A METHOD OF ANALYSING THE CONSISTENCY OF TIME SERIES FOR CAPITAL AND INVESTMENT*

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When independent time-series for capital and investment are used in econometric analyses it is important to know if the two sets of data are consistent, if the reported investments can "explain" the growth in capital when the other factors that also affect the capital stock are taken into account.[1] This note presents a method for the analysis of such a question and applies it to capital and investment data for Norwegian Mining and Manufacturing at the two digit level and the years 1951–1959.

The change in capital value during a particular period can be thought of as consisting of three elements; gross investment, depreciation, and price change. We can, therefore, write:

$$(1) \qquad K_{i,t} - K_{i,t-1} = J_{i,t} - \Delta_{i,t}K_{i,t-1} + \eta_{i,t}K_{i,t-1}$$

where $K_{i,t-1}$ and $K_{i,t}$ are the values of the capital stock at the beginning and at the end of the year respectively, $J_{i,t}$ is gross investment during the year, $\Delta_{i,t}$ is depreciation ratio, $\eta_{i,t}$ is the price change ratio, and i and t are the industry and time subscripts respectively. If everything in this equation were measured correctly it would be an identity in all the variables.[2]

If one had independent information about the appropriate depreciation and price-change ratios, one could compute the right side of relation (1) and thus have a direct check of the consistency of the two (capital and investment) sets of data provided, of course, that the depreciation and price change ratios were correct. Since this last requirement may not be fulfilled, one may prefer an approach which does not depend on *a priori* knowledge of these ratios, allowing the data to determine them instead.

If the depreciation ratio and the price change ratio were to vary along both of the available sample dimensions—industry and time—we would not have enough degrees of freedom to compute all of the ratios on the basis of the data available to us. We make, therefore, what we believe are reasonable restrictions on these parameters and assume that: (a) depreciation ratios are independent of time but they may be different for different industries, and (b) price-change ratios are independent of industry but may be different for different years.

Dividing through by $K_{i,t-1}$ and introducing the following dummy variables:

$$y_j = 1 \text{ when } j = i, \; y_j = 0 \text{ otherwise}$$

$$z_\tau = 1 \text{ when } \tau = t, \; z_\tau = 0 \text{ otherwise}$$

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[1]This problem does not arise often. Usually one of the series, e.g., "capital", is "manufactured" from the investment data, as in the perpetual inventory approach, and the identities are satisfied provided no computational errors were made.

[2]We presume that investment expenditures are reported on the basis of original costs, that is: No depreciation or price change on capital that is less than one year old. This seems to be the common way of measuring investment expenditures, and it corresponds to the definition of investment in the data we are going to use.

we can write relation 1 in the following way:

$$(2) \qquad \frac{K_{i,t} - K_{i,t-1}}{K_{i,t-1}} = \alpha \frac{J_{it}}{K_{i,t-1}} - \sum_{j=1}^{I} \Delta_j y_j + \sum_{\tau=1}^{T} \eta_\tau z_\tau \qquad \begin{array}{l} i = 1 \ldots I \\ t = 1 \ldots T \end{array}$$

where I is the number of industries and T is the number of years in our sample.

We have allowed the coefficient of $J_{i,t}/K_{i,t-1}$ to differ from one in (2), both because we have made simplifying assumptions about the depreciation and price-change ratios and because there may be errors of measurement present in both the capital and investment data sets.

We shall estimate the parameters of this relation using ordinary least squares procedures.[3] Since the simplifying assumptions we made are unlikely to lead to any systematic bias in the estimate of α, we shall argue that the capital and investment data are inconsistent if α is significantly different from one.

As mentioned above, we are applying this procedure to industry data in Norwegian Mining and Manufacturing. They are taken from the Central Bureau of Statistics' Industrial Production Statistics, Annual Survey. Between 1949 and 1950 there is a "break" in the data due to a revision of the lower bound for the size of the establishments included in the annual statistics, and 1959 was the last year in which the capital data were collected. We have then data for nine years; 1951 through 1959.[4] In our analysis we have twenty-one industries, based on the two digit ISIC code.[5]

The Industrial Production Statistics for the years under consideration provide data on the full fire insurance value and on investment expenditures for three types of capital: Buildings, Other Construction, and Machinery. We have estimated relation (2) for Buildings and Machinery separately, and for Total Capital consisting of all three types of capital mentioned.

