ON REVISIONS IN NATIONAL ACCOUNTS ESTIMATES[1]

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This paper treats three subjects:

(1) In Sections II and III there is given a general analysis of revisions in national income data, namely the sources of revisions are enumerated and the conclusions that might be drawn are discussed in some detail.

(2) Section IV gives a description of the history of revisions in national income estimates for the FRG (Federal Republic of Germany) for the period 1949–1965. The general revisions are classified according to their causes.

(3) Sections V to VII give a statistical analysis of the revisions described in Section IV. Revisions in the level and in the linear annual change are characterized by their mean and their mean absolute deviation. Theil's coefficient of inequality is computed and on the basis of its decomposition a kind of analysis of variance is carried out.

I

Revisions in national income estimates occur frequently, and are discussed at some length in the literature dealing with the accuracy of these data. An appraisal of the quality of quarterly national accounts estimates is made entirely on the basis of the characteristics discovered in the revisions of those data (cf. the reports delivered at the eighth conference of the International Association for Research in Income and Wealth).

This paper treats the following three subjects:

1. a general analysis of revisions in national income data,
2. a description of the history of revisions in national income estimates for the FRG (Federal Republic of Germany),
3. a statistical analysis of the revisions mentioned in [2].

II

Revisions in national income estimates may be of two kinds: statistical revisions, resulting from time-lags in the preparation of data, and conceptual revisions, resulting from changes in definitions.

Statistical revisions occur because not all basic statistics are available when the first national income estimates are prepared; as a result, the first estimates are often rendered obsolete as soon as the missing basic statistics appear. Such

[1]This paper is a revised and shortened version of chapter 3 from the author's Ph.D. thesis, "Das Sozialprodukt, Unzulänglichkeiten des Konzepts und Ungenauigkeiten der Schätzung", which was submitted to the Technical University of Berlin in July 1967.

The author wishes to thank Professor David Durand (M.I.T.) for an introduction to triangular graph paper and some pointers on English style.

statistical revisions are unavoidable unless one is willing to wait until the final basic statistics are ready for use. But should one insist on waiting so long, the national income data, though of interest to the historian, would be virtually useless to the government policy maker, since the time-lag might be five years or more.

A closer examination of these statistical revisions will show that they are of two types. Type I is a permanent revision of estimates. It results from a relatively short time-lag in the preparation of the underlying data. For instance, statistics of income taxes for each period always appear with a time-lag of one or two years. Type II revisions occur when the underlying data are prepared at long intervals. Examples are data from the census of population and housing (approximately every ten years) or from the censuses concerning the structure of costs in the different sectors of industry (every four years in the FRG). For the intercensal years, one has first to extrapolate from the last available census; then, when the new census is completed, one can interpolate between the two census years. The preliminary extrapolations usually turn out to be wrong. These statistical revisions of Type II are characterised by two features:

1. They affect not only the estimates for the period just closed, but those for several past periods.
2. While revisions of Type I refer only to the estimate for the last period, and thus cause a change in the apparent rate of growth, revisions of Type II cause primarily a shift in a series of adjacent estimates and secondarily a change in the rate of growth.

Conceptual revisions result when changes in concepts or definitions in the "ideal form"[2] or in the "working form"[2] of an item in the national income accounts are applied retroactively—usually to a number of periods. Like the statistical revisions of Type II, conceptual revisions take place only after a longer period. These two kinds of revisions are ordinarily made simultaneously, as empirical studies show (cf. Section IV of this paper).

There are also revisions resulting from a change in the method of estimation, and these occupy an intermediate position between the statistical and the conceptual revisions. Their effects are similar to those of the Type II revisions; for if one makes use of a new method of extrapolation, interpolation, deflation, or seasonal adjustment, the new method is generally extended backwards so that the older estimates will be comparable to current and future estimates.

III

In studying revisions in the national income estimates to obtain insight into the accuracy of estimates, the student would like to know what part of any given revision is ascribable to changes in statistics and what part to changes in definitions. The Federal Statistical Office of the FRG which is responsible for estimates and revisions in Germany, does not furnish this information. One should

[2]We take these terms in the sense of I. Ohlsson, *On National Accounting*, Stockholm, 2nd Impression, 1961, p. 2.

therefore be careful in drawing conclusions about the accuracy of estimates from the revisions. As data referring to the United States and published by the Office of Business Economics (U.S. Department of Commerce) have shown, statistical and conceptual revisions might well go in opposite directions.

Even if the breakdown between statistical and conceptual revisions were available, judging the quality of the estimates from a study of the revisions would be difficult. To be sure, frequent large changes in an estimate cast doubt on the quality of the earlier estimates. But the converse is not true. When estimates are revised infrequently and in small amounts—or even when they are not revised at all—there is no implication that these estimates are reliable. A lack of revision may simply mean that improved underlying statistics to support a revision have never become available. One hopes, of course, that revisions will improve quality and reduce errors, but there is no reason to think that they always do.

In technical statistical language, one can say that estimates are precise when they tend to cluster closely around a central point. The estimates may be very inaccurate, however, because they may all reflect a large systematic error. Precision implies a small dispersion about some one value, which may be very far from the true value; accuracy implies a small dispersion about the true value.

IV

The Federal Statistical Office of the FRG publishes annual national income estimates twice a year as a rule. This text refers only to the undeflated (current price) annual estimates for 1949–1965; an example for Gross National Product at Market Prices appears in Table 1. Estimates for half-year periods are also available, but only in the form of an interrupted time series. Quarterly estimates with revisions are prepared by the DIW (German Institute for Economic Research, Berlin) on the basis of the annual estimates of the Federal Statistical Office, but these behave very much like the annual estimates—except for seasonal variation—and will not be discussed.

