.319
1967	631.1	1.160	625.4	1.161	0.044	424.326	1.190	206.903	1.097	0.310

TABLE 4

PRIVATE DOMESTIC LABOR INPUT, 1929–1967 (CONSTANT PRICES OF 1958)

Year	1. Private Domestic Persons Engaged (Millions)	2. Educational Attainment Per Person (Index)	3. Private Domestic Hours Per Person (Thousands Per Year)	4. Effective Labor Input Per Hour (Index)	5. Private Domestic Labor Input, Quantity Index	6. Private Domestic Labor Input, Price Index
1929	44.151	0.836	2.579	0.858	173.3	0.324
1930	41.898	0.840	2.530	0.875	165.4	0.311
1931	36.948	0.844	2.494	0.910	158.2	0.273
1932	35.686	0.848	2.409	0.916	141.7	0.236
1933	35.533	0.852	2.395	0.921	141.6	0.219
1934	37.854	0.855	2.210	0.974	148.0	0.238
1935	39.014	0.859	2.260	0.960	154.4	0.248
1936	40.765	0.863	2.326	0.941	163.5	0.263
1937	42.484	0.867	2.372	0.927	172.0	0.285
1938	40.039	0.871	2.297	0.950	161.5	0.281
1939	41.443	0.875	2.334	0.939	168.6	0.290
1940	43.149	0.879	2.340	0.937	176.5	0.300
1941	46.576	0.886	2.361	0.931	192.4	0.337
1942	49.010	0.893	2.416	0.914	205.1	0.398
1943	49.695	0.900	2.465	0.898	210.1	0.459
1944	48.668	0.907	2.489	0.896	208.8	0.494
1945	47.136	0.914	2.427	0.911	202.1	0.511
1946	49.950	0.922	2.308	0.946	213.4	0.540
1947	52.350	0.929	2.252	0.962	223.6	0.594

1948	53.336	0.936	2.228	0.969	228.2	0.639
1949	51.469	0.942	2.223	0.970	221.3	0.647
1950	52.972	0.948	2.197	0.978	228.8	0.683
1951	55.101	0.954	2.185	0.981	239.0	0.742
1952	55.385	0.960	2.187	0.980	241.7	0.782
1953	56.226	0.965	2.159	0.986	245.2	0.827
1954	54.387	0.971	2.139	0.990	237.4	0.846
1955	55.718	0.977	2.161	0.986	245.9	0.880
1956	56.770	0.982	2.151	0.988	251.6	0.930
1957	56.809	0.988	2.121	0.995	251.5	0.978
1958	55.023	1.000	2.099	1.000	245.1	1.000
1959	56.215	1.012	2.122	0.995	254.9	1.042
1960	56.743	1.020	2.126	0.994	259.6	1.074
1961	56.211	1.028	2.110	0.998	258.1	1.103
1962	57.078	1.036	2.117	0.996	264.6	1.144
1963	57.540	1.043	2.117	0.996	268.5	1.180
1964	58.508	1.051	2.122	0.995	275.4	1.229
1965	60.055	1.058	2.134	0.992	285.3	1.271
1966	62.130	1.067	2.126	0.994	297.4	1.335
1967	63.162	1.077	2.126	0.994	305.0	1.387

shifts in relative hours per man among components of the labor force, and the sum of the last two terms is the change in total effective man-hours. Two types of "quality" adjustments are required to convert total man-hours to an index of aggregate labor input—one based on shifts in composition of the labor force and the other based on changes in relative hours worked.

Quality adjustments of effective man-hours required to obtain an index of labor input are not available in the detail that would be desirable. Kendrick distinguishes different categories of labor by industry; Jorgenson and Griliches distinguish labor by years of schooling completed.[29] Both adjustments account for changes in quality associated with changes in the composition of the labor force. We have used the quality adjustment provided by Jorgenson and Griliches and extended by Griliches[30] to adjust for changes in the quality of labor due to changes in the educational composition of the labor force. Our measure of labor services is based on the stock of labor as measured by persons engaged, adjusted for effective hours per person and for changes in the composition of the labor force by educational attainment. The cost of labor services index is calculated by dividing total labor compensation by the quantity index of labor services. The number of persons engaged, the index of quality change, actual hours per worker, effective labor input per man-hour, and the quantity of labor input for 1929–1967 are given in Table 4. The price of labor services implicit in private domestic labor compensation is also given in Table 4. It would obviously be desirable to incorporate additional aspects of labor force composition in adjusting the stock of labor for quality change. It would also be desirable to adjust the number of hours per man for changes in the relative number of hours worked by persons differing in educational attainment.

In a previous paper [31] we have constructed a quantity index of capital input. The starting point for such an index is the measurement of capital stock corresponding to each type of capital services. We have used the perpetual inventory method[32] to estimate the level of capital stock for seven types of assets—land, residential structures, non-residential structures, producers' durable equipment, nonfarm business inventories, farm inventories, and consumers' durable equipment. We have allocated each class of assets among four sectors of the private domestic economy—corporations, non-corporate business, households, and institutions.