The estimates of α are presented in Table 1a. Since the results for the industry dummies indicated that there were few significant differences between the depreciation ratios for different industries, we also estimated relation (2) assuming the same depreciation ratio for all industries. The main effect of this is a reduction in the estimated standard deviation of α.

The conclusion from both sets of results is that the capital—and investment—data are *not* consistent either for Buildings and Machinery or for Total Capital, since in all cases except one we can reject the hypothesis that $\alpha = 1$.[6]

What, then, is wrong with these data? We know that there have been some minor changes in the lower bound on the size of establishments included in the annual survey, and also some regrouping between two-digit industry groups during the period under consideration. This is reflected in the relatively poor fit of the estimated relation and it might also have had a systematic effect on the estimate of α. But it is difficult to believe that this is the only cause of our findings of inconsistency.

[3]We have to exclude one y-variable and one z-variable to avoid singularity. This implies that we cannot identify the different industry depreciation ratios or the price-change ratios of different years without additional information—such as the depreciation ratio of one industry or the price-change ratio for one year. But using the dummy-variables method we can detect and allow for *differences* in depreciation ratios between industries and in price-change ratios between years.

[4]Since the data on capital at the beginning of 1950 are before the "break", this year is dropped from the analysis. The data on capital at the end of 1950 (at the beginning of 1951) and investment during 1950 are after the "break" and hence usable.

[5]Groups 11–19, Mining and Quarrying, are considered as one industry. The twenty two-digit manufacturing industry groups 20 through 39 are each considered as one industry.

[6]At the 5 per cent level. The hypothesis is not rejected for Buildings when industry dummies are included. But since the hypothesis is rejected when these dummies are excluded and since the "acceptance margin" is very slight the conclusion of inconsistency appears to be valid also for Buildings.

Since the capital stock data are "full fire insurance values", the inconsistency could be due to a "lag" effect; it may take some time before investment expenditures are "registered" as stocks of capital. If this conjecture is correct we would expect a positive and significant coefficient for lagged investment, both when it is included in relation (2) *together* with unlagged investment and when it is introduced *instead* of current investment. The results of these two tests are presented in Tables 1b and 1c, respectively. They indicate rather clearly that the coefficient of the same year's investment is not significantly different from zero for any type of capital when lagged investment is included, and that the coefficient of lagged investment is not significantly different from one whether it is included alone or together with unlagged investment.[7]

TABLE 1

ESTIMATES OF A RELATION EXPLAINING THE RELATIVE GROWTH IN REPORTED CAPITAL VALUES*

Table 1.*a*

	Buildings		Machinery		Total Capital	
$\frac{J_{i,t}}{K_{i,t-1}}$	0.558 (0.243)	0.588 (0.162)	0.132 (0.063)	0.223 (0.052)	0.089 (0.053)	0.167 (0.046)
Dummies for years	Yes	Yes	Yes	Yes	Yes	Yes
Dummies for industries	Yes	No	Yes	No	Yes	No
Intercept	0.016	0.018	0.070	0.024	0.072	0.040
R	0.474	0.396	0.485	0.426	0.554	0.459
M.SQ	0.013	0.013	0.024	0.023	0.007	0.007

Table 1.*b*

	Buildings		Machinery		Total Capital	
$\frac{J_{i,t}}{K_{i,t-1}}$	0.168 (0.240)	−0.026 (0.210)	0.008 (0.069)	0.043 (0.063)	−0.024 (0.055)	0.005 (0.051)
$\frac{J_{i,t-1}}{K_{i,t-1}}$	1.360 (0.275)	1.033 (0.239)	1.240 (0.319)	0.884 (0.192)	1.005 (0.222)	0.826 (0.142)
Dummies for years	Yes	Yes	Yes	Yes	Yes	Yes
Dummies for industries	Yes	No	Yes	No	Yes	No
Intercept	−0.021	0.004	−0.014	−0.036	0.010	−0.008
R	0.573	0.486	0.549	0.519	0.621	0.580
M.SQ	0.012	0.012	0.022	0.020	0.006	0.006

*Footnote overleaf.