A cursory inspection of the arrangement in Table 1 will reveal two distinct patterns. There are diagonally arranged revisions, lying close under the original estimates, and horizontally arranged revisions between two rows running across the table. These diagonal and horizontal revisions differ in magnitude, direction, and basic significance.

The diagonal revisions, which pertain to the first preliminary estimates, usually occur within six months or a year following the original estimates—though they may extend into the second year. Sometimes, of course, there may be two or more successive revisions—as if the revision had not been completed on the first try. From the short official text commenting on the revisions, one surmises that these diagonal revisions belong almost exclusively to the statistical category—presumably Type I (permanent). If one ignores the few points in the table where the diagonal and horizontal revisions run together, one finds that the diagonal revisions are relatively the smaller. In Table 1, the first revisions of Gross National Product are never more than $\pm$ 1 per cent of the estimate first published; these revisions represent the net effect of all first revisions in the national accounts. The first revisions of some components, however, are often large (see Section VI of

TABLE 1

GROSS NATIONAL PRODUCT AT MARKET PRICES OF THE FEDERAL REPUBLIC OF GERMANY[a]

Date of Publication	Year of Reference							
	1949	1950	1951	1952	1953	1954	1955	1956
1950								
July	81.798							
Dec.	81.798							
1951								
April	81.265	92.016						
Sept.	81.265	91.088						
1952								
April	—	90.279	113.381[b]					
Sept.	79.776	90.279	113.523					
1953								
April	79.776	90.279	113.523	125.564[b]				
Sept.	79.471	89.862	113.719	126.120				
1954								
April	79.377	89.765	113.596	126.018	133.666[b]			
Sept.	—	89.765	113.596	126.018	134.315			
1955								
April	79.377	89.765	113.596	126.018	134.315	145.293[b]		
Sept.	—	89.765	113.596	126.018	134.315	145.460		
1956								
April	79.377	89.765	113.596	126.018	134.315	145.460	164.000[b]	
Sept.	—	—	—	126.018	134.315	145.460	164.000	
1957								
March	—	97.200	119.600	134.200	143.750	153.950	—	
May	—	97.200	119.600	134.200	143.750	153.950	175.600	
July	—	97.200	119.600	134.200	143.750	153.950	175.600	192.450[b]
1958								
Jan.	—	97.200	119.600	134.200	143.750	153.950	175.600	192.450
May	—	97.200	119.600	134.200	143.750	153.950	175.600	193.400
1959								
Jan.	—	97.2	119.6	134.2	143.8	154.0	175.6	193.4
May	—	97.200	119.600	134.200	143.750	153.950	175.600	193.400
1960								
Jan.	—	97.2	118.6	135.6	145.5	156.4	178.3	196.4
March	—	97.200	118.600	135.600	145.500	156.400	178.300	196.400
Sept.	—	—	—	—	—	—	—	—
1961								
Jan.	—	97.200	—	—	—	156.400	—	—
Sept.	—	97.200	—	—	—	—	178.300	—
1962								
Jan.	—	—	—	—	—	—	—	—
Sept.	—	97.200	—	—	—	—	—	—
1963								
Jan.	—	—	—	—	—	—	—	—
Oct.	—	97.900	119.500	136.600	147.100	157.900	180.400	198.800
1964								
Jan.	—	—	—	—	—	—	—	—
Sept.	—	—	—	—	—	—	—	—
1965								
Jan.	—	—	—	—	—	—	—	—
Sept.	—	—	—	—	—	—	—	—
1966								
Jan.	—	—	—	—	—	—	—	—
Sept.	—	—	—	—	—	—	—	—

[a]Figures are given in billion DM; the results of the revisions made in the autumn of 1963 for the years 1960–1962 relate to the FRG including Berlin (West) and the Saar, while all other figures exclude the Saar and Berlin.

1957	1958	1959	1960	1961	1962	1963	1964	1965
207.0[c]								
209.600[b]								
209.6[b]	222.3[c]							
209.600[b]	222.300[b]							
213.6	227.3[b]	244.4[c]						
213.600	227.300[b]	244.400[c]						
213.600	228.200[b]	247.000[c]						
213.600	228.500	247.900[b]	275.800[c]					
—	228.500	247.900[b]	276.600[b]					
213.600	228.500	247.900	277.700[b]	310.400[c]				
—	228.500	247.900	277.700[b]	310.400[b]				
—	228.500	247.900	277.700	310.400[b]	336.800[c]			
216.300	231.500	250.900	296.800	326.400[b]	355.100[b]			
—	—	—	296.8	326.2	354.8[b]	376.8[c]		
—	—	—	296.800	326.200	354.500[b]	376.500[c]		
—	—	—	296.800	326.200	354.500	376.500[b]	412.500[c]	
—	—	—	296.800	326.200	354.500	376.800[b]	413.400[b]	
—	—	—	296.800	326.200	354.500	377.600	414.600[b]	449.700[c]
—	—	—	296.800	326.200	354.500	377.600	413.800[b]	448.800[b]

[b]preliminary estimate.
[c]first preliminary estimate.
— no figure published.

this paper). In direction of change, the first revisions vary by period and by component, but there seems to be a slight tendency towards upward revisions; i.e., the first estimates tend to be too low. One may surmise that national income statisticians are conservative in preparing first estimates—not only in the FRG but elsewhere. The effect of the first revisions on apparent rates of growth and on the structural composition of Gross National Product is usually minor.