The second step in the construction of a quantity index of capital input is to separate price and quantity components of the value of property compensation for each sector of the economy. Our method of imputation is based on the equality of the value of an asset and the discontinued value of its services. Total property compensation or the value of all capital services is equal to the

[29]See Kendrick [26] and Jorgenson and Griliches [23].

[30]See Jorgenson and Griliches [23] and Griliches [21]. We have extended Griliches' estimates back to 1929, using relative earnings for 1939 and estimates of the educational attainment of the labor force for 1930 and 1940 by Folger and Nam [14].

[31]Christensen and Jorgenson [5].

[32]The perpetual inventory method is discussed by Goldsmith [18] and employed extensively in his *Study of Saving* [20] and more recent studies of U.S. national wealth [16, 17, 19]. This method is used in the OBE *Capital Goods Study* [22] and in the study of capital stock for the United States, 1900–1962, by Tice [35].

sum of the values of the individual capital services. Each capital service flow may be expressed as the sum of four terms, depending on the rate of return, the rate of replacement, the rate of capital losses accrued, and the tax structure. The rate of return for each sector is imputed from total property compensation.

The final step in construction of a quantity index of capital input is the measurement of actual quantities of each type of capital service utilized. For land, inventories, residential structures, and consumers' durables we assume that actual capital services are equal to potential services. For non-residential structures and producers' durables we adjust the potential quantities of capital services on the corporate and non-corporate sectors to reflect changes in relative utilization. Our estimates of relative utilization are based on the consumption of electricity relative to installed horsepower of electric motors.

Our measure of capital services is based on capital stock for each asset, weighted by potential service prices, and adjusted for relative utilization of capital. The quantity index of capital input for 1929–1967 is given in Table 5. The price of capital services implicit in private domestic property compensation is also given in Table 5. To provide the basis for comparison of sources of growth of capital input with those for labor input we present data on capital stock, potential service flow per unit of capital stock, and the relative utilization of capital in Table 5. Capital stock is a Divisia index of capital stock for each class of asset—consumers' durables, non-residential structures, producers' durables, residential structures, non-farm inventories, farm inventories, and land. The potential service flow per unit of capital stock is the ratio of the quantity of potential gross private domestic capital input to the index of capital stock. The relative utilization of capital is the ratio of the quantity of actual to potential gross private domestic capital input.

We can combine estimates of labor and capital services into an estimate of real factor input for the U.S. private domestic economy. The basic data on labor input—number of persons engaged, educational attainment per person, and hours per person—are presented in Table 4. The corresponding data on capital input—capital stock, potential service flow per unit of stock, and the relative utilization of capital—are presented in Table 5. Persons engaged is an unweighted stock of labor. The index of educational attainment per person provides an adjustment for the aggregation bias that results from combining different types of labor into an unweighted aggregate. Persons engaged, adjusted for educational attainment, must be multiplied by hours per person to obtain the flow of labor services. Similarly, capital stock is an unweighted aggregate; the index of potential capital services per unit of the capital stock provides an adjustment for the aggregation bias that results from combining different types of capital by adding together capital services weighted by asset prices rather than service prices. Potential capital services must be adjusted for relative utilization to obtain the actual flow of capital services.

We construct price and quantity index numbers of factor input by combining Divisia indexes of labor and capital input into a Divisia index of total factor input. The weights for labor and capital are the relative shares of labor and property compensation in the value of total factor outlay. Price and quantity

TABLE 5

GROSS PRIVATE DOMESTIC CAPITAL INPUT, 1929–1967 (CONSTANT PRICES OF 1958)

