DEPRECIATION, OBSOLESCENCE, AND THE MEASUREMENT OF THE AGGREGATE CAPITAL STOCK OF THE UNITED STATES 1900–1962

By Helen Stone Tice

Mount Holyoke College

This paper is concerned with the sensitivity of estimates of the aggregate capital stock of the United States to the statistician's choice of depreciation method. The usual depreciation charge can be shown to include allowances both for physical deterioration and for obsolescence. If one interprets the gross stock as the stock of surviving assets, then the various net stocks defined by depreciation accounting may be interpreted as a revaluation of these assets by means of an index of embodied technical change. Estimates of the United States capital stock were generated under eight sets of assumptions. These estimates are compared with respect to level, trend, and implications for other aggregate statistical indicators. The conclusion is reached that the assumptions which define a country's stock of tangible capital are of considerably greater importance than has often been supposed.

I. Introduction

In recent years there has been a growing interest in obtaining and in using estimates of the stock of tangible capital. Such data provide a natural complement to the income and product flows to which we have become accustomed; and they have been widely applied in studies of productivity, in the analysis of the market for both capital and consumer goods, in financial analysis, in studies of aggregate economic structure, and in the analysis of and planning for economic growth.[1]

With the growing analytical interest in stock concepts there has come a growing concern for the further development of stock measurements. The conference volume of the 1957 meeting of the International Association for Research in Income and Wealth contains a summary of the wealth estimates then available. Eighteen countries were represented, and most of these data were benchmark estimates derived by a varied assortment of methods.[2] By

1. Tibor Barna, "Alternative Methods of Measuring Capital," in *Income and Wealth*, Series VIII (Chicago: Quadrangle Books, 1959), pp. 36–41; Richard and Nancy Ruggles, "Concepts of Real Capital Stocks and Services," in *Output, Input, and Productivity Measurement*, Studies in Income and Wealth, Volume Twenty-five (Princeton: Princeton University Press, 1961), p. 387; Wealth Inventory Planning Study, *Measuring the Nation's Wealth: Materials developed by the Wealth Inventory Planning Study*, The George Washington University (Washington, D.C.: U.S. Government Printing Office, 1964), pp. 15–18.

2. R. W. Goldsmith and Christopher Saunders, "A Summary Survey of Wealth Estimates," in *Income and Wealth*, Series VIII, pp. 1–6.

contrast, the 1965 United Nations proposals for revising international standards in national accounting included explicit recommendations that capital consumption allowances be estimated at straight-line depreciated replacement cost and further indicated that national and sectoral balance sheets were to be included in the new system.[3] In the United States the work of private estimators has been augmented recently by studies undertaken by the Office of Business Economics; and the report of the Wealth Inventory Planning Study addressed itself directly to the problems and to the best procedures involved in incorporating wealth estimates into the regular federal statistical program.[4]

There is an extensive literature, both theoretical and technical, on the subject of measuring capital.[5] Despite the desirability of estimates based on census or survey methods, in order to construct a time series it is generally necessary to employ the perpetual inventory method in which the stock is taken to be equal to the sum of net (or gross) investment over a suitable period of years. The reliability of such estimates depends on the quality of the investment data being cumulated, on the accuracy of the service life assumption, and on the validity of the method by which assets are assumed to depreciate over their service lives.

It is primarily to questions relating to the last of these that this paper is directed; it attempts to assess the impact of the assumptions made about the pattern of physical decay and obsolescence both on the estimates of the value of depreciable assets and on the results of analyses in which such estimates are used as inputs. These assumptions affect not only the stock estimates but also the estimates of capital consumption allowances derived from them; thus, to the extent that these statistics are sensitive to the choice of assumptions, there

3. United Nations Economic and Social Council, *A System of National Accounts (Proposals for the Revision of SNA, 1952)* (E/CN.3/320, 9 February 1965) (New York: 1965).

4. See R. W. Goldsmith, *A Study of Saving in the United States* (3 vols.; Princeton: Princeton University Press, 1955–1956); and *The National Wealth of the United States in the Postwar Period* (Princeton: Princeton University Press, 1962). Also see Simon Kuznets, *Capital in the American Economy: Its Formation and Financing* (Princeton: Princeton University Press, 1961); and the works summarized in Daniel Creamer, "An Appraisal of Long-Term Capital Estimates: Some Reference Notes," in *Output, Input, and Productivity Measurement, op. cit.*, pp. 413–444. For more recent work see *Measuring the Nation's Wealth, op. cit.*, p. xvl, and George Jaszi, Robert C. Wasson, and Lawrence Grose, "Expansion of Fixed Business Capital in the United States: Rapid Postwar Growth—Rise Slackens," *Survey of Current Business* (November 1962), pp. 9–18, 28.

5. See *Output, Input, and Productivity Measurement, op. cit.*, papers by Griliches, Goldsmith, Nutter, Domar, Ruggles and Ruggles, and Denison; F. A. Lutz and D. C. Hague, Eds., *The Theory of Capital* (London: Macmillan, 1963), papers by Barna and Hague; *Income and Wealth*, Series VIII, *op. cit.*, papers by Barna, Goldsmith and Saunders, and Aukrust and Bjerke; *A Critique of the United States Income and Product Accounts, Studies in Income and Wealth*, Volume Twenty-two (Princeton: Princeton University Press, 1958), papers by Jaszi and Schiff; *Measuring the Nation's Wealth, op. cit.*, pp. 75–84, 229–290; Goldsmith, *The National Wealth . . . , op. cit.*, p. 3; Kuznets, *op. cit.*, pp. 58–60; E. F. Denison, *The Sources of Economic Growth in the United States and the Alternatives Before Us*, Supplementary Paper No. 13 (New York: Committee for Economic Development, 1962), pp. 94–98; R. M. Solow, *Capital Theory and the Rate of Return* (Amsterdam: North Holland, 1963), pp. 13–14; J. W. Kendrick, *Productivity Trends in the United States* (Princeton: Princeton University Press, 1961), pp. 35–36; J. W. Kendrick, "Some Theoretical Aspects of Capital Measurement," *American Economic Review*, LI (May 1961), pp. 102–105; Eugene Grant and Paul T. Norton, *Depreciation* (New York: Ronald Press, 1955), pp. 11–18, 33–41, 269–281.

exists the real possibility that much of the aggregative statistical description of an economy is determined by the assumptions made by statisticians or by enterprise accountants in defining the book, i.e. depreciated, value of tangible capital.

II. Theoretical Relationships in the Measurement of Capital

A. *Capital as a Weighted Sum of Past Investments*

It is obvious that whatever else one may say about "the stock of capital"—whatever magical properties it is assumed to possess—its value as measured is simply a weighted sum of past gross investment flows. This is true certainly of any perpetual inventory calculation, and it should be apparent that this is in fact the process by which the responses to census and survey questions have been derived. Since this is the case, differences among estimates derived under the various definitions of *the* capital stock can be analysed in terms of the differences in the weights attached to the gross investment of a given year or a given period of years.

These weights are the product of two factors. The first of these is essentially the probability that an asset of a given age will still be in existence in the year for which the stock is being calculated. Thus this component of the weight gives the fraction of the gross investment undertaken "j" years ago which is still physically present; the weights for the full range of years constitute, of course, a survivor curve, and the difference in weights for adjacent years represents depreciation in the sense of physical decay, retirement, etc.

The second component of the weight had to do with the fraction of their original value retained by those items still physically present in the stock. If the value of this element is one for all years, then we are of course dealing with a measure of the gross stock; if the asset is included, it is included at its full original value. If, on the other hand, this factor has a value of less than one, we are dealing with a net stock notion in which allowance has been made for obsolescence.

It is obviously desirable to be able to distinguish the allowance for physical decay from the revaluations caused by obsolescence, market forces and the like. Solow's surrogate capital stock and embodiment hypothesis attempt to do this by assuming that technical progress embodied in investment goods proceeds at a constant rate over time independently of depreciation.[6] It will be demonstrated below that an analogous interpretation can be given the results of the depreciation accounting methods currently permissible under U.S. income tax regulations; thus some conclusions can be reached about the rate of embodied technical change implicit in business accounting practice and tax law, and thus in the capital stock estimates based upon them.

Parenthetically, the embodiment hypothesis is a most adaptable concept. In a recent paper, Jorgenson demonstrated that the index of the quality of investment goods and thus of embodied technical change has the same computational characteristics—"factual implications"—as errors of measurement in

6. Solow, *op. cit.*, Ch. II, III.

the price of investment goods.[7] Since that which we call depreciation can also be related neatly to embodiment, it is clear that any weighting system is susceptible to a wide range of possible interpretations, and that a given capital estimate is also able to support quite a variety of hypotheses about economic relationships.

We may summarize the nature of the calculations involved in measuring the stock of capital in the following fashion:

Let I_t = gross investment during year t in current prices
P_t = price index of investment goods during year t
K_t = stock of capital at end of year t valued at original cost
D_t = depreciation charges during year t valued at original cost
R_i = per cent of asset's original value remaining at the end of its ith year of life
d_i = rate of depreciation during the ith year of an asset's life
n = the number of years over which the cumulation takes place
$\overline{K}_t$, $\overline{D}_t$ = capital stock and depreciation valued at constant cost
$\overline{\overline{K}}_t$, $\overline{\overline{D}}_t$ = capital stock and depreciation valued at replacement cost

Then

$$K_t = \sum_{i=1}^{n} R_i I_{t+1-i} \tag{2.1}$$

$$D_t = I_t - (K_t - K_{t-1}) \tag{2.2}$$

$$\overline{K}_t = \sum_{i=1}^{n} R_i (I/P)_{t+1-i} \tag{2.3}$$

$$\overline{D}_t = (I/P)_t - (\overline{K}_t - \overline{K}_{t-1}) \tag{2.4}$$

$$\overline{\overline{K}}_t = \overline{K}_t * (P_t + P_{t+1})/2 \tag{2.5}$$

$$\overline{\overline{D}}_t = \overline{D}_t * P_t \tag{2.6}$$

Thus the stock estimates require data on I_t and P_t, some assumption about "n", and the form of R_i which embraces the two aspects of the weight previously discussed, the survivor curve and the revaluation of assets as the result of obsolescence. It is obvious that the original cost and constant cost procedures are the same, given the proper gross investment series; hereafter, although the discussion will be given in original cost terms, it is understood that the derivation can be applied to constant cost estimates as well. Notice that depreciation is defined as the difference between gross and net investment during the year, with net investment being measured by the first difference of the stock.