The horizontal revisions in Table 1 take place at certain points of time and affect all estimates made up to that point. They are mainly statistical revisions of Type II (non-permanent), or revisions caused by altering the methods of estimation or by changes in concepts. During the period under consideration (1949–1966) there were four general revisions:

1. The general revision of 1953 is hard to distinguish from the first revisions because it was carried out in the beginning of the period under consideration here. It resulted from replacing the conventions and methods of estimation and of presentation used up to then by those laid down in the OEEC "Standardised System". This general revision represented mainly a change in concept.
2. By contrast, the general revision of 1957 illustrates the statistical Type II; it resulted from a change in the method of estimation. Up to 1957 national accounting in the FRG was based entirely on structural relations valid in the pre-war period, since surveys revealing the post-war economic structure of West Germany did not exist in 1948–1949, when national accounting began. When, in 1957, surveys covering all parts of the West German economy had been made, it became possible to update the statistical basis of national income estimates.
3. A third general revision occurred in 1960, when the Federal Statistical Office of the FRG began to present national income estimates in the form of a new and very detailed system of accounts. While preparing data for past periods, to extend the new system backwards, statisticians discovered that many past estimates had become obsolete, and that surveys performed since 1957 gave better information. This general revision like that of 1957 was a statistical revision resulting from new methods of estimation and the adoption of a more sophisticated system of national accounts.
4. A fourth general revision, in the autumn of 1963, followed the completion of several surveys concerning the structure of costs in the main industries of the FRG. All figures were revised back to 1950.

The Federal Statistical Office says that there will be general revision in the near future, pending the results of several major surveys.

V

In analysing revisions, it is not enough to determine the magnitude (and direction) of change in the original published estimates. Also of interest are the secondary effects of these changes on economic forecasts, estimated structural parameters (propensity to consume, foreign trade multipliers, and so on), or

econometric models. But these secondary effects, however interesting, are beyond the scope of this paper.[3]

As for the immediate effects, there are two questions of interest: the magnitude (and direction) of change between the first published estimate and the first revision; second, the magnitude (and direction) of change between the first estimate and the last revision.[4] By the first revision, we mean the first change that actually occurs in the original published estimates, whether that change takes place in the period immediately following original publication or in some later period; the first revision is thus unambiguous. About the last revision, however, there may be doubts, because there is always a chance that further revisions will occur at some future time. The only feasible procedure is to accept as final those revisions occuring several years after the original publication—assuming that no further revision appears imminent. For this paper—as for many others appearing in the literature on national accounts—the last revisions are the last ones actually made before the paper was submitted. One must keep in mind, of course, that the first revision and the last revision are sometimes identical, especially for recent periods. Then, of course, the change from the original estimate to the first revision is the same as the change from the original estimate to the last revision.

In the following sections we want to construct statistical measures describing the effect of revision on the level and on the linear annual change of Gross National Product and of its components.

VI

Revisions in the Level of National Accounts Estimates

Two questions may be asked:

1. What is the average amount of the first (or the last available) revision of the estimates, and what is the direction of this revision?
2. What is the average amount of the first (or the last available) revision of the estimates if one takes the difference between the first and the revised estimates without regard to sign?

We introduce the following symbols:

E_i —first estimate of a certain item for period i, with $i = 1, 2, \ldots, n$,
R_i —first revised estimate of a certain item for period i,
R_i^* —last revised estimate of a certain item for period i.

A statistical measure to answer the first question, is:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} (R_i - E_i), \tag{1}$$

[3]A project consisting of feeding an econometric model for Canada with a set of first, unrevised estimates and then with a set of revised estimates and finally studying the discrepancies, had been carried out by F.T. Denton and J. Kuiper in 1962–1963, c.f.: *The Review of Economics and Statistics*, Vol. XLVII, 1965, pp. 198–206.

[4]In answering these two questions we consciously abstain from describing the history of an estimate from the first figure given for it to the last one available, though it might be of great interest to see how these intermediate values behave statistically.

or, if one takes a measure having no dimension,

$$\bar{e} = \frac{\sum_{i=1}^{n} (R_i - E_i)}{\sum_{i=1}^{n} R_i}. \tag{2}$$

x reflects the amount and direction of bias in the first estimates as compared to the first revised estimates.

A statistical measure answering the second question, is:

$$\bar{x}' = \frac{1}{n} \sum_{i=1}^{n} |R_i - E_i|, \tag{3}$$

or, taking a measure without dimension,

$$\bar{e}' = \frac{\sum_{i=1}^{n} |R_i - E_i|}{\sum_{i=1}^{n} |R_i|}. \tag{4}$$

$\bar{x}'$, which reflects the uniformity of the first estimates as compared to the revised estimates, is a measure of dispersion. If $\bar{x}' = 0$, then $\bar{x} = 0$, too, but not *vice versa*. One can, of course, substitute $R_i = R_i^*$ in (1) to (4).

The level of the first estimates (or the last) is called absolutely stable if $\bar{e}$ and $\bar{e}'$ are both zero, or approximately zero. The stability of the first estimates is great if the values of $\bar{e}$ and $\bar{e}'$ are close together, for this means that the revisions of the first estimates are always or nearly always in the same direction.