Year	1. Private Domestic Capital Stock	2. Potential Capital Input per unit of Capital Stock	3. Relative Utilization of Capital	4. Private Domestic Capital Input, Quantity Index	5. Private Domestic Capital Input, Price Index
1929	888.9	0.116	0.880	87.8	0.533
1930	904.0	0.116	0.848	87.8	0.437
1931	900.2	0.116	0.818	84.0	0.402
1932	883.6	0.116	0.780	78.3	0.312
1933	851.4	0.114	0.804	76.6	0.318
1934	823.7	0.112	0.836	76.0	0.326
1935	805.3	0.112	0.870	77.7	0.396
1936	800.4	0.112	0.896	79.1	0.423
1937	805.5	0.113	0.888	80.0	0.447
1938	817.6	0.114	0,840	77.6	0.411
1939	809.8	0.114	0.892	81.4	0.442
1940	814.1	0.114	0.944	87.0	0.465
1941	830.3	0.115	1.013	96.2	0.528
1942	857.9	0.117	1.053	104.4	0.589
1943	851.4	0.116	1.118	110.0	0.656
1944	834.6	0.116	1.123	107.8	0.686
1945	819.3	0.116	1.081	102.1	0.702
1946	812.3	0.117	1.031	97.2	0.774
1947	851.3	0.119	1.050	105.9	0.805
1948	888.3	0.122	1.042	113.0	0.828
1949	934.6	0.124	0.995	114.9	0.805
1950	964.6	0.126	1.028	124.1	0.908
1951	1021.4	0.127	1.036	134.5	0.965
1952	1068.5	0.129	1.019	139.7	0.959
1953	1100.3	0.129	1.037	147.4	0.932
1954	1134.6	0.130	1.007	148.9	0.955
1955	1163.2	0.131	1.040	158.6	0.996
1956	1213.9	0.132	1.042	167.1	0.971
1957	1255.5	0.134	1.026	171.9	0.983
1958	1287.9	0.135	1.000	173.1	1.000
1959	1305.8	0.135	1.038	182.5	1.028
1960	1341.4	0.136	1.042	189.0	1.024
1961	1373.9	0.137	1.034	194.1	1.043
1962	1399.1	0.137	1.056	202.3	1.091
1963	1436.7	0.138	1.062	205.4	1.139
1964	1477.8	0.140	1.086	215.9	1.158
1965	1524.4	0.141	1.091	225.0	1.235
1966	1582.2	0.144	1.096	236.2	1.285
1967	1645.3	0.146	1.096	247.9	1.245

index numbers for gross private domestic input may be represented in the form:

$$\log p_t - \log p_{t-1} = \bar{v}_L[\log p_t^L - \log p_{t-1}^L] + \bar{v}_K[\log p_t^K - \log p_{t-1}^K],$$

$$\log X_t - \log X_{t-1} = \bar{v}_L[\log L_t - \log L_{t-1}] + \bar{v}_K[\log K_t - \log K_{t-1}]$$

where p is the price index and X the quantity index, $\bar{v}_L$ and $\bar{v}_K = 1 - \bar{v}_L$ are arithmetic averages of the relative shares of labor and property compensation in total factor outlay in the two periods, p^L and p^K are the price indexes of labor and capital input, and L and K are the corresponding quantity indexes. Price

and quantity indexes for 1929–1967 are given in Table 6. The relative share of property compensation for the same period is also given in Table 6.

To provide a detailed accounting for the sources of growth in real factor input, we can separate the growth of quantity indexes of labor and capital input into the growth of the stock, growth in the quantity of input due to shifts in composition of such unweighted aggregates as persons engaged and capital stock or "quality change,"[33] and growth in relative utilization. The growth in labor input is the sum of growth in the number of persons engaged, the quality of the labor force, and the effective number of hours per person. The growth in capital input is the sum of growth in capital stock, the quality of capital, and relative utilization. Geometric average annual rates of growth for 1929–1967 and for the sub-periods 1929–1948 and 1948–1967 are given for each component of the growth of labor and capital input in Table 7.

The sources of growth in factor input may be seen from a different perspective through a similar decomposition of growth in factor prices. Considering factor price indexes that result from dividing total labor and property compensation by stocks of capital and labor, we obtain "stock" factor prices. These prices do not represent the cost of factor services since they fail to take into account the aggregation biases and variations in relative utilization that must be eliminated in order to measure the actual cost of factor services. We may adjust labor and capital stock for quality change; dividing total labor and property compensation by the resulting potential flows of factor services, we obtain "potential" service prices. Finally, adjusting labor and capital for relative utilization we obtain the actual service prices. All three sets of factor prices are given in Table 8. The actual prices are, of course, the price indexes of labor and capital services from Tables 4 and 5, respectively.

From these data it is apparent that estimates of the growth in labor and capital costs and the change in relative factor prices depend critically on the method of measurement. Consider, for example, the growth in labor cost. If we measure labor cost as labor compensation per person engaged, the stock price of labor from Table 8, we obtain rates of growth of 4.03 per cent from 1929–1948, 4.72 per cent from 1948–1967, and 4.37 per cent from 1929–1967; these rates of growth are given in Table 9 along with the growth of labor costs taking into account changes in the quality of the labor force, the labor cost for potential labor services and costs taking into account relative utilization of the labor force, the cost for actual labor services.

Estimates of the growth of capital cost or the rental price per unit of capital input may be analyzed in a similar way. The rental price per unit of capital stock grows at the average annual rate of 3.66 per cent from 1929–1948, 3.03 per cent from 1948–1967, and 3.34 per cent for 1929–1967. Capital costs taking into account changes in the quality of capital, the potential flow rental price, grows

[33]"Quality change" in this sense is equivalent to aggregation bias. Aggregation bias may be removed by treating the components of aggregate factor input separately, weighting each component in proportion to its relative price. This is not to imply that any proposed adjustment for quality change is legitimate. The appropriateness of each adjustment must be judged on the basis of evidence on the movement of separate components of aggregate factor input and the relative prices of the components. For further discussion, see Jorgenson and Griliches [23], especially pages 259–260.