The replacement cost calculation is simply a reflation of the constant cost estimate; thus it is a measure of the cost of duplicating the asset itself, not of duplicating its function except insofar as R_i revalues old assets in terms of the latest technique. We shall now define R_i in somewhat greater detail.

7. D. W. Jorgenson, "The Embodiment Hypothesis," *Journal of Political Economy*, LXXIV (February 1966), pp. 8–9.

1. The Survival Assumptions

The traditional assumption made on this score is that of the rectangular distribution of survivors; an asset is assumed to have a fixed life span at the end of which the original investment ceases to exist, though none of it has previously departed. This is of course more familiarly known as the one-horse-shay method of depreciation accounting.

There is, however, the existence of the mortality distribution to contend with. It is quite likely that the maximum realized life will be at least twice the mean life, especially if the asset accounts are nonhomogeneous (and they are *very* nonhomogeneous in calculations of this type).[8] If the mortality distribution is normal, then an estimate of the gross stock based on a one-horse-shay assumption and employing the mean life will be less than an estimate which takes account of the full distribution. The discrepancy will not exceed 5 per cent, however, and it will be less than 3 per cent if the product of the rate of growth of investment and the average life is below 0.5 or above 4.5.[9]

Statistical procedures are often used in estimating average service lives; the logistic curve has been found to describe some observed survivor curves quite well.[10] Barna, whose study included questions on the mortality experience of his respondents, concluded that his survival curve could be described as well by a linear function as by any other, though since he had little information on higher age ranges and since the curve started from a value of 100 per cent at ages ranging from three to five years, he was unwilling to rule out the logistic curve.[11]

In recent years, however, exponential depreciation has become increasingly popular in theoretical models.[12] This assumption has great analytic convenience, since it makes the stream of production possibilities implicit in the stock independent of its age structure.[13] Under this assumption a fixed fraction, "d," of the stock of capital vanishes each year; thus if "d" is a mortality rate the average lifetime of capital goods is $(1/d)$ years.[14] This implies that

$$(2.7) \qquad K_t = I_t + (1 - d)K_{t-1}$$
$$= I_t + (1 - d)I_{t-1} + (1 - d)^2 I_{t-2} + \ldots + (1 - d)^t K_0$$

This is also familiar; it is in fact a version of declining balance depreciation accounting in which the undepreciated balance at the end of the mean service

8. Grant and Norton, *op. cit.*, p. 408.

9. Eric Schiff, "Gross Stocks Estimated from Past Installations," *Review of Economics and Statistics*, XL (May 1958), p. 176, cited in Goldsmith, *The National Wealth . . .* , *op. cit.*, p. 26.

10. Grant and Norton, *op. cit.*, pp. 44–86, contains a discussion of statistical mortality studies.

11. Barna, *op. cit.*, "On Measuring . . . ," pp. 85–89.

12. See, for example, James Meade, *A Neoclassical Theory of Economic Growth* (London: Macmillan, 1961); Edmund S. Phelps, "The New View of Investment: A Neoclassical Analysis," *Quarterly Journal of Economics*, LXXVI (November 1962), pp. 548–567; Paul A. Samuelson, "The Evaluation of 'Social Income': Capital Formation and Wealth," in *The Theory of Capital*, *op. cit.*, pp. 32-57; and R. M. Solow, *Capital Theory and the Rate of Return* (Amsterdam: North Holland Publishing Company, 1963).

13. Solow, *Ibid.*, p. 29.

14. Phelps, *op. cit.*, p. 554.

life is not written off; and thus no year's gross investment ever completely vanishes from the stock, though indeed its weight will in time become, to all intents and purposes, zero.

In the remainder of this study only two survival curves will be considered, the traditional rectangular or one-horse-shay assumption, and that provided by exponential or declining balance depreciation. For the first case "n" is taken to be "L," the service life in (2.1) and (2.3). Under the second assumption "n" is set equal to "t" and the value of the initial stock is assumed to be zero. This enables us to look at the conventional depreciation accounting methods as variations on these two assumptions, variations which are analogous to the assumption of an index of embodied technical change.

2. Definition of R_t: Depreciation Accounting

Four methods will be considered in this study: one-horse-shay, straight line, sum of digits, and declining balance at various multiples of the straight line rate. All are currently permissable for purposes of U.S. income tax accounting, and thus may be presumed to have some empirical validity as a description of actual accounting practice. To the extent that this is the case, the investigation of their consequences for stock estimation would shed some light on the implications of estimates of the stock of capital and of depreciation charges derived from accounting records.

The one-horse-shay method has already been discussed; it is the basis for the conventional definition of the gross stock. Straight line depreciation accounting is also familiar as the manner in which the aggregate net stock has been defined; although formerly quite common as a business accounting procedure, it has been losing favor in recent years as the tax advantages of more rapid amortization formulae have become more widely appreciated. There is also some evidence that it gives a poorer approximation to the market value of older assets than do the sum-of-digits and double-declining balance methods.

The chief advantage of the sum-of-digits methods lies in the rapidity with which an asset is written off; three fourths of the cost will have been depreciated by the time half the expected life has expired; it does not share the ease of computation or of interpretation exhibited by either the straight line or declining balance estimates. The declining balance rate is generally defined in terms of some multiple of the straight line rate. In addition, then, to declining balance at the straight line rate which defines one of our survival assumptions, we also employ multiples of this rate of 1.5 and 2.0, both of which are permissible for tax purposes.

Expressions for R_t for the depreciation accounting variants are shown in Table 2.1. The abbreviations which will be used henceforth are quite obvious.

OH = one-horse-shay

SL = straight line

SD = sum of digits

DM1 = declining balance at the straight line rate ($d = 1/L$)

DM5 = declining balance at 1.5 times the straight line rate ($d = 1.5/L$)

DM2 = double declining balance ($d = 2/L$)

TABLE 2.1

DEFINITION OF R_i FOR THE METHODS OF DEPRECIATION ACCOUNTING

Variant	R_i	Period
OH	1	$i = 1, \ldots, L$
SL	$1 - (2i - 1)/2L$	$i = 1, \ldots, L$
SD	$1 - \dfrac{L(2i-1) - (i-1)^2}{L(L+1)}$	$i = 1, \ldots, L$
DM1	$(1 - 1/2L)(1 - 1/L)^{i-1}$	$i = 1, \ldots, L, \ldots$
DM5	$(1 - 1.5/2L)(1 - 1.5/L)^{i-1}$	$i = 1, \ldots, L, \ldots$
DM2	$(1 - 2/2L)(1 - 2/L)^{i-1}$	$i = 1, \ldots, L, \ldots$

All are defined in terms of the half year convention, a further borrowing from accounting practice. All assets installed during the year are assumed to have been put in place on July 1 so that the depreciation charge in the first year is one-half the normal depreciation charge on an annual basis.

3. Definition of R_i: Embodied Technical Change

So far we have defined two possible gross stocks—OH and DM1. These are gross in the sense that under the assumptions about the mortality of capital goods, the only deductions made from gross investment are made because the assets in question have ceased to exist. There are for each of these gross stocks two net stocks; these are net in the sense that the deductions from gross investment reflect asset revaluation as well as physical depreciation. We can now round out our collection of net stock variants by defining for each gross stock a Solow-type surrogate capital stock incorporating a rate of embodied technical change.

If investment goods are assumed to improve in quality at a constant rate of λ per cent per annum, then $I'_t = I_t\,(1 + \lambda)^{t-c}$ where c is the year whose technology serves as the basis of the comparison. We can thus define the stock incorporating the rectangular survival assumption and embodied technical change, COH, as

$$(2.8)\qquad K_t = \sum_{i=1}^{L} I_{t+1-i}(1 + \lambda)^{t-c+1-i}$$

$$= (1 + \lambda)^{t-c} \sum_{i=1}^{L} I_{t+1-i}(1 + \lambda)^{-(i-1)}$$

Similarly the stock incorporating exponential depreciation and embodied technical change, CDB, can be defined as

$$(2.9)\qquad K_t = (1 + \lambda)^{t-c}(1 - 1/2L) \sum_{i=1}^{t} I_{t+1-i} \left\{\frac{(1 - 1/L)}{(1 + \lambda)}\right\}^{i-1}$$

In other words, the value of these stocks in a given year is the product of the stock measured in terms of that year's technology and of a power of $(1 + \lambda)$ which converts it to the technology of the base year. Therefore the multiplication of these time series by $(1 + \lambda)^{-(t-c)}$ will generate a series measuring the stock in terms of the technology of year t. Indeed this is quite analogous to the reflation by which constant cost estimates are converted to replacement cost estimates. Thus we can define R_i for the COH calculation as $(1 + \lambda)^{t-c}$ $(1 + \lambda)^{-(t-1)}$ and for CBD as

$$(1 + \lambda)^{t-c}(1 + \lambda)(1 - 1/2L) \left\{\frac{(1 - 1/L)}{(1 + \lambda)}\right\}^{i}.$$

4. The Existence of an Aggregate Capital Stock

So far we have demonstrated that the common forms of depreciation accounting can be considered to be variations on two survival assumptions for which we can in turn define stocks incorporating embodied technical change. Before continuing our investigation of the relationships among those various net and gross stock estimates it is well to mention one of the byproducts of the embodiment discussion, namely the conditions for the existence of an aggregate capital stock.[15] In the context of this study, the question is not whether a capital aggregate exists, but whether the estimates generated here are legitimately interpreted in such a manner for the analysis of production functions.