The computations have been carried out for the following 21 items:

BSP,M —Gross National Product at Market Prices,
NSP,M—Net National Product at Market Prices,
NSP,F —Net National Product at Factor Costs (National Income);

contribution of the following industries to Net Domestic Product at Factor Costs (net output approach):

LuF —Farming and Forestry,
WG —Manufacturing,
H —Wholesale and Retail Trade,
V —Transportation,
ÖV —Government,
W —Housing,
KuV —Banking and Insurance;

items of the expenditure approach of Gross National Product:

Cpr —Private Consumption,
Cst —Government Consumption,
Ibr —Gross Fixed Investment,
Iv —Change in Business Inventories,
Ex–Im —Net Exports of Goods and Services,

D —Depreciation;

items of the income-distributed approach of National Income:

L —Wages, i.e. compensation of employees,
G —Profits, i.e. income from entrepreneurship and property,
Gh —Profits of Private Households,
Gu —Undistributed Profits of Business Corporations,
Gs —Profits of Government.

Of course these 21 items are only a sample of all those shown in the national accounts of the FRG. They were chosen primarily because their definitions have not changed considerably in the course of time.

Tables 2 and 3 show the results of computation. In some cases, one will note, the number of observations (n) is greater than the number of years—17 for 1949–1965, or 16 for 1950–1965. This apparent incongruity results because there were, in several years, two first and revised estimates referring to different regions: e.g., FRG with or without West-Berlin and/or Saar.

A plus sign in columns 3 and 4 means that the first estimate was too low with respect to the revision. These plus signs predominate, implying a negative bias for the first estimates (cf. formulas (1) and (2)). The arrangement of plus signs is essentially the same for the first and the last revisions. The magnitudes, however, often differ greatly—note especially the difference between −2.4 (column 4, Table 2) and −10.8 (column 4, Table 3) for item 21 (Profits of Government). For

TABLE 2

REVISIONS IN THE LEVEL (COMPARISON BETWEEN FIRST ESTIMATE AND FIRST REVISED ESTIMATE)

	n	$\frac{1}{n}\sum R_i$	$\bar{x}$	$\bar{e}\cdot 100$	$\frac{1}{n}\sum \lvert R_i\rvert$	$\bar{x}'$	$\bar{e}'\cdot 100$
	1	2	3	4 = 3 : 2·100	5	6	7 = 6 : 5·100
1. BSP,M	19	238,620	+1,259	+0.5	238,620	1,591	0.7
2. NSP,M	19	217,417	+1,339	+0.6	217,417	1,795	0.8
3. NSP,F	19	183,677	+768	+0.4	183,677	1,136	0.6
4. LuF	18	13,730	−96	−0.7	13,730	192	1.4
5. WG	18	91,247	−135	−0.1	91,247	541	0.6
6. H	17	21,080	+823	+3.9	21,080	901	4.3
7. V	17	11,579	−41	−0.4	11,579	93	0.8
8. ÖV	17	16,152	+239	+1.5	16,152	350	2.2
9. W	17	2,949	−133	−4.5	2,949	190	6.4
10. KuV	16	5,325	+410	+7.7	5,325	518	9.7
11. Cpr	19	138,196	+1,649	+1.2	138,196	1,908	1.4
12. Cst	19	34,711	+97	+0.3	34,711	616	1.8
13. Ibr	19	57,102	+333	+0.6	57,102	495	0.9
14. Iv	19	4,717	0	0	4,717	560	11.9
15. Ex-Im	19	4,763	+107	+2.2	5,274	266	5.0
16. D	19	21,638	+275	+1.3	21,638	814	3.8
17. L	18	119,563	+189	+0.1	119,563	235	0.2
18. G	18	70,629	+648	+0.9	70,629	905	1.3
19. Gh	17	55,497	+835	+1.5	55,497	963	1.7
20. Gu	17	11,374	+136	+1.2	11,374	276	2.4
21. Gs	17	3,368	−82	−2.4	3,368	103	3.1

Column 2, 3, 5, 6—Million DM; Column 4, 7—%.

TABLE 3

REVISIONS IN THE LEVEL (COMPARISON BETWEEN FIRST ESTIMATE AND LAST REVISED ESTIMATE)

	n	$\frac{1}{n}\Sigma R_i^*$	$\bar{x}^*$	$\bar{e}^* \cdot 100$	$\frac{1}{n}\Sigma \lvert R_i^* \rvert$	x'^*	$\bar{e}'^* \cdot 100$
	1	2	3	4 = 3 : 2·100	5	6	7 = 6 : 5·100
1. BSP,M	18	251,689	+5,686	+2.3	251,689	5,874	2.3
2. NSP,M	18	228,325	+4,408	+1.9	228,325	4,682	2.1
3. NSP,F	18	193,697	+4,210	+2.2	193,697	4,412	2.3
4. LuF	17	14,178	−16	−0.1	14,178	262	1.8
5. WG	17	94,203	−568	−0.6	94,203	2,019	2.1
6. H	16	22,406	+1,226	+5.5	22,406	1,720	7.8
7. V	16	11,670	−320	−2.7	11,670	320	2.7
8. ÖV	16	16,955	+421	+2.5	16,955	461	2.7
9. W	16	3,049	−161	−5.3	3,049	203	6.7
10. KuV	15	5,487	+308	+5.6	5,487	941	17.1
11. Cpr	18	146,125	+4,925	+3.4	146,125	5,057	3.5
12. Cst	18	35,866	−4	0	35,866	684	1.9
13. Ibr	18	59,710	+676	+1.1	59,710	855	1.4
14. Iv	18	4,647	−132	−2.8	4,647	1,261	27.1
15. Ex-Im	18	5,303	+244	+4.6	5,530	389	7.0
16. D	18	23,362	+1,192	+5.1	23,362	1,742	7.5
17. L	18	119,863	+489	+0.4	119,863	525	0.4
18. G	18	73,836	+3,855	+5.2	73,836	4,056	5.5
19. Gh	17	57,027	+2,365	+4.1	57,027	2,695	4.7
20. Gu	17	11,251	+13	+0.1	11,251	310	2.8
21. Gs	17	3,114	−335	−10.8	3,114	335	10.8