TABLE 6
GROSS PRIVATE DOMESTIC FACTOR INPUT, 1929–1967 (CONSTANT PRICES OF 1958)

Year	1. Gross Private Domestic Factor Input, Quantity Index	2. Gross Private Domestic Factor Input, Price Index	3. Property Compensation, Relative Share
1929	261.5	0.394	0.455
1930	253.1	0.355	0.428
1931	242.2	0.318	0.439
1932	220.8	0.262	0.423
1933	218.5	0.254	0.441
1934	223.1	0.269	0.414
1935	230.2	0.300	0.446
1936	240.2	0.318	0.438
1937	247.9	0.342	0.422
1938	236.0	0.328	0.413
1939	247.1	0.344	0.424
1940	260.7	0.358	0.434
1941	285.7	0.405	0.439
1942	307.2	0.466	0.431
1943	318.2	0.530	0.428
1944	314.6	0.563	0.418
1945	301.5	0.581	0.410
1946	305.2	0.624	0.395
1947	324.8	0.672	0.391
1948	337.5	0.710	0 391
1949	333.8	0.707	0.392
1950	350.7	0.767	0.419
1951	371.6	0.827	0.423
1951	380.0	0.850	0.415
1953	391.6	0.869	0.404
1954	385.7	0.889	0.415
1955	404.4	0.927	0.422
1956	419.1	0.946	0.410
1957	423.5	0.980	0.408
1958	418.4	1.000	0.414
1959	437.6	1.036	0.414
1960	448.9	1.052	0.410
1961	452.0	1.078	0.416
1962	466.8	1.122	0.422
1963	479.0	1.150	0.425
1964	498.6	1.180	0.425
1965	519.3	1.234	0.434
1966	545.2	1.285	0.433
1967	566.8	1.292	0.422

more slowly than the stock rental price, reflecting increases in the quality of the capital stock. Most of this improvement in quality took place during the period 1948–1967, so that the potential service price follows the capital stock price rather closely during the period 1929–1948. Finally, the relative utilization of capital has grown during the period 1929–1967, so that the actual flow rental price grows more slowly than the potential flow rental price. Most of the growth in relative utilization took place during the period 1929–1948, so that the actual service price follows the potential service price during the period 1948–1967.

Estimates of the responsiveness of factor proportions to relative factor prices also depend on the method of measurement. The average elasticity of

TABLE 7
SOURCES OF GROWTH IN FACTOR INPUT, 1929–1967 (ANNUAL PERCENTAGE RATES OF GROWTH)

	1929–1948	1948–1967	1929–1967
1. Capital input			
a. Stock	0.00	3.24	1.62
b. Quality change	0.30	0.94	0.62
c. Relative utilization	0.89	0.26	0.58
2. Labor input			
a. Stock	0.99	0.89	0.94
b. Quality change	0.59	0.74	0.67
c. Relative utilization	−0.13	−0.11	−0.12

substitution is defined as the ratio of the average rate of growth in capital services relative to labor services to the average rate of growth in the wage rate relative to the capital service price. Estimates of the average elasticity of substitution are given for each of the alternative methods of measurement in Table 9. For the actual flows of labor and capital services, the average elasticity of substitution is −0.25 for the period 1929–1948, 1.30 for 1949–1967, and 0.79 for the period as a whole. For comparison estimates of the average elasticity of substitution based on man-hours of labor and the stock of capital, the conventions used by Solow and subsequently adopted by Arrow, Chenery, Minhas, and Solow, are −0.20 for the period 1929–1948, 1.35 for 1948–1967, and 0.77 for the period as a whole.[34]

It is useful to compare the growth of product prices with the growth of factor costs. Price indexes for investment and consumption goods product are given in Table 3 above. The price of investment goods product grows at the rate of 2.22 per cent per year from 1929–1948, 1.81 per cent from 1949–1967, and 2.02 per cent for the period as a whole. The corresponding rates of growth for the price of consumption goods product are 2.22 per cent per year from 1929–1948, 1.97 per cent from 1948–1967, and 2.05 per cent for the period as a whole. Estimates of the responsiveness of the composition of output to relative prices of these two types of product may be obtained from the average elasticity of transformation. The average elasticity of transformation is defined as the ratio of the average rate of growth in investment goods product relative to consumption goods product to the average rate of growth in the investment goods price relative to the consumption goods price. Rates of growth of product prices and average elasticities of transformation for 1929–1967 and for the two sub-periods, 1929–1948 and 1948–1967, are given in Table 9.

6. TOTAL FACTOR PRODUCTIVITY

The main application of estimates of real product, real factor input, and their prices is to the study of production. We have illustrated the use of relative

[34]See Solow [32] and Arrow, Chenery, Minhas, and Solow [2]. Their data are for private non-farm gross national product for the period 1909-1949. Their estimate of total factor productivity for the period 1929-1948 rises from 1.251 to 1.761 on a base of unity in 1909, for an average rate of growth of 1.8 per cent per year.