In essence the argument can be stated in the following manner: since the production process is defined in terms of production functions specific to particular vintages of capital goods, we may aggregate across vintages, i.e. across time, only if certain conditions are met. These conditions take the form of being able to express vintage production functions, whose arguments are vintage capital and the associated labor, in terms of an equivalent aggregate production function, whose arguments are labor and some transformation of vintage capital. These conditions are not very restrictive, particularly in the case of a constant returns to scale production function; in this case it is necessary only that the capital goods of the various vintages can be related by a set of multiplicative constants. This is frequently expressed as a requirement that all technical change be capital augmenting. If returns to scale are not constant, the drift of the argument is the same, but the weights which combine the capital of the various vintages are monotonic functions rather than constants.

Two comments are in order. First, the authors of such papers take great pains to point out that their analysis should only be taken to apply to the case of embodied, not disembodied, technical change. In the light of what has been said in this chapter on the measurement of capital as a weighted sum of past gross investment, it seems clear that it will indeed be difficult to find

15. F. M. Fisher, "Embodied Technical Change and the Existence of an Aggregate Capital Stock," *Review of Economic Studies*, XXXIII (October 1965), pp. 263–288; P. Diamond, "Technical Change and the Measurement of Capital and Output," *Review of Economic Studies*, XXXIII (October 1965), pp. 289–298.

any measure of capital which cannot be interpreted as containing some amount of embodied technical change.

Second, although the conditions do not seem particularly restrictive, there seems to be no reason for the particular weighting pattern associated with a given estimate of the aggregate capital stock to be compatible with the form of the aggregate production function which the investigator has selected as a replacement for the vintage production functions.

C. *Some Theoretical Relationships Among the Estimates*

1. The Relative Levels of the Estimates

In the following discussion asset lives are assumed to be no less than one year, as required by the definition of a capital good. The declining balance depreciation rate is defined as "m/L", where m may equal 1.0, 1.5, or 2.0. All other terminology has been defined previously. If we assume that investment is the same in each year, or $I_t = I_0$ for all t, then we can define the following values for the stock under the cumulation methods which we are investigating.[16]

(2.10) $$\text{KOH}_t = I_0 L$$

(2.11) $$\text{KSL}_t = I_0 L/2$$

(2.12) $$\text{KSD}_t = I_0(2L + 1)/6$$

(2.13) $$\text{KCOH}_t = I_0 \frac{(1+\lambda)}{\lambda}[1 - (1+\lambda)^{-L}]$$

(2.14) $$\text{KDM}_t = I_0 \frac{(2L - m)}{2M}[1 - (1 - m/L)^t]$$

(2.15) $$\text{KCDB}_t = I_0 \frac{(2L-1)(1+\lambda)}{2(1+L\lambda)}[1 - (1 - 1/L)^t(1+\lambda)^{-t}]$$

A bit of manipulation of these equations indicates that the following relationships hold among the stock estimates generated by the techniques of depreciation accounting.

(2.16) $$\text{KOH} \gtreqless \text{KSL} \quad \text{as } L \gtreqless L/2 \text{ or as } L \gtreqless 0$$

(2.17) $$\text{KOH} \gtreqless \text{KSD} \quad \text{as } L \gtreqless (2L+1)/6 \text{ or as } L \gtreqless \tfrac{1}{4}$$

(2.18) $$\text{KSL} \gtreqless \text{KSD} \quad \text{as } L/2 \gtreqless (2L+1)/6 \text{ or as } L \gtreqless 1$$

(2.19) $$\text{KOH} \gtreqless \text{KDM} \quad \text{as } L \gtreqless \frac{(2L-m)}{2m}[1 - (1 - m/L)^t]$$

16. It is assumed that $t = c$ in the case of COH and CDB. For additional details see H. S. Tice, "Depreciation, Obsolescence, and the Measurement of the Aggregate Capital Stock of the United States 1900–1962" (unpublished doctoral dissertation, Yale University, 1967).

(2.20) $$\text{KSL} \gtreqless \text{KDM} \quad \text{as } L/2 \gtreqless \frac{(2L - m)}{2m} [1 - (1 - m/L)^t]$$

(2.21) $$\text{KSD} \gtreqless \text{KDM} \quad \text{as } \frac{(2L + 1)}{6} \gtreqless \frac{(2L - m)}{2m} [1 - (1 - m/L)^t]$$

Conditions 2.16 and 2.17 indicate that KOH will exceed both KSL and KSD for all asset lives considered here. Condition 2.18 implies that KSD and KSL will be equal for assets with lives of one year, but that the straight line estimate will be the greater for all lives greater than one. It is also obvious that within the set of declining balance estimates, or exponential depreciation, the stock will be the larger as m is the smaller.

Conditions 2.19, 2.20, and 2.21 may be rewritten as

(2.19a) $$L \gtreqless \frac{-m [1 - (1 - m/L)^t]}{2m - 2[1 - (1 - m/L)^t]}$$

(2.20a) $$L \lesseqgtr \frac{-m [1 - (1 - m/L)^t]}{2 - m - 2(1 - m/L)^t}$$

(2.21a) $$L \lesseqgtr \frac{m [4 - 3(1 - m/L)^t]}{2 [3 - m - 3(1 - m/L)^t]}$$

Condition 2.19a indicates that one-horse-shay stock estimates will exceed all declining balance estimates so long as L is positive. For conditions 2.20a and 2.21a we must first consider the value of the expression on the right hand side both at $t = L$ and as $t \to \infty$, since in the latter case $(1 - m/L)^t$ approaches zero. Furthermore, we must, at both of these values of t, examine the condition for the three values of m which are used in the calculations. Making use of the approximate values of $(1 - m/L)^L$ of .35 for $m = 1.0$, .20 for $m = 1.5$, and .11 for $m = 2.0$, we derive the results summarized in Table 2.2 below.

TABLE 2.2

RELATIONSHIP BETWEEN KSL, KSD, AND KDM

	m	at $t = L$	as $t \to \infty$
KSL $\gtreqless$ KDM	1.0	$L \lesseqgtr 2.17$	$L \lesseqgtr 1.0$
	1.5	$L \lesseqgtr 12.0$	$L \lesseqgtr 3.0$
	2.0	$L \lesseqgtr -8.0$	$L \gtreqless -\infty$
KSD $\gtreqless$ KDM	1.0	$L \lesseqgtr 1.55$	$L \lesseqgtr 1.0$
	1.5	$L \lesseqgtr 2.8$	$L \lesseqgtr 2.0$
	2.0	$L \lesseqgtr 5.5$	$L \lesseqgtr 4.0$

Thus stocks generated with the assumption of declining balance at the straight line rate ($m = 1$) will generally exceed both the straight line and the sum-of-digits estimates except in the case of very short-lived assets; indeed the only exception is the asset with a life of one year. Estimates made by using declining balance at 1.5 times the straight line rate will nearly always exceed sum-of-digits estimates and will exceed straight line estimates so long as asset lives are not too short. Double declining balance produces estimates between those generated by the straight line and the sum-of-digits assumptions unless again asset lives are short, in which case this procedure will yield the lowest estimates of all.

Therefore, the techniques of depreciation accounting will produce estimates of the stock of assets which lie in the range bounded by the one-horse-shay and the sum-of-digits methods, with some possible exceptions in the case of short-lived assets. In most cases both of the higher declining balance estimates considered here will exceed what is generally taken to be the net stock, i.e. the net stock defined under straight line assumptions.

When we examine the two cases of embodied technical change, a rather interesting result appears; the relationship between this allowance for obsolescence and that implicit in the formulae of depreciation accounting depends almost entirely on the relationship between λ and L. Indeed there is a curious tendency for the relationship between stock estimates generated under these two techniques to depend approximately on the relationship between λ and the straight line rate.

Turning first to exponential depreciation,

$$(2.22) \quad \text{KDM} \gtreqless \text{KCDB as } \frac{(2L-m)}{2m}\left[1-(1-m/L)^t\right] \gtreqless \frac{(2L-1)(1+\lambda)}{2(1+L\lambda)}\left[1-\frac{(1-1/L)^t}{(1+\lambda)^t}\right]$$

As $t \to \infty$ 2.22 can be rewritten as

$$(2.22\text{a}) \qquad \lambda \gtreqless \frac{2L(m-1)}{2L^2-3Lm+m}$$

Thus when $m = 1.0$, any positive λ will result in estimates of KCDB less than the gross stock under this survival assumption. For $m = 1.5$, the condition in 2.22a becomes $\lambda \gtreqless 2L/(4L^2-9L+3)$; for $m = 2.0$ it reduces to $\lambda \gtreqless L/(L^2-3L+1)$. For large L this implies that if λ much exceeds one-half the straight line rate for $m = 1.5$ and if it much exceeds the straight line rate in the case of $m = 2.0$, the declining balance estimates of conventional depreciation accounting will exceed these generated under the assumption of embodied technical change. Furthermore if $\lambda = (m-1)/(L-m)$ the expressions in square brackets on either side of 2.22 will be equal for all t, and the condition defined in 2.22a will hold. Since, however,

$$\frac{2L(m-1)}{2L^2-3L+m} > \frac{m-1}{L-m}$$

for all $L > 1$, if this second condition is met, then KCDB will inevitably exceed both of the smaller declining balance depreciation accounting estimates. For cases in which $\lambda \neq (m - 1)/(L - m)$, we can again approximate for $t = L$ as in 2.22b, though the condition is little changed from 2.22a.

$$\text{(2.22b)} \qquad \lambda \gtreqless \frac{1}{L-1}\left\{\frac{m(2L-1)}{2L-m}\left[\frac{1 - .35(1+\lambda)^{-L}}{1-(1-m/L)^{L}}\right] - 1\right\}$$

We now consider "embodied technical change" under the rectangular survival assumption.

$$\text{(2.23)} \qquad \text{KOH} \gtreqless \text{KCOH} \quad \text{as } L \gtreqless \frac{(1+\lambda)}{\lambda}[1-(1+\lambda)^{-L}]$$

$$\text{(2.24)} \qquad \text{KSL} \gtreqless \text{KCOH} \quad \text{as } \lambda \gtreqless \frac{2}{L}[1-(1+\lambda)^{-L}]$$

$$\text{(2.25)} \qquad \text{KSD} \gtreqless \text{KCOH} \quad \text{as } \lambda \gtreqless \frac{6}{2L+1}[1-(1+\lambda)^{-L}]$$

KOH, the "gross stock," will exceed KCOH, though the latter will exceed both the straight line and sum-of-digits estimates so long as λ is less than or equal to the straight line rate. As positive departures from this rate of improvement become too large, however, it is quite likely that KSL will exceed KCOH and not impossible that KSD should do so.