Column 2, 3, 5, 6—Million DM; Column 4, 7—%.

items 4, 5, 12, and 17 the differences between Table 2 and Table 3 are small—perhaps because the basic statistics available at the time of the first estimates were already reasonably satisfactory, thus leaving little room for improvement.

Regarding the level of estimates one can draw the following conclusions:

1. The most precise estimates (with precision implying that the revised estimates differ little from the first estimates) are those for Wages (item 17), probably because the estimates cannot be better founded than in the moment of their first estimation.
2. The least precise estimates are the following: Changes in Business Inventories (Iv), despite a bias of zero in Table 2, Banking and Insurance (KuV), Housing (W), Depreciation (D), Profits of Government (Gs), and Net Exports (Ex–Im). Of these Iv is imprecise because the first estimate of it is mainly derived from the balance of the National Product account. The next four—KuV, W, D, and Gs—are all very difficult to define and to estimate. The last, Ex–Im, had to be computed from statistics that were very poor during the early fifties for the FRG.
3. The effect of revisions on the level of global aggregates such as items 1, 2, and 3 in Tables 2 and 3 is small in comparison with the effect on the various components—a phenomenon often taken as evidence of compensating errors. This interpretation is strictly valid, however, only when it refers to that part of the error caused by deficiences at the time of first

estimation; it need not be valid for the whole error of estimation defined as the difference between the estimated and the true, but unknown value.

4. Conclusions about stability as defined in Section III above can be drawn by comparing columns 4 and 7 of Tables 2 and 3. The only item that is absolutely stable is Wages. Regarding the last revision (Table 3) the following items are not stable at all: LuF, WG, H, W, KuV, Cst, Iv, Ex–Im, D, and Gu.

Revisions in the Linear Annual Change of National Income Estimates

There are two ways of defining a first estimate of linear annual change as is shown in the following table.

Period of Estimation	Period of Reference 1	2	3	4	5
2	x_{21}	—	—	—	—
3	x_{31}	x_{32}	—	—	—
4	x_{41}	x_{42}	x_{43}	—	—
5	x_{51}	x_{52}	x_{53}	x_{54}	—
6	x_{61}	x_{62}	x_{63}	x_{64}	x_{65}

The first estimates used here for linear annual change in x are the differences $(x_{32}-x_{21})$, $(x_{43}-x_{32})$, $(x_{54}-x_{43})$, and so on. In the literature on this subject one often finds another definition of first estimate of linear annual change: $(x_{32}-x_{31})$, $(x_{43}-x_{42})$, $(x_{54}-x_{53})$, and so on. This last definition has the advantage that the difference, (in the form $x_{tj}-x_{t,j-1}$), can be computed from the data in the same issue of the source publication; it has the disadvantage that the subtrahend $x_{t,j-1}$ normally has already undergone a revision, so that the difference $x_{tj}-x_{t,j-1}$ is not a pure first unrevised estimation of change.[5]

The measures describing the effect of revision of linear annual change are similar in construction to those on pages 235-236.

$$\bar{a} = \frac{1}{n-1}\sum_{i=2}^{n}[(R_i-R_{i-1})-(E_i-E_{i-1})], \tag{5}$$

$$\bar{\epsilon} = \frac{\sum_{i=2}^{n}[(R_i-R_{i-1})-(E_i-E_{i-1})]}{\sum_{i=2}^{n}(R_i-R_{i-1})}, \tag{6}$$

$$\bar{a}' = \frac{1}{n-1}\sum_{i=2}^{n}|(R_i-R_{i-1})-(E_i-E_{i-1})|, \tag{7}$$

[5]The author is quite aware of the fact that a great part of his readers would have chosen the second definition of linear annual change, for this is the only one economists and politicians care about. Our decision was influenced by arguments lying in the sphere of definitions as indicated above.

$$\bar{\epsilon}' = \frac{\sum_{i=2}^{n} |(R_i - R_{i-1}) - (E_i - E_{i-1})|}{\sum_{i=2}^{n} |R_i - R_{i-1}|}. \tag{8}$$

Substituting R_i by R_i^* in (5) to (8) gives $\bar{a}^*$, $\bar{\epsilon}^*$, $\bar{a}'^*$, and $\bar{\epsilon}'^*$. Formulas (5) and (6) measure direction and amount of bias in the first estimate of linear annual change; formulas (7) and (8) measure dispersion.

If one carries out the computations one will see that there is a slight dominance of negative signs, especially for the global aggregates. This means that, on the average, the annual change—as defined here—is overestimated at first, in contrast to the underestimation exhibited in Tables 2 and 3. The computations will also show a frequent change in signs between first and last revision—also in contrast to Tables 2 and 3. Stability of estimated change turns out to be much lower than that in Tables 2 and 3.