[7]Using *F*-statistics with 20 and 159 degrees of freedom based on the results of relation 2 and the results of this relation when assuming a common depreciation rate for all industries we cannot reject the hypothesis of a common depreciation rate at 5 per cent level, either for Buildings, Machinery or Total Capital. The results are the same when lagged investment is substituted for current investment. This corresponds quite well with other evidence on deprecation rates, suggesting that at the two digit level and during this period the differences among such rates were rather insignificant in Norwegian Mining and Manufacturing industries.

Table 1.c

	Buildings		Machinery		Total Capital	
$\frac{J_{i,t-1}}{K_{i,t-1}}$	1.423 (0.260)	1.013 (0.176)	1.257 (0.282)	0.966 (0.150)	0.963 (0.198)	0.834 (0.119)
Dummies for years	Yes	Yes	Yes	Yes	Yes	Yes
Dummies for industries	Yes	No	Yes	No	Yes	No
Intercept	−0.013	0.003	−0.014	−0.036	0.010	−0.008
R	0.571	0.486	0.549	0.517	0.621	0.580
$M.SQ$	0.011	0.011	0.022	0.020	0.006	0.006

*Yes means that the dummy variables concerned are included in the regression. No means that the dummy variables concerned are not included in the regression.

The intercept is the sum of the coefficients of the two dummy variables excluded from the regression (see footnote 3), that is $-\Delta_I+\eta_T$ where Δ_I is the depreciation ratio of industry 39, Miscellaneous manufacturing industries, and η_T is the price change ratio of the year 1959. When industry dummies are not included in the regression the intercept is $-\Delta+\eta_T$ where Δ is the common depreciation ratio.

R is the multiple correlation coefficient and $M.SQ$ is the mean square value of the estimated residual.

These findings imply strongly the existence of a lag between the purchase of a capital object and its emergence as a part of the capital stock. According to our results this lag is more than one year on the average.[8]

Thus, we conclude that after all, the consistency between the capital and investment data sets is not as poor as the first results for relation (2) indicated. We do not have consistency between the change in capital in a particular year and the investment expenditures of the same year, but we have consistency between the change in capital and the investment expenditures of the previous year. Taking this into consideration when applying these data in contexts where consistency is important, they should for most purposes be as good as any other sets of data on capital and investment.

[8]There are probably two major sources of the observed lag between investments and growth in capital stock: (a) While all investment costs of a year are reported, the value of uncompleted investment projects at the end of the year is not reported as part of the capital stock. (b) There may be a general sluggishness in the adjustment of "full fire insurance value" which, as pointed out, is the measure of the current value of the capital stock. If the latter cause is dominating we would expect the estimated price-change ratios to show a lag also, compared with the price-change ratios implied by a current price index of capital.

To investigate this we computed the price-change ratios for Total Capital from the relation with lagged investments instead of current investments and a common depreciation rate for all industries. We cannot identify the *level* of the price-change ratios, by our method of estimation, but this does not matter in this context. These estimates were compared with the price-change ratios implied by a price index for Total Capital of the Mining and Manufacturing industries. The latter index is based on price indices for different categories of gross investment chained together with the amounts of corresponding categories of capital as weights. This comparison gives an indication of a lag of about one year between the two sets of price-change ratios in the period 1951 through 1953, while for the following years they have fairly similiar movements year by year. Thus, this comparison does not provide particular support to either of the two main causes of lag mentioned. There is a slight suggestion of a twist of the relative importance over time of the two causes—the effects of "sluggishness" are reduced in relation to the effects of "incompleted investment projects." The basis for this suggestion is, however, rather weak and it is difficult to find any clear evidence of it from other sources.