The two measures of the stock employing the concept of embodied technical change are related in much the same way as are the other estimates generated by the two survival assumptions. KCOH will exceed KCDB for $L \geqq 1$.

$$\text{(2.26)} \qquad \text{KCOH} \gtreqless \text{KCDB as}$$

$$L \lesseqgtr \frac{2[1-(1+\lambda)^{-L}] + \lambda[1-(1-1/L)^{t}(1+\lambda)^{-t}]}{2\lambda[(1+\lambda)^{-L} - (1-1/L)^{t}(1+\lambda)^{-t}]}$$

and as $t \to \infty$ this reduces to $L \lesseqgtr [(2+\lambda)(1+\lambda)^{L} - 2]/2$. This condition is somewhat less restrictive for smaller values of t.

Finally we must examine the relationship between each of the improvement factor estimates and those yielded by the application of the techniques of depreciation accounting to the alternative mortality assumption.

$$\text{(2.27)} \qquad \text{KCOH} \gtreqless \text{KDM as}$$

$$\lambda \lesseqgtr \frac{2m[1-(1+\lambda)^{-L}]}{(2L-m)[1-(1-m/L)^{t}] - 2m[1-(1+\lambda)^{-L}]}$$

which becomes, as $t \to \infty$

$$\lambda \lesseqgtr \frac{2m[1-(1+\lambda)^{-L}]}{(2L-m) - 2m[1-(1+\lambda)^{-L}]}$$

Thus so long as the improvement rate is less than the declining balance rate (approximately), KCOH will exceed KDM. The condition is somewhat less restrictive for smaller values of t.

$$\text{(2.28)} \qquad \text{KOH} \gtreqless \text{KCDB as } \lambda \gtreqless \frac{(2L-1)[1-(1-1/L)^t(1+\lambda)^{-t}]-2L}{2L^2-(2L-1)[1-(1-1/L)^t(1+\lambda)^{-t}]}$$

which becomes, as $t \to \infty$, $\lambda \gtreqless -1/(2L^2 - 2L + 1)$. Thus KOH will exceed KCDB for positive λ. The condition is much the same for smaller values of t.

$$\text{(2.29)} \qquad \text{KCDB} \gtreqless \text{KSL as } \lambda \lesseqgtr \frac{(2L-1)[1-(1-1/L)^t(1+\lambda)^{-t}]-L}{L^2-(2L-1)[1-(1-1/L)^t(1+\lambda)^{-t}]}$$

As $t \to \infty$ this reduces to $\lambda \lesseqgtr 1/(L-1)$, and thus the straight line estimates will be less than those generated under the assumption of exponential depreciation with embodied technical change so long as the rate of improvement is no greater than $1/(L-1)$.

Sum-of-digits estimates will as usual be quite low.

$$\text{(2.30)} \qquad \text{KCDB} \gtreqless \text{KSD as}$$

$$\lambda \lesseqgtr \frac{4(L-1)-(6L-3)(1-1/L)^t(1+\lambda)^{-t}}{(2L-3)(L-1)+(6L-3)(1-1/L)^t(1+\lambda)^{-t}}$$

which becomes $\lambda \lesseqgtr 4/(2L-3)$ as $t \to \infty$. Thus so long as λ is no greater than $2/L$, the sum of digits estimates will lie below KCDB.

2. The Trend of the Estimates

In all cases, the trend of the stock series is strongly influenced by the trend in gross investment. Substituting for our previous assumption of constant investment a flow which grows at a constant geometric rate "g" per cent per annum so that $I_t = I_0 (1+g)^t$, we then define

$$\text{(2.31)} \qquad K_t = \sum_{i=1}^{n} R_i I_{t+1-i} = I_0(1+g)^{t+1} \sum_{i=1}^{n} R_i(1+g)^{-i}$$

where $n = L$ for the rectangular survival assumption and $n = t$ for all declining balance estimates.[17]

It is obvious that all four estimates based on the assumption of rectangularly distributed survivals will have growth rates which are identical and equal to "g", the rate of growth of gross investment.

$$\text{(2.32)} \qquad K_t = I_0(1+g)^{t+1} \sum_{i=1}^{L} R_i(1+g)^{-i}$$

$$K_{t-1} = I_0(1+g)^{t} \sum_{i=1}^{L} R_i(1+g)^{-i}$$

17. For derivation of the expressions contained in this section see Tice, *op. cit.*, Appendix A. Again the results for COH and CDB assume that $t = c$ or that the series has been converted to this assumption by multiplication by the appropriate power of $(1 + \lambda)$.

and therefore

$$\rho = \frac{K_t}{K_{t-1}} - 1 = g.$$

In the case of the declining balance estimates, over time all ρ's converge to g, though they will differ from g initially by an amount dependent on the depreciation and/or improvement rates assumed. Letting $d = m/L$,

$$K_t = \frac{I_0(1+g)(1-d/2)}{g+d}\left[(1+g)^t - (1-d)^t\right] \tag{2.33}$$

$$K_{t-1} = \frac{I_0(1+g)(1-d/2)}{g+d}\left[(1+g)^{t-1} - (1-d)^{t-1}\right]$$

and

$$\rho = \frac{g(1+g)^{t-1} + d(1-d)^{t-1}}{(1+g)^{t-1} + (1-d)^{t-1}}.$$

ρ converges to g as $t \to \infty$; it will exceed g so long as $d > -g$; and its value declines as m increases. For CDB we can define

$$\rho = \frac{g(1+\lambda)^t(1+g)^{t-1} + (d+\lambda)(1-d)^{t-1}}{(1+\lambda)^t(1+g)^{t-1} - (1+\lambda)(1-d)^{t-1}} \tag{2.34}$$

to which the same general comments apply. In this case $d = 1/L$.[18]

3. The Service Life Assumption

It is clear that a great deal depends on the life that is assumed. In the case of the trends in the stock series, not so much is at stake. ρ is invariant to the life assumed in the case of rectangularly distributed survivals, and a reduction in L will simply accelerate the approach of ρ to g in the declining balance cases. However, in the matter of the level of the stock estimate, the consequences of a change in the life assumption are somewhat more dramatic. Returning to the stock values derived in (2.10)–(2.15), we derive the results shown in Table 2.3.

TABLE 2.3

EFFECTS OF CHANGES IN THE SERVICE LIFE ON STOCK ESTIMATES

Method	$\Delta K/\Delta L$
OH	I_0
SL	$I_0/2$
SD	$I_0/3$
DM	$I_0(1/m)$ as $t \to \infty$. Less than this for smaller t.
COH	$I_0[(1+\lambda)^{-(L-1)} \ln(1+\lambda)/\lambda]$ or approximately $I_0(1+\lambda)^{-(L-1)}$
CDB	$I_0[(1+\lambda)(2+\lambda)/2(1+L\lambda)^2$ as $t \to \infty$. Less for smaller t.

18. The growth rates derived above for COH and CDB refer to the series generated when the adjustment is made which expresses each stock in terms of its own year's technology. The stock valued in terms of the technology of year c grows at a rate $\rho = g + \lambda + g\lambda$.

For both survival assumptions the effects of variation in this parameter are most pronounced for the "gross" stocks; for each year's change in the life estimate, we change the stock by one year's investment. Double declining balance and straight line estimates are both effected to the extent of half a year's investment. If the rate of growth of gross investment is non-zero, the declining balance estimates are somewhat less sensitive to changes in the life assumption than are their counterparts based on the rectangular survival assumption. So long as λ is fixed so that $d\lambda/dL = 0$, both of the embodied technical change estimates are potentially less sensitive than anything short of sum-of-digits estimates; sensitivity will be less, of course, the higher is λ.

D. *Summary*

Much of the controversy over the "proper" measure of capital has at its base a difference of opinion on the proper allowance to be made for obsolescence as opposed to physical decay. We have demonstrated that the methods of traditional depreciation accounting can be treated as variations on two survival assumptions; thus we have two gross stocks, OH and DM1, and a set of net stocks corresponding to each of the gross stocks.

These net stocks are shown to incorporate rates of embodied technical change, either implicitly or explicitly; for COH and CDB, λ is given explicitly, while for the other methods, there is an implicit λ which is a function of the straight line depreciation rate. For DM5 the implicit λ is approximately one-half the straight line rate; for DM2 and SL this implicit λ is approximately equal to the straight line rate; and for SD estimates, the implicit rate of embodied technical change is approximately twice the straight line rate.

Not only, then, do existing measures of the value of the capital stock incorporate, for all intents and purposes, the embodiment hypothesis, they also involve rates of technical change which are in many cases quite high. Furthermore, to the extent that the theoretical issues raised by the embodiment question are relevant, the issues cannot be resolved empirically by accepting disembodiment as an article of faith.