VII

As a statistical measure of the agreement between the first estimate and some revision, the ordinary correlation coefficient (Bravais-Pearson) is hardly satisfactory because it measures conformity to *any* linear relation between the estimate and the revision. Far better is H. Theil's[6] coefficient of inequality:

$$U = \frac{\sqrt{1/n \sum (R_i - E_i)^2}}{\sqrt{1/n \sum R_i^2} + \sqrt{1/n \sum E_i^2}}, 0 \leqslant U \leqslant 1, \tag{9}$$

which measures lack of conformity to the linear relation implying perfect agreement between E_i and R_i (or R_i^*)—that is,

$$E_i = R_i \quad \text{or} \quad E_i = R_i^*, \tag{10}$$

which is represented graphically by a 45°-line through the origin. When $E_i = R_i$ for all i, U = 0; when $R_i = -\alpha E_i$ for all i and $\alpha > 0$, $U = 1$. (One may, of course, substitute R_i for R_i^* in all equations which follow.)

Theil's inequality coefficient has been frequently used in econometrics to appraise the quality of forecasting by measuring the disagreement between the forecast and the actual values subsequently realised. But it does more than this. Its square can be decomposed into components associated with various causes of discrepancy.

Although several decompositions of U^2 are possible, one is particularly appropriate for this analysis. If U^2 is written N^2/D^2, then the following is an algebraic identity easily verified:

$$U^2 = N^2/D^2 = 1/D^2[(\bar{R} - \bar{E})^2 + (s_R - s_E)^2 + 2(1-r)s_E s_R] \tag{11}$$

[6]H. Theil, *Economic Forecasts and Policy*, Amsterdam, 1958, pp. 31ff.

where $\bar{R}$ and $\bar{E}$ are the arithmetic means of E_i and R_i, where s_E and s_R are standard deviations defined by

$$s_E^2 = \frac{1}{n}\sum_{i=1}^{n}(E_i-\bar{E})^2 \quad \text{and} \quad s_R^2 = \frac{1}{n}\sum_{i=1}^{n}(R_i-\bar{R})^2$$

and where r is the Pearsonian correlation coefficient between E_i and R_i. If (11) is divided through by U^2, to reduce the components to fractions of the whole, the following results:

$$1 = 1/N^2[(\bar{R}-\bar{E})^2+(s_R-s_E)^2+2(1-r)s_E s_R] \tag{12}$$
$$= U^{(m)}+U^{(s)}+U^{(k)}.$$

In (12) the fractional component $U^{(m)}$ represents any bias in the differences (R_i-E_i)—a bias that would arise if national income statisticians generally tended to make their first estimates either too low or too high. The fraction $U^{(s)}$ represents any consistent difference in the variability of the two series; this is a more subtle bias than that in the differences (R_i-E_i). Finally, the residual

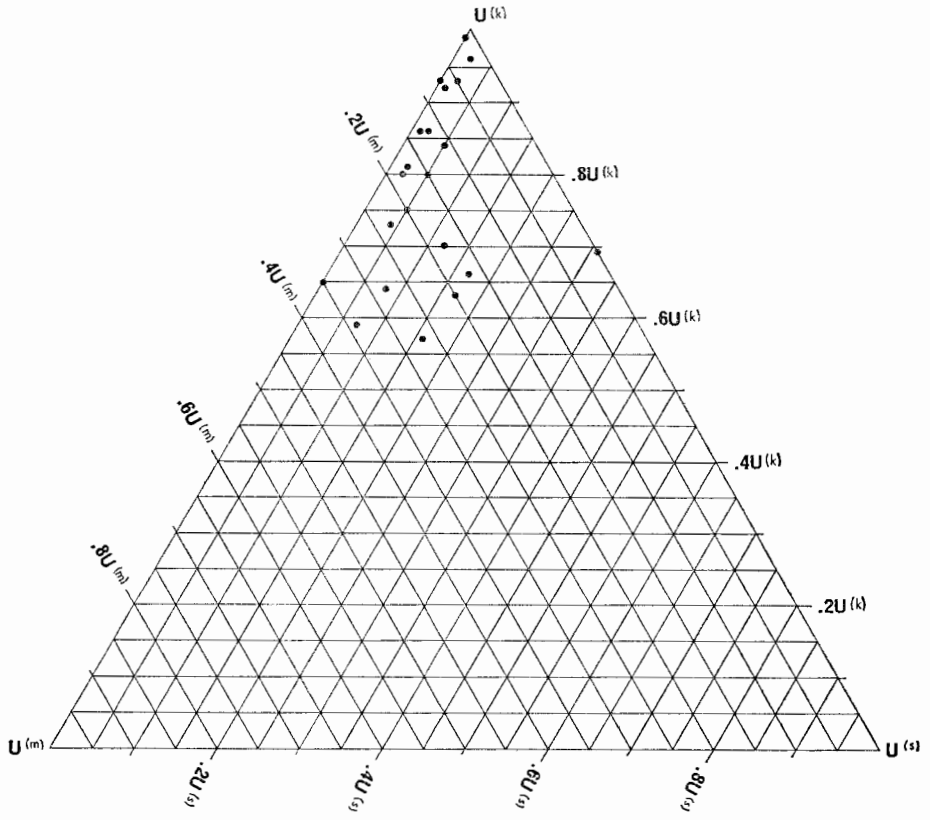

Figure 1

fraction $U^{(k)}$ represents the unsystematic differences between R_t and E_t, which is due to imperfect covariation in both series.[7]

We do not want to reproduce the numerical values obtained and to discuss them in detail, but rather to look for any tendencies toward bias in the Federal Statistical Office's techniques of forming preliminary estimates and then revising them. For this purpose we plotted the components of U^2 in (12) on triangular graph paper, which is superbly suited for representing three quantities necessarily adding to unity.