TABLE 8

GROSS PRIVATE DOMESTIC FACTOR PRICES, 1929–1967 (1958 = 1.000)

Year	1. Labor Cost "Stock"	2. Labor Cost from "Potential" Flow	3. Labor Cost from "Actual" Flow	4. Capital Cost from "Stock"	5. Capital Cost from "Potential" Flow	6. Capital Cost from "Actual" Flow
1929	0.286	0.342	0.324	0.391	0.456	0.518
1930	0.275	0.328	0.311	0.315	0.365	0.431
1931	0.249	0.295	0.273	0.279	0.324	0.397
1932	0.210	0.248	0.236	0.206	0.240	0.308
1933	0.196	0.230	0.219	0.213	0.253	0.314
1934	0.209	0.244	0.238	0.224	0.270	0.323
1935	0.220	0.256	0.248	0.284	0.342	0.393
1936	0.236	0.274	0.263	0.311	0.374	0.418
1937	0.259	0.299	0.285	0.331	0.395	0.444
1938	0.255	0.292	0.281	0.291	0.343	0.408
1939	0.265	0.303	0.290	0.331	0.391	0.438
1940	0.275	0.313	0.300	0.370	0.435	0.461
1941	0.313	0.353	0.337	0.337	0.532	0.525
1942	0.373	0.418	0.398	0.534	0.617	0.586
1943	0.436	0.484	0.459	0.631	0.730	0.653
1944	0.475	0.524	0.493	0.660	0.767	0.683
1945	0.492	0.538	0.511	0.652	0.757	0.700
1946	0.518	0.562	0.540	0.689	0.795	0.771

1947	0.570	0.614	0.594	0.745	0.842	0.802
1948	0.614	0.656	0.638	0.784	0.862	0.827
1949	0.626	0.665	0.647	0.736	0.797	0.801
1950	0.662	0.699	0.683	0.869	0.930	0.905
1951	0.723	0.758	0.742	0.945	0.999	0.964
1952	0.766	0.798	0.782	0.933	0.977	0.959
1953	0.809	0.838	0.827	0.929	0.967	0.932
1954	0.829	0.854	0.846	0.932	0.961	0.955
1955	0.872	0.893	0.880	1.011	1.037	0.996
1956	0.925	0.942	0.930	0.955	1.010	0.970
1957	0.972	0.983	0.978	1.001	1.009	0.983
1958	1.000	1.000	1.000	1.000	1.000	1.000
1959	1.060	1.048	1.042	1.069	1.067	1.028
1960	1.103	1.081	1.074	1.074	1.066	1.023
1961	1.137	1.106	1.103	1.095	1.079	1.043
1962	1.190	1.149	1.144	1.173	1.151	1.091
1963	1.236	1.185	1.180	1.211	1.179	1.110
1964	1.298	1.236	1.229	1.258	1.213	1.116
1965	1.356	1.281	1.271	1.356	1.291	1.183
1966	1.435	1.344	1.335	1.426	1.336	1.219
1967	1.504	1.397	1.387	1.395	1.283	1.171

TABLE 9

SOURCES OF GROWTH IN FACTOR PRICES AND PRODUCT PRICES; ELASTICITIES OF SUBSTITUTION AND TRANSFORMATION, 1929–1967 (ANNUAL PERCENTAGE RATES OF GROWTH)

	1929–1948	1948–1967	1929–1967
1. Labor cost			
a. Stock	4.03	4.72	4.37
b. Potential flow	3.43	3.98	3.71
c. Actual flow	3.56	4.09	3.83
2. Capital cost			
a. Stock	3.66	3.03	3.34
b. Potential flow	3.35	2.10	2.72
c. Actual flow	2.47	1.83	2.15
3. Elasticity of substitution			
a. Stock	−2.69	1.40	0.66
b. Potential flow	−16.15	1.36	0.64
c. Actual flow	−0.25	1.30	0.79
d. ACMS	−0.20	1.35	0.77
4. Consumption goods price	2.13	1.97	2.05
5. Investment goods price	2.22	1.81	2.02
6. Elasticity of transformation	6.13	−2.07	−16.10

factor proportions and relative factor prices in analyzing the responsiveness of factor proportions to factor price changes. We have also analyzed the responsiveness of product proportions to product price changes. We now consider the application of real product and real factor input to the measurement of total factor productivity. We present a number of alternative estimates of total factor productivity based on alternative conventions about the measurement of real factor input. We begin with an estimate of total factor productivity based on the actual flow of labor and capital services. We compare this estimate with alternatives based on potential flows of labor and capital services and on stocks of labor and capital.

The services of consumers' durables and producers' durables used by institutions are allocated directly to final demand so that growth in the quantities of these services does not affect growth of total factor productivity. Similarly, the services of owner-occupied dwellings and institutional structures are allocated directly to final demand. In evaluating the relative importance of growth of real factor input and of total factor productivity as sources of economic growth, it is useful to compare the relative proportions of each on the growth of real product, including and excluding capital services from the household sector. We present estimates of the relative importance of the sources of economic growth for gross private domestic product as we have defined it and for analogous gross product measures excluding household durables and structures.