III. A Comparison of Stock Estimates for the United States 1900–1962

A. *The Nature of the Data*

The gross investment and deflator series used in this study are those developed by R. W. Goldsmith.[19] Goldsmith's procedure was used to extend the data to 1962 in all but a few cases.[20] These estimates were taken from statistical publications of the U.S. Departments of Commerce, Labor, and

19. R. W. Goldsmith, *A Study of Saving*, *op. cit.*, and *The National Wealth . . .* , *op. cit.*

20. An account of these procedures and of the ways in which the stock estimates reported here differ from the published Goldsmith series may be found in Tice, *op. cit.*, pp. 43–45, and Appendix B.

Agriculture, wherever possible. Service life estimates are those used by Goldsmith in his more recent work; they were based in most instances on the U.S. Treasury's "Bulletin F" which set service lives permissible for the computation of depreciation allowances for business income tax deductions. Values of λ, the rate of embodied technical change, were assumed to be 2% for structures and 4% for equipment; these values were selected because they were among the higher rates investigated by Solow.[21]

B. Estimates of the United States Capital Stock 1900–1962

1. Expected Results

Although we expect (1) that the levels of the estimates will be quite sensitive to changes in computational assumptions and (2) that the trend will be little affected, the existence of the irregularities characteristic of any actual gross investment series make the empirical situation somewhat less predictable and therefore somewhat less comfortable than we should like. With the exception of the straight-line and one-horse-shay methods, the weight given any year's investment varies; and thus large departures from a simple trend in gross investment will be transmitted to the stock at somewhat different times. For the United States during the period under investigation, this becomes really significant only for the depression and war years; otherwise the trends in the series are on the whole quite similar.

We should expect, in general, that the estimates will lie in a range bounded on the one hand by the one-horse-shay estimate and on the other by the series generated under the sum-of-digits assumption. Within this range, so long as λ does not depart too much from the straight-line rate, we should expect the following ordering in descending order of size:

OH DM1 COH DM5 SL CDB DM2 SD

Broadly speaking, as λ departs from $1/L$ in an upward direction, the revaluation for obsolescence becomes quite extreme relative to that provided by the formulae of depreciation accounting; it becomes quite possible to get:

OH DM1 DM5 SL COH DM2 CDB SD

As λ departs from $1/L$ in the opposite direction, so that the obsolescence deduction implied by depreciation accounting becomes relatively large, then an ordering of:

OH COH DM1 CDB DM5 SL DM2 SD

is not unlikely. All this assumes, of course, that asset lives are not so short as to make the relationship between the two sets of depreciation accounting estimates depart from the normal case.

21. R. M. Solow, *Capital Theory and the Rate of Return, op. cit.*, pp. 80 ff.; "Technical Progress, Capital Formation, and Economic Growth," *American Economic Review*, LII (May 1962), pp. 76–86.

To summarize, since fixed improvement rates of 2% for structures and 4% for equipment were used in the calculations presented here, the precise relationship among the estimates will depend in large degree upon the service life composition of the investment series being cumulated. This will also hold true for the relationship among the estimates generated under the various methods of depreciation accounting, though this can only become important for certain items of equipment. The trend in the series, however, should be immune to changes in the method by which we allow for physical decay and obsolescence, since it is so dominated by the trend in gross investment.

In any empirical case, however, we must allow for the effects of the departures from trend value which are common to gross investment. While these are generally quite mild—i.e. the fluctuations have rather low amplitude—such phenomena as wars and depressions must be allowed for. It is these events and the different patterns by which their effects are transmitted to the stock that provide the cases which are of the most interest to the statistician and of the least comfort to the potential user of the estimates. Total fixed investment including government assets and consumer durables was characterized by quite dramatic changes in trend from 1921–1946 with little annual variation around the trend. Both before and after this period, investment tended upward with some departures from smooth trends; the departure was pronounced for the pre-World War I years and almost non-existent in the post World War II period. If we examine a fixed investment concept comparable with that used in the OBE income and product accounts the same pattern is evident.

Turning to an asset break-down of the total we find that the four components have somewhat different characteristics. Residential construction is characterized by rather long, smooth waves with little short-term fluctuation until after World War II. Nonresidential construction, on the other hand exhibited considerable short-term departure from trend before 1921, large smooth waves from then until the late 1940's, and an almost perfectly constant percentage rate of increase in the postwar period. Investment in both producer and consumer durables, however, has been characterized by considerable year-to-year movement around a generally upward trend; the fall in expenditure on consumer durables in the early 1930's was less than that in expenditure on producer durables, though the war had a somewhat greater effect on the former than on the latter.

On the basis of a simple inspection of the gross investment series, therefore, we should expect (1) that if the capital stock is at all sensitive to the computational assumptions underlying its measurement, this sensitivity should be exhibited in the roughly twenty years of depression and war, and (2) that durables, whose investment pattern is somewhat more irregular than is that of structures, might be expected to show somewhat greater sensitivity, on balance, than would structures. The second expectation is reinforced by the fact that for durables, asset lives are shorter, depreciation (and obsolescence) rates are higher, and therefore the differential effects of sudden large departures from trend in the investment series will be more pronounced.

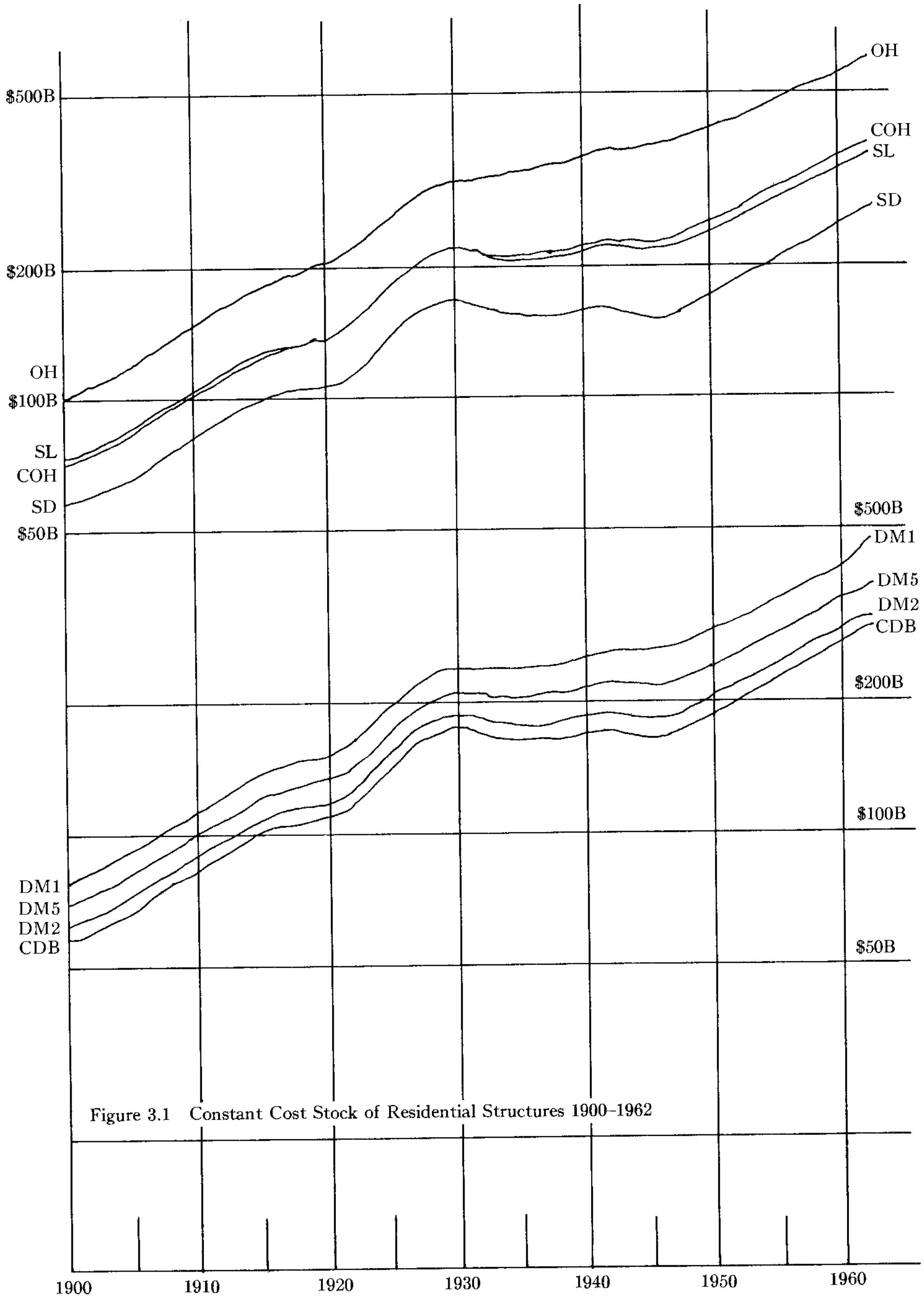

Figure 3.1 Constant Cost Stock of Residential Structures 1900–1962

2. Residential Structures

For residential structures (Figure 3.1) we find that the estimates explicitly incorporating the notion of "embodied technical change" are quite low; COH is less than SL well into the 1920's and never rises very far above it, while CDB is the lowest of all the declining balance estimates and is barely above SD during the early years. Here the 2% rate of improvement is clearly excessive relative to the implicit λ given by the asset lives. The series is heavily dominated by one-to-four family housing for which lives of 80, 73, and 66 years were used, depending on the period. It is not until the last of these lives begins to be employed and the dominance of such structures begins to be challenged by the increasing importance of somewhat shorter-lived assets that this extreme application of embodied technical change does not yield grossly understated estimates in comparison with the other methods.

As far as the trend is concerned all the series show broadly the same pattern, but the two gross measures behave somewhat differently after 1929. The OH stock continues to rise slightly throughout the 1930's, while DM1 remains virtually constant over a good deal of the period. After 1945, DM1 tends to rise at a somewhat more rapid rate. Furthermore, within each of the two survival assumption groupings, the rate of change, either positive or negative, seems to increase slightly as the level decreases.