When all the points are concentrated around one apex of the triangle, the corresponding component dominates. There is a close concentration around $U^{(k)}$ in Figure 2, for example, and a somewhat weaker concentration in Figure 1.

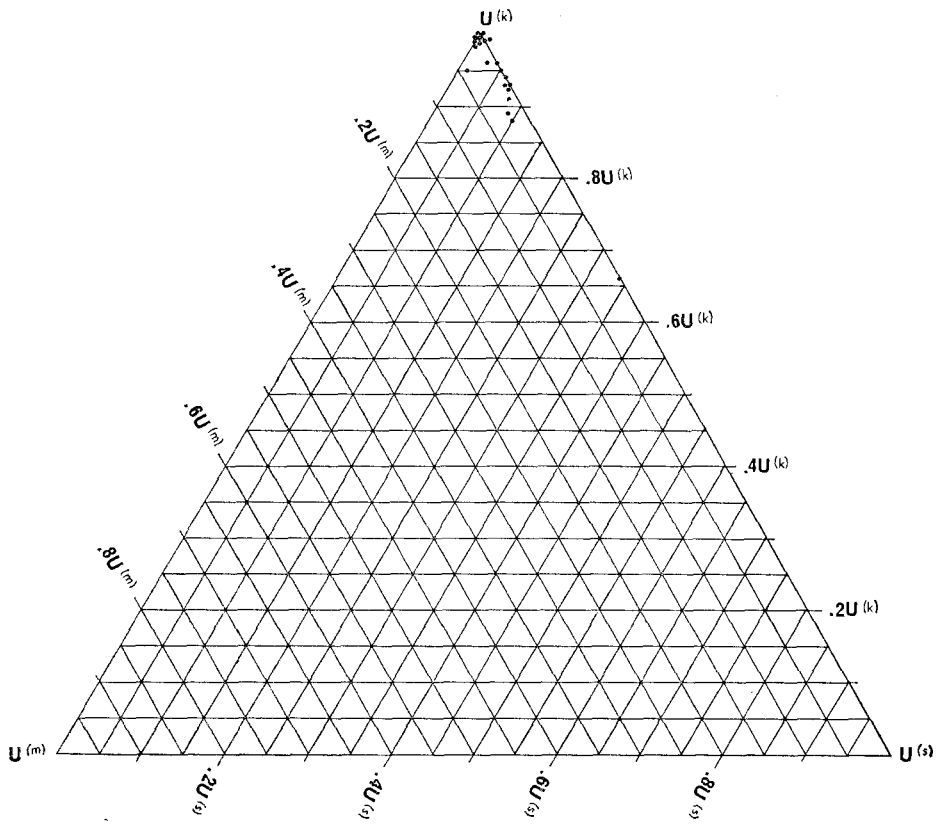

Figure 2

[7]Another decomposition of U^2 is:

$$1 = 1/N^2[(\bar{R}-\bar{E})^2+(s_E-r\cdot s_R)^2+(1-r^2)s_R{}^2]. \tag{13}$$

The first term in (13) has the same meaning as $U^{(m)}$ in (12) above, the second term measures the deviation between the actually found slope of the regression line of R_t on E_t and the ideal slope the value of which is unity. The last term measures the variation about the line of regression. A transformation like this makes sense only if one can assume the existence of an equation of regression between R_t and E_t; but as both, R_t and E_t, are random variables a regression analysis is not justified mathematically.

When the points lie along one edge and close to it, then two components dominate jointly. In Figure 4 the points are strung out along the $U^{(k)}$–$U^{(s)}$ edge, so that these two components dominate jointly (with $U^{(k)}$ somewhat more important); the $U^{(m)}$ component at the opposite apex plays virtually no role at all. An equal distribution of points across the triangle means that if one takes all items together no component is of major importance.

There is a marked difference between Figures 1 and 2 on the one hand and Figures 3 and 4 on the other, as well as between Figure 3 and Figure 4. In Figures