Total factor productivity is defined as the ratio of real product to real factor input or, equivalently, as the ratio of the price of factor input to the product price. Growth in total factor productivity has a counterpart in growth of the price of factor input relative to the price of output. We may define a Divisia index of total factor productivity, say P, as:

$$\log \frac{P_t}{P_{t-1}} = \log \frac{Y_t}{Y_{t-1}} - \log \frac{X_t}{X_{t-1}},$$

where Y is the quantity index of total product and X is the quantity index of total factor input. Equivalently, the index of total factor productivity may be defined as:

$$\log \frac{P_t}{P_{t-1}} = \log \frac{p_t}{p_{t-1}} - \log \frac{q_t}{q_{t-1}},$$

where p is the price index of total factor input and q is the price index of total product.[35] The index of total factor productivity for 1929–1967 corresponding to the quantity index of gross private domestic product from Table 3 and the quantity index of gross private domestic factor input from Table 6 is given in Table 10.

The conventions for measurement of factor services underlying our concept of gross private domestic factor input have been employed by Jorgenson and Griliches. Our estimates differ from theirs in two significant respects: First, we have converted their index of relative utilization to an annual basis and reduced the scope of adjustments of potential flows of capital services for changes in relative utilization. Second, we have measured the flow of capital services for sectors distinguished by legal form of organization in order to provide a more detailed representation of the tax structure. These differences have an important impact on the estimate of total factor productivity.

Our conventions for the measurement of factor services are not the only ones employed in the measurement of total factor productivity. Denison and Solow use a stock concept of capital input, measuring neither changes in relative utilization nor changes in the quality of capital services due to changes in the composition of the capital stock.[36] Denison weights persons engaged by an index of labor quality that incorporates the effects of growth in educational attainment but differs in a number of important respects from the index we have used.[37] Denison also adjusts man-hours for changes in labor efficiency that accompany changes in hours per man.[38] Solow uses unweighted man-hours, omitting the effects of changes in the composition of the labor force on the quantity of labor input.[39] Kendrick adjusts labor and capital input for changes in the industrial composition of labor force and capital stock.[40] However, changes within an industrial sector due to shifts in composition are not included in his measures of real factor input.

To provide a basis for comparison of our estimates of total factor productivity with estimates that result from alternative conventions for the measurement of real factor input, we present measures of total factor productivity based on potential service flows and on stocks of labor and capital in Table 10. The first variant on our estimate of total factor productivity omits the relative utilization adjustment for capital, the second omits the relative utilization adjustment for

[35]For further discussion of this index of total factor productivity, see Jorgenson and Griliches [23], especially pages 250–254. The Divisia index of total factor productivity described in the text is a discrete approximation to the continuous Divisia index discussed by Jorgenson and Griliches.

[36]See Denison [10], pages 94–99, and Solow [32], page 315.

[37]See Denison [10], especially pages 67–72.

[38]See Denison [10], especially pages 35–41.

[39]See Solow [32], page 315.

[40]See Kendrick [26], especially pages 252–289.

TABLE 10

TOTAL FACTOR PRODUCTIVITY, 1929–1967 (1958 = 1.000)

Year	1. Labor and Capital Services	2. Actual Labor Services; Potential Capital Services	3. Potential Labor and Capital Services	4. Potential Labor Services; Capital Stock	5. Labor and Capital Stock	6. Actual Labor Services; Capital Stock	7. Unweighted Man-hours; Capital Stock
1929	0.726	0.685	0.707	0.664	0.599	0.644	0.530
1930	0.680	0.631	0.652	0.614	0.555	0.595	0.496
1931	0.657	0.600	0.628	0.591	0.536	0.565	0.483
1932	0.614	0.550	0.567	0.533	0.484	0.517	0.445
1933	0.604	0.548	0.564	0.527	0.480	0.511	0.443
1934	0.636	0.586	0.596	0.552	0.504	0.543	0.487
1935	0.668	0.627	0.640	0.593	0.543	0.581	0.518
1936	0.714	0.679	0.696	0.645	0.592	0.629	0.556
1937	0.738	0.699	0.719	0.669	0.615	0.650	0.571
1938	0.734	0.679	0.695	0.649	0.599	0.634	0.567
1939	0.763	0.724	0.743	0.694	0.642	0.676	0.601
1940	0.788	0.766	0.786	0.736	0.682	0.716	0.638
1941	0.826	0.828	0.851	0.799	0.744	0.777	0.692
1942	0.839	0.855	0.882	0.832	0.778	0.807	0.715
1943	0.872	0.912	0.941	0.888	0.834	0.860	0.758
1944	0.925	0.969	1.005	0.946	0.893	0.913	0.807
1945	0.944	0.973	1.004	0.945	0.896	0.916	0.822
1946	0.898	0.908	0.930	0.878	0.836	0.857	0.790