The period covered falls rather neatly into three subperiods: 1900–1929, 1929–1945, and 1945–1962. In the first period the basic investment data are weak; in many cases they were estimated by interpolative methods. The second period covers the war and depression in which the movement of all investment series is somewhat erratic. The third period covers the postwar years; the basic data are sounder than before and they are not subject to the violent fluctuations of the previous period. Annual average percentage rates of growth in the stock of residential structures at constant cost valuation are shown for the three periods in Table 3.1. Since they were calculated as geometric means of annual indexes, they are of course subject to bias arising from the selection of initial and terminal years. Used in conjunction with Figure 3.1, however, they are quite useful.

TABLE 3.1

ANNUAL AVERAGE PERCENTAGE RATES OF GROWTH OF THE CONSTANT COST STOCK OF RESIDENTIAL STRUCTURES

Period \ Method	OH	SL	SD	COH	DM1	DM5	DM2	CDB
1900–1929	3.96%	3.74%	3.74%	3.85%	3.87%	3.84%	3.82%	3.85%
1929–1945	1.21	0.12	−0.66	0.31	0.65	0.26	−0.11	−0.32
1945–1962	2.70	3.00	3.47	3.12	3.07	3.18	3.32	3.50

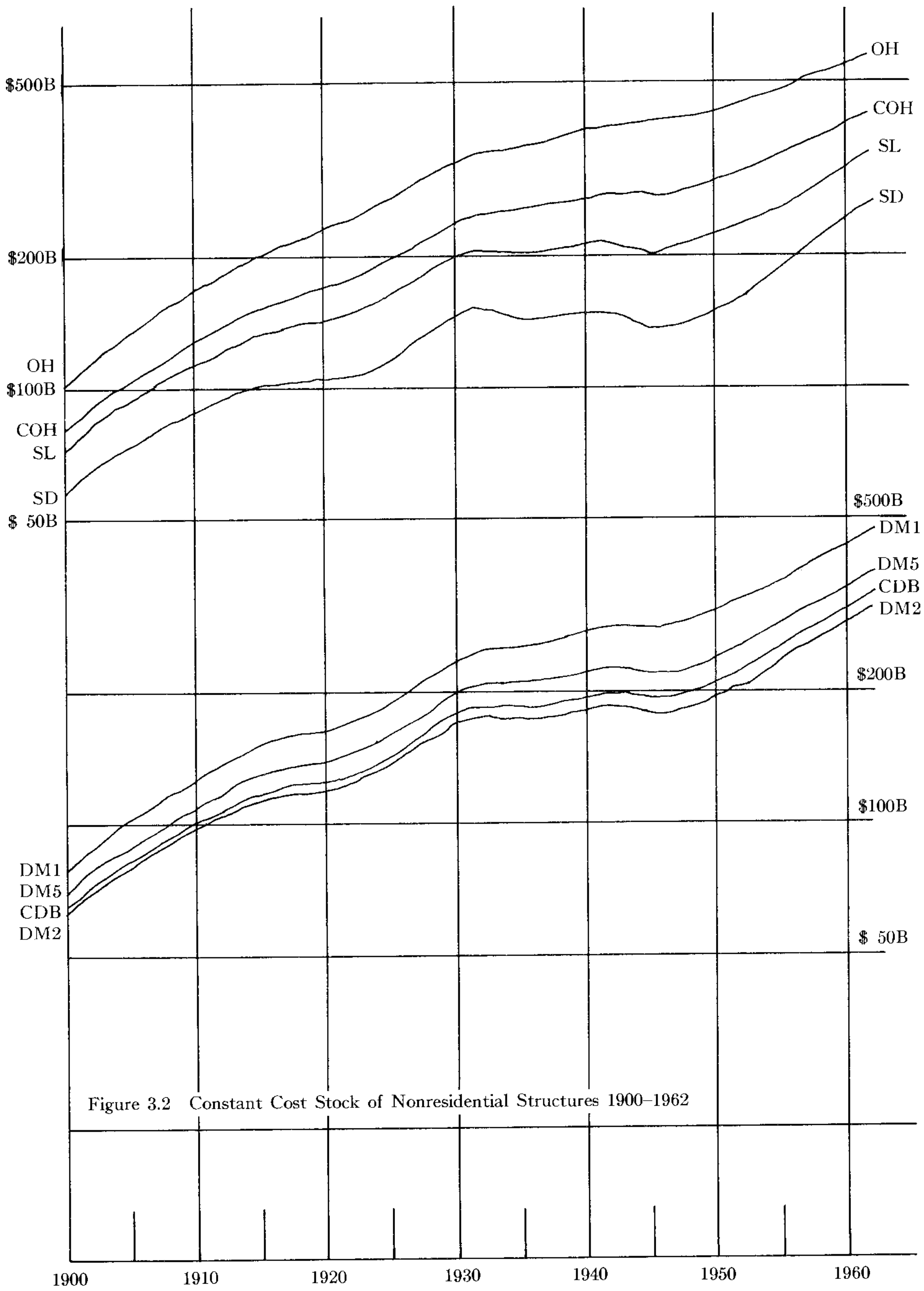

Figure 3.2 Constant Cost Stock of Nonresidential Structures 1900–1962

The growth rates in the first period do not differ much; this happy state of affairs does not obtain during the middle period, however, and even during the postwar years trend rates differ by almost one-half a percentage point.

3. Nonresidential Structures

In this case the 2% rate of embodied technical change is not very different from that implicit in the service lives; indeed if anything it is on the low side. Looking at Figure 3.2 we find that COH exceeds the straight line estimate by a fairly substantial amount in the period following 1930, though the difference is not so large in the earlier years when gross investment contained a somewhat larger proportion of long-lived assets. CDB is below DM5, but it exceeds the double declining balance estimate. The difference again becomes larger as the combined asset lives become shorter.

Again the trends in the various series, while basically similar, are not quite identical. COH initially slightly exceeds DM1, but it falls below it at an increasing rate after 1945; the same situation obtains for SL and DM5; and indeed the SL and CDB estimates come increasingly close together after 1945. The two gross stocks, OH and DM1, do not follow precisely the same time trend, since before 1945 OH grows more rapidly than does DM1 and the converse is true after 1945. Within the group of stock estimates defined by the assumption of rectangularly distributed survivals, there seems to be an inverse relationship between level and trend rate after 1945 which constitutes a reversal of the previous relationship between the two; the same relationship also exists for the declining balance estimates, though a to less obvious degree. This relationship is probably the result of the nearly zero rate of growth in gross investment in nonresidential structures during the early years of this century, which yields fairly high and virtually constant depreciation charges for one-horse-shays, but has less impact as the amount previously written off increases. Growth rates for the constant cost stock of nonresidential structures are shown in Table 3.2 for the three periods previously discussed. Again the trends are much more similar in the early period than is the case in either of the later periods. Furthermore in both the first and the third period the range over which the trend rates vary is much larger for nonresidential structures than for the residential structures discussed above.

TABLE 3.2

ANNUAL AVERAGE PERCENTAGE RATES OF GROWTH OF THE CONSTANT COST STOCK OF NONRESIDENTIAL STRUCTURES

Period \ Method	OH	SL	SD	COH	DM1	DM5	DM2	CDB
1900–1929	3.99%	3.38%	3.09%	3.68%	3.75%	3.55%	3.38%	3.44%
1929–1945	1.40	0.38	−0.25	0.97	1.23	0.81	0.44	0.60
1945–1962	2.13	3.04	3.85	2.62	2.98	3.15	3.38	3.32

4. Producer Durables

The estimates of producer durables given in Figure 3.3 are among the most interesting in terms of the present analysis and are the most nightmarish, therefore, from the point of view of the consumer of such estimates. The asset lives involved are short, and thus the difference between the weights attached to the investment expenditures of adjacent years are somewhat greater than in the cases we have been considering heretofore. This interacts with the high degree of volatility in the gross investment series to produce the results portrayed in Figure 3.3. The 4% rate of embodied technical change, which seems quite high in the abstract, is actually fairly low when compared with that implicit in the asset lives and the formulae of depreciation accounting. Thus COH exceeds SL by quite a large amount, and CDB is considerably closer to DM5 than to DM2.

It is in the trends of these estimates, however, that the real interest lies. Trend effects have so far been present, but in a relatively undramatic form; in the case of producer durables, this is no longer true. Furthermore the drama is not confined to the depression and war years where we are somewhat better prepared to live with it, but it characterizes the entire period of observation. In general, the series are not so smooth as are those for structures, and even small fluctuations tend to have a somewhat different timing as we go from method to method.

Looking first at the set of estimates based on the rectangular survival assumption, we see that while the OH stock increased throughout the recession of the early 1920's, COH stayed constant for a year before increasing, SL declined for one year and stayed constant for another before increasing, and SD fell sharply for two years, then rose sharply for the remainder of the decade. Stocks increased during 1930 for the first two methods, remained constant for the third, and declined slightly for the fourth. At this point some interesting developments occur. All four series fall until 1935 with the rate of decline inversely related to the level of the estimate; the two higher stocks continue to decline until the end of 1939; the other two begin to recover, however, with the SD series increasing at an extremely rapid rate. In the postwar period all series increase again at rates which are at first inversely related to the level of the estimates and then diminish to different degrees as investment falls off in the late 1950's.

Differences among the declining balance estimates are not so sharp. The double declining balance stock declines during 1921 and 1937, years in which the other estimates are constant, and there is again the pronounced tendency for the absolute value of the stock's growth rate to vary inversely with the level of the estimate. In the postwar period, however, the CDB series moves closer to DM5, a maneuver which seems to have been accomplished between 1945 and 1948.