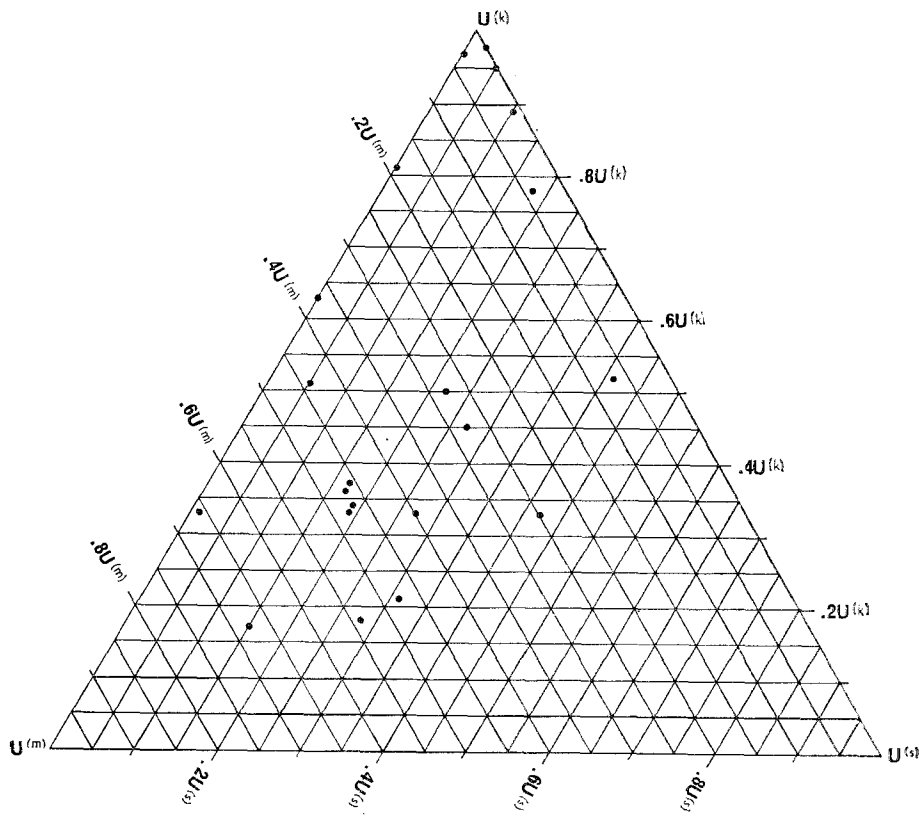

Figure 3

1 and 2, the unsystematic component $U^{(k)}$ dominates, implying lack of systematic tendencies in the preparation of the first revisions. But in Figures 3 and 4, which refer to last revisions, there is evidence of systematic components. Particularly in Figure 3 the component $U^{(m)}$ plays a significant role, thus suggesting systematic differences between the averages of the two series. Figures 3 and 4 seem to confirm the author's hypothesis on revision policy of national accounts statisticians, as follows:

If the estimate of a certain item for a certain period is revised considerably, as it may be in general revisions (Figures 3 and 4), this revision is accompanied by other revisions. First of all the estimates of that item for adjacent years have to be

raised or lowered to eliminate suspicious-looking jumps or discontinuities. Evidence in favour of such an interpretation would be a large value of $U^{(m)}$, or even a relatively small value of $U^{(s)}$. Up to here the interpretation is valid only for the position of an individual point in the triangle, but not for the population of points. The position of the population may be explained by a cross-section

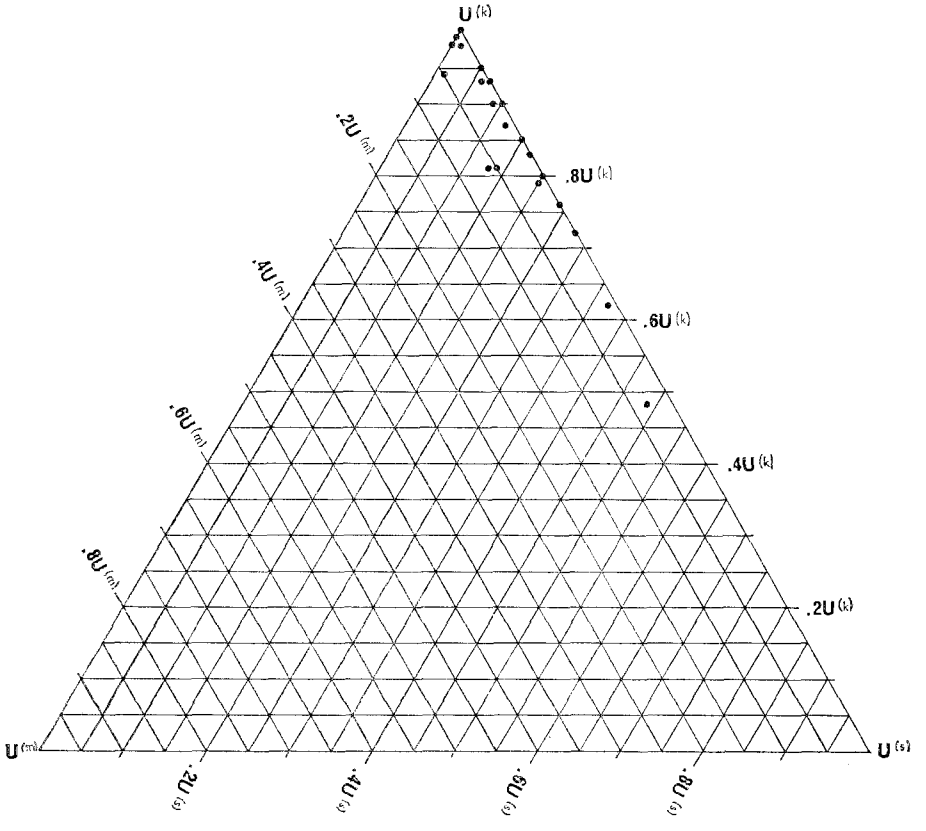

Figure 4

analysis. A substantial revision of the estimate of one aggregate in national income accounts calls for a similar revision of other aggregates because a considerable short-run shift in the structure of Gross National Product seems improbable. The composition of Gross National Product in the short run is relatively stable, so that a parallel shift of the estimates for one aggregate causes a similar parallel shift in other ones.

If, as is maintained here, shifts in the values of the different items of Gross National Product are more or less parallel, the shifting will not exert a systematic effect on the apparent growth of these items because growth, at least in this study, is always measured by linear annual change and is not expressed as percentage. So all items in Figure 4 are situated near $U^{(k)}$, thus indicating irregularity in preparing revisions of annual change.

The interpretation or hypothesis developed here explains *uno acto* the observations made in Figures 3 and 4. It would be of interest to see whether revisions of national accounts estimates of other countries behave like those of the FRG or not.

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