1947	0.862	0.878	0.895	0.852	0.815	0.836	0.782
1948	0.882	0.896	0.911	0.876	0.843	0.862	0.814
1949	0.892	0.890	0.904	0.875	0.845	0.861	0.817
1950	0.938	0.948	0.961	0.935	0.906	0.922	0.882
1951	0.946	0.960	0.971	0.949	0.923	0.938	0.902
1952	0.949	0.956	0.967	0.949	0.927	0.938	0.904
1953	0.968	0.982	0.990	0.974	0.954	0.966	0.938
1954	0.974	0.977	0.982	0.969	0.953	0.964	0.942
1955	1.006	1.022	1.031	1.020	1.006	1.012	0.989
1956	0.993	1.010	1.018	1.011	1.001	1.004	0.986
1957	0.998	1.009	1.012	1.009	1.002	1.006	0.996
1958	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1959	1.018	1.034	1.038	1.039	1.046	1.035	1.039
1960	1.019	1.036	1.040	1.043	1.056	1.039	1.048
1961	1.032	1.046	1.048	1.054	1.072	1.053	1.068
1962	1.061	1.085	1.088	1.097	1.120	1.094	1.114
1963	1.076	1.104	1.106	1.119	1.147	1.116	1.141
1964	1.091	1.130	1.134	1.151	1.185	1.147	1.177
1965	1.115	1.157	1.162	1.187	1.226	1.181	1.215
1966	1.129	1.174	1.178	1.211	1.258	1.207	1.249
1967	1.114	1.157	1.162	1.204	1.256	1.199	1.247

labor; the second variant is based on potential service flows for both labor and capital input. The third variant omits the quality adjustment for capital, while the fourth omits the quality adjustment for labor, providing a stock measure of total factor productivity. Two final variants provide combinations of alternative measures of labor input with the stock measure of capital. The fifth combines actual labor input with the stock of capital, while the sixth combines unweighted actual man-hours with capital stock.

TABLE 11
GROWTH IN TOTAL FACTOR PRODUCTIVITY, 1929–1967 (AVERAGE ANNUAL RATES OF GROWTH)

	1929–1948	1948–1967	1929–1967
1. Actual labor and capital services	1.03	1.23	1.13
2. Actual labor services; potential capital services	1.42	1.35	1.38
3. Potential labor and capital services	1.34	1.28	1.31
4. Potential labor services; capital stock	1.46	1.67	1.56
5. Labor and capital stock	1.80	2.10	1.95
6. Actual labor services; capital stock (Denison)	1.54	1.74	1.64
7. Man-hours and capital stock (Solow and ACMS)	2.26	2.25	2.25

It is obvious from a comparison of the alternative estimates of total factor productivity given in Table 10 that the results are highly sensitive to the choice of conventions for measuring real factor input. The effects of varying the conventions are summarized for the periods 1929–1948, 1948–1967, and 1929–1967 in Table 11; geometric average annual rates of growth are given for each variant of total factor productivity.

Finally, to evaluate the relative importance of growth in real factor input and growth in total factor productivity as sources of economic growth, we consider the relative proportion of growth in real factor input for two alternative concepts of real product—including and excluding the capital input of the household sector. Geometric average annual rates of growth are given for real product and real factor input, including and excluding household capital services, for 1929–1967 in Table 12. The relative proportion of growth in total factor productivity in the growth of real product is also provided for both concepts of real product.[41]

We find that the growth in real factor input predominates in the explanation of the growth of real product for the period 1929–1967 and for each of the sub-periods, 1929–1948 and 1948–1967. These findings are directly contrary to those of Abramovitz [1], Kendrick [26], and Solow [32], in earlier studies of productivity change. We have estimated real factor input on the basis of capital stock and actual man-hours, the conventions used by Solow and subsequently adopted by Arrow, Chenery, Minhas, and Solow [2], for 1929–1967. The resulting

[41]Denison [10], pages 148–149, employs real national income, Solow [32], page 315, employs private, non-farm, gross national product, and Kendrick [26], pages 328–342, employs both gross national product and net national product.

TABLE 12

THE RELATIVE IMPORTANCE OF PRODUCTIVITY CHANGE, 1929–1967 (AVERAGE ANNUAL RATES OF GROWTH)

	1929–1948	1948–1967	1929–1967
1. Gross private domestic product			
Real product	2.37	3.96	3.16
Real factor input	1.34	2.73	2.04
Total factor productivity	1.03	1.23	1.13
Relative proportion of productivity change	0.43	0.31	0.36
2. Gross private domestic product, excluding household capital services			
Real product	2.54	3.70	3.12
Real factor input	1.54	2.28	1.91
Total factor productivity	1.00	1.42	1.21
Relative proportion of productivity change	0.39	0.38	0.39

estimates of the distribution of the growth of real product between growth in real factor input and total factor productivity are comparable to those of Solow's earlier study. On this basis total factor productivity grows at the average rate of 2.25 per cent per year while real factor input grows at 0.91 per cent per year. Our estimates, given in Table 12, are that total factor productivity grows at 1.13 per cent per year and real factor input at the rate of 2.04 per cent per year. Total factor productivity accounts for 36 per cent of the growth of real product, while real factor input accounts for 64 per cent of output growth.