Comparing the two sets is even less encouraging. Though the two "gross" stock estimates have roughly the same trend from 1905 until the end of 1928, the path taken by DM1 is much more like that of SD than like OH. Thus,

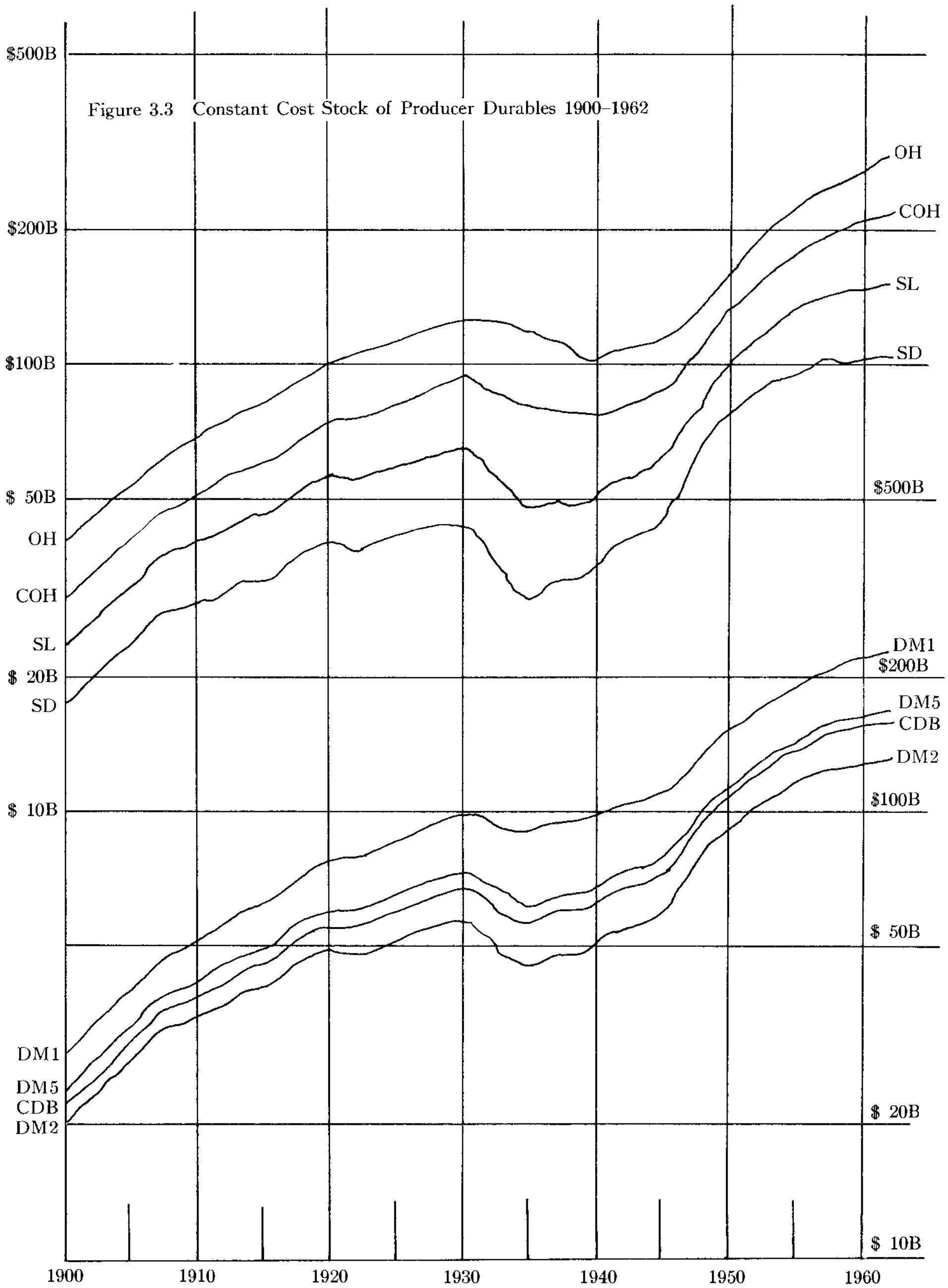
Figure 3.3 Constant Cost Stock of Producer Durables 1900–1962
$500B
$200B
$100B
$ 50B
OH
COH
SL
$ 20B
SD
$ 10B
DM1
DM5
CDB
DM2
OH
COH
SL
SD
$500B
DM1
$200B
DM5
CDB
DM2
$100B
$ 50B
$ 20B
$ 10B
1900
1910
1920
1930
1940
1950
1960

though the two gross stocks are of roughly the same magnitude in the late 1940's, DM1 grows somewhat more slowly throughout the postwar period. COH is initially above DM1, falls below it in 1915, and diverges from it increasingly until after World War II when it begins to approach it closely again. Interestingly enough, the straight line estimate, though initially above all but the DM1 estimate, falls below DM5 in 1907 and below CDB in 1927, maintaining its new position throughout the remainder of the period investigated; for a few years in the late 1930's and the early 1940's it was not much above the double declining balance estimate. Very little of this stock of producer durables belongs to the government; the bulk of it, with all of its interesting properties, is "private capital." Quite a bit has been done with the straight line version of the net stock of these assets.

Again we can compare trends in these series for the three sub-periods; results are shown in Table 3.3. Here the range of the growth rates is even larger than before; the maximum and minimum rates differ by a full percentage point in the early period, and by even greater amounts in the two later periods.

TABLE 3.3

Annual Average Percentage Rates of Growth of the Constant Cost Stock of Producer Durables

Period \ Method	OH	SL	SD	COH	DM1	DM5	DM2	CDB
1900–1929	3.95%	3.55%	3.32%	3.87%	4.32%	3.96%	3.70%	3.92%
1929–1945	−0.48	−0.14	0.29	−0.09	0.88	0.50	0.32	0.61
1945–1962	5.63	5.26	4.94	5.45	4.27	4.55	4.72	4.67

5. Consumer Durables

Consumer durables present a somewhat more encouraging picture. Fluctuations in gross investment were less pronounced than in the case of producer durables, and thus the rather alarming results of the previous section are not to be found in Figure 3.4. However, COH is quite close to OH and CDB now exceeds DM5 as a result of the very short service lives involved. Indeed this tendency might have been even more pronounced had the estimated service life for automobiles not increased over time, since the share of automobiles in gross investment in consumer durables is very large.

We may again note that the absolute value of the growth rate varies inversely with the level of the estimate, and that while OH and COH stocks fell relatively little during the depression, the others fell and recovered at progressively greater rates. The COH stock—above DM1 until 1920—fluctuates around DM1 thereafter. The straight line stock ranks below both CDB and DM5 over the entire period. Growth rates for the various estimates of the stock of consumer durables are given in Table 3.4.

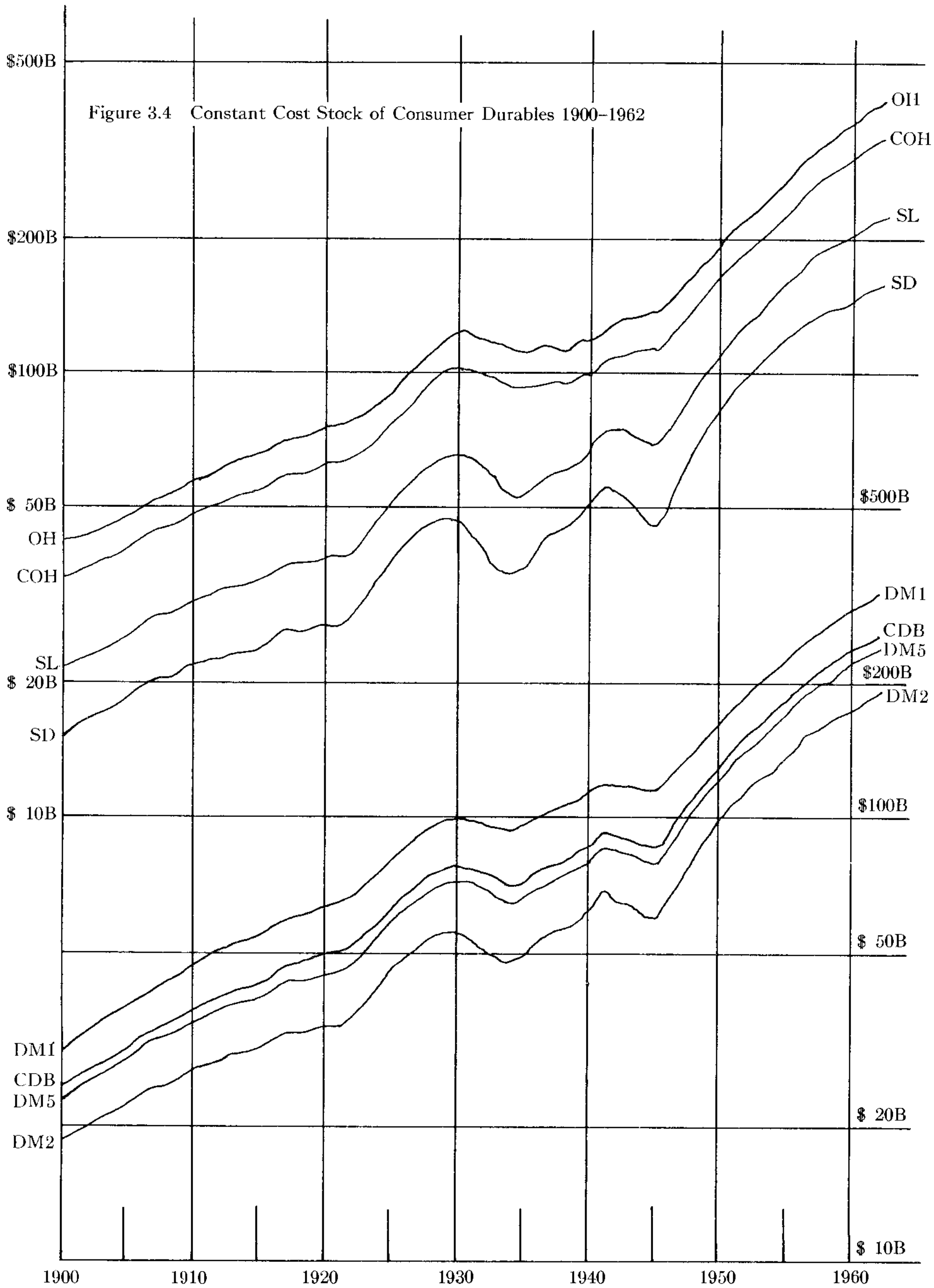
Figure 3.4 Constant Cost Stock of Consumer Durables 1900–1962
$500B
$200B
$100B
$ 50B
$ 20B
$ 10B
OH
COH
SL
SD
DM1
CDB
DM5
DM2
1900
1910
1920
1930
1940
1950
1960