We have also extended estimates of real factor input based on capital stock and actual labor input, the conventions adopted by Denison [10], through 1967. Denison's estimates of the growth of labor input are conceptually similar to our own and his empirical results are closely comparable to ours. We find that estimates of real factor input based on the conventions used by Denison suggest that total factor productivity grows at the average rate of 1.64 per cent per year while real factor input grows at 1.52 per cent per year. The discrepancy between our estimates, given in Table 12, and those of Denison is accounted for almost entirely by our adjustments of the measure of capital input for quality change and relative utilization. Denison has incorporated about half the growth in real factor input over and above the growth of capital stock and actual man-hours into his estimates of real factor input.

Finally, although growth in real factor input predominates in the growth of real product, we estimate that changes in total productivity are substantial for 1929–1967 and for both the sub-periods we have considered. The conclusion of Jorgenson and Griliches [23] that productivity growth is negligible must be revised accordingly. The main differences between our estimates and those of Jorgenson and Griliches are in the measurement of capital. We have incorporated the effects of taxation in greater detail through separation of property compensation by legal form of organization. However, the discrepancy between our empirical results and those of Jorgenson and Griliches is primarily accounted for by our measurement of the relative utilization of capital. We have reduced the scope of the adjustment for relative utilization by confirming it to depreciable assets in the corporate and non-corporate sectors. Second, incorporation of

annual estimates of capacity to consume electricity and actual electricity consumption results in the allocation of the total growth in relative utilization for the period 1929–1967 to the period 1929–1948. In the relative utilization adjustment of Jorgenson and Griliches, almost all of the growth in relative utilization was allocated to the period 1945–1965.

7. Summary and Conclusion

In this paper we have attempted to provide a conceptual basis for separating social product and social factor input into price and quantity components. To test the feasibility of our accounting framework we have measured real product and real factor input for the United States from 1929–1967. We conclude that estimates of real factor input paralleling the real product estimates in the United States national accounts are feasible. The data required for estimation of real product are the same as those required for perpetual inventory estimates of capital stock together with data on property compensation by legal form of organization and information on the tax structure for property income.

Fully satisfactory estimates of real factor input will require much additional research. In measuring labor input, data on persons engaged should include estimates of the number of unpaid family workers, such as those of Kendrick [25, 26]. Estimates of man-hours for the different components of the labor force should be compiled on a basis consistent with data on persons engaged, as Kendrick [25, 26] has done. The weakest link in the chain of imputations linking labor input to the underlying data on man-hours and employment is the adjustment of labor input for the intensity of effort, along the lines suggested by Denison [10]. Additional evidence on this adjustment is given by Denison [11] for the United States and for Europe. The validity of estimates of intensity of effort must be tested through the study of variations in labor income by hours worked, holding other characteristics of labor input constant. Finally, the quality adjustments for the labor force should be expanded to incorporate changes in the relative number of hours worked. The quality adjustments should also incorporate characteristics of the labor force other than educational attainment such as age, race, sex, occupation, and industry. Similar improvements in the measurement of capital input are discussed in our previous paper.[42]

Detailed accounting measurements of real product and real factor input will open up many new possibilities for the study of production. We have analyzed the responsiveness of factor proportions to changes in relative factor prices and the responsiveness of product proportions to changes in relative product prices. Average elasticities of substitution between factors and transformation between products vary considerably between the sub-periods 1929–1948 and 1948–1967. Estimates of these elasticities depend critically on the method for measurement of factor input. Our estimates of the elasticity of substitution, based on actual flows of labor and capital input, are strikingly similar to those of Arrow, Chenery, Minhas, and Solow [2], based on very different conventions of measurement. However estimates of the elasticity of substitution based on stocks

[42]Christensen and Jorgenson [5].

of labor and capital or potential flows of labor and capital services differ substantially from these estimates.

We have measured total factor productivity in the United States for the period 1929–1967. This study extends the analysis of productivity change by Jorgenson and Griliches [23]. First, we have provided measurements for a considerably longer time period than the time period 1945–1965 used in their study. Second, we have analyzed the growth of real factor input in more detail. One important change is the refinement of the measurement of relative utilization of capital by incorporation of annual data on capacity to consume electricity and on actual electricity consumption. A second important change is the separation of property compensation by legal form of organization. This change enables us to incorporate the effects of taxation of income from capital in a more satisfactory way.

Although growth in real factor input predominates in the growth of real product, we estimate that changes in total factor productivity are substantial for 1929–1967 and for both the sub-periods we have considered. The conclusion of Jorgenson and Griliches that productivity growth is negligible must be revised accordingly.

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