Figure 3.5 Constant Cost Stock of All Civilian Assets 1900–1962

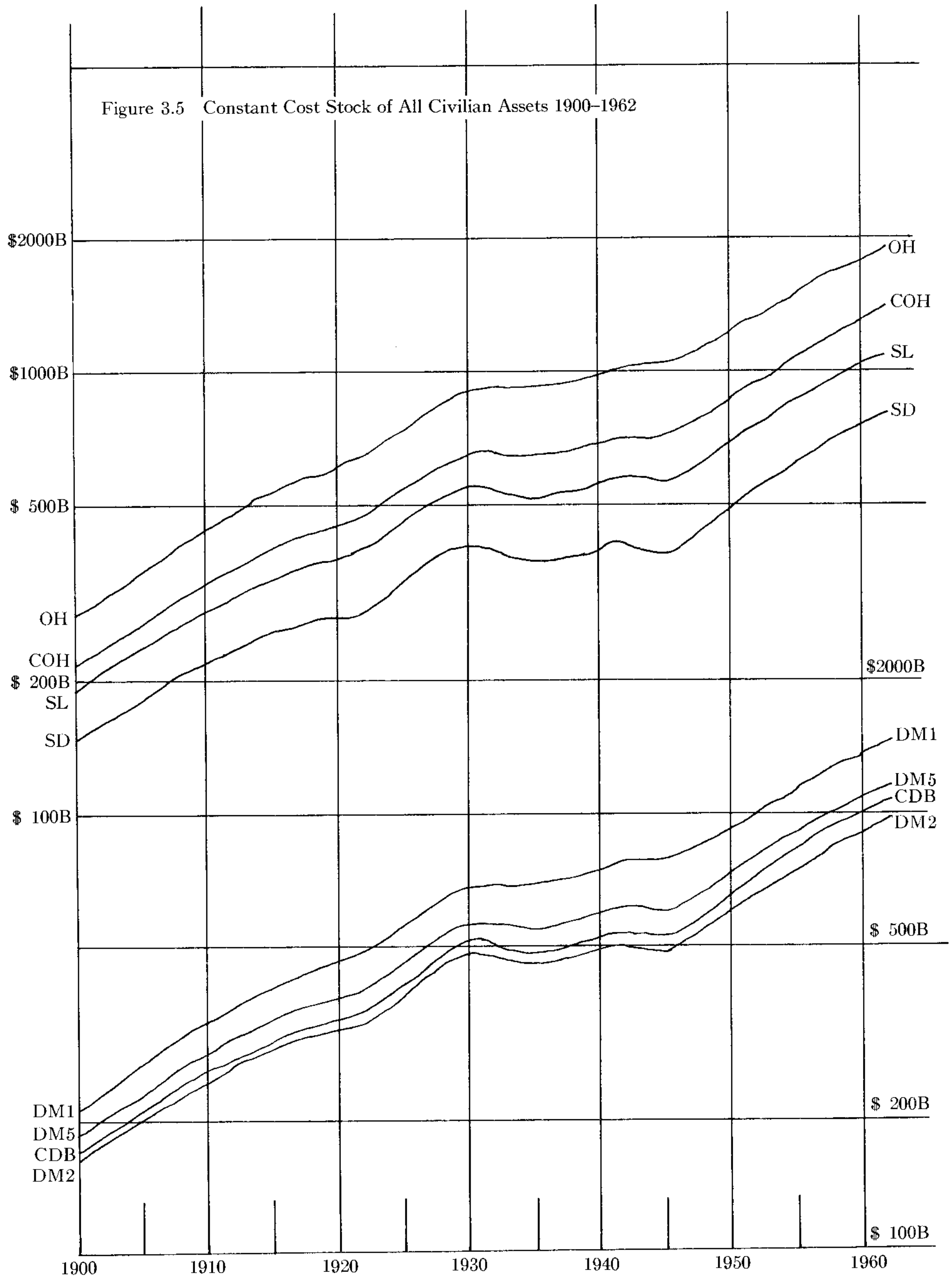

TABLE 3.4

ANNUAL AVERAGE PERCENTAGE RATES OF GROWTH OF THE CONSTANT COST STOCK OF CONSUMER DURABLES

Period \ Method	OH	SL	SD	COH	DM1	DM5	DM2	CDB
1900–1929	3.65%	3.93%	3.98%	3.76%	4.25%	4.00%	3.90%	4.10%
1929–1945	0.96	0.30	−0.19	0.66	1.03	0.61	0.30	0.58
1945–1962	6.59	7.09	7.50	6.69	6.16	6.71	7.19	6.58

6. The Constant Cost Stock of All Assets

When total depreciable assets are summed for the United States, the discrepancies among the estimates become much less pronounced, since the factors which tend to create the problems with even the highly aggregated series previously discussed—primarily departures from trend in the gross investment series—tend to be mutually offsetting for aggregate investment. Constant price estimates of the stock of all assets are shown in Figure 3.5. COH initially exceeds DM1 when nonresidential structures dominate and it later diverges from DM1, falling below it as other longer lived assets become more prominent, i.e. residential structures. The straight line stock estimate exceeds DM5 until 1922, but lies below it thereafter. In general the ranking of the estimates conforms to that expected when λ is close to $1/L$.

The growth rates for all assets are shown in Table 3.5. Although it is obvious from the shifting of rank ordering that all series do not have identical time paths, still this aggregate is on the whole much less trend sensitive than are the components previously discussed. The spread between maximum and minimum rates of growth is lower for all assets than for any of the components other than residential structures.

TABLE 3.5

ANNUAL AVERAGE PERCENTAGE RATES OF GROWTH OF THE CONSTANT COST STOCK OF ALL CIVILIAN ASSETS

Period \ Method	OH	SL	SD	COH	DM1	DM5	DM2	CDB
1900–1929	3.93%	3.60%	3.47%	3.78%	3.94%	3.77%	3.66%	3.75%
1929–1945	1.03	0.21	−0.33	0.56	0.95	0.54	0.19	0.29
1945–1962	3.57	3.96	4.41	4.00	3.79	3.95	4.11	4.23

IV. Further Implications

Since one of the products of the capital stock calculation is a set of depreciation estimates consistent with the stock series, the assumptions used to define the stock also in some sense define net income and net output. Furthermore such summary descriptive measures as factor shares, capital-output ratio, and the rate of return on capital are not independent of the assumptions used to define the stock.

Assume that gross output at factor cost grows at a constant percentage rate "g" and that gross investment is always a constant fraction "s" of output. Then

$$Y_t = Y_0 (1 + g)^t \tag{4.1}$$

$$I_t = s Y_t = s Y_0 (1 + g)^t \tag{4.2}$$

Further assume that non-labor income accounts for a fraction "a" of gross output, with labor's share being of course $(1 - a)$. It can then be demonstrated that even though gross output and its functional distribution are taken as given, net output, its functional distribution, and certain productivity measures are by no means invariant to the assumptions underlying the stock calculation.

We must first define K and D for the assumption of a constant rate of growth in gross investment. These expressions are shown in Tables 4.1 and 4.2.

TABLE 4.1

Stock of Capital at End of Year t When Gross Investment Grows at g Per Cent Per Annum

Method	Stock of Capital (K_t)
OH	$I_0(1 + g)^{t+1} \left[\frac{1 - (1 + g)^{-L}}{g}\right]$
SL	$I_0(1 + g)^{t+1} \left[\frac{(1 + g)^L(2Lg - g - 2) + (2 + g)}{2Lg^2(1 + g)^L}\right]$
SD	$I_0(1 + g)^{t+1} \left[\frac{(1 + g)^L[2 + g + Lg(Lg - 2)] - (2 + g)}{L(L + 1)g^3(1 + g)^L}\right]$
COH	$I_0(1 + g)^{t+1} \left[\frac{(1 + \lambda)[1 - (1 + g)^{-L}(1 + \lambda)^{-L}]}{\lambda + g + g\lambda}\right]$
DM	$I_0(1 + g)^{t+1} \left[\frac{(2L - m)[1 - (1 - m/L)^t(1 + g)^{-t}]}{2(Lg + m)}\right]$
CDB	$I_0(1 + g)^{t+1} \left[\frac{(1 + \lambda)(2L - 1)[1 - (1 - 1/L)^t(1 + \lambda)^{-t}(1 + g)^{-t}]}{2[1 + L(g + \lambda + g\lambda)]}\right]$

It is clear from Tables 4.1 and 4.2 that we can write

$$K_t = I_0(1 + g)^{t+1}(K) \tag{4.3}$$

$$D_t = I_0(1 + g)^t(D) \tag{4.4}$$

where (K) and (D) are the expressions in brackets in Tables 4.1 and 4.2 or the

TABLE 4.2

DEPRECIATION CHARGES DURING YEAR t WHEN INVESTMENT GROWS AT g PER CENT PER ANNUM

Method	Depreciation Charges (D_t)
OH	$I_0(1+g)^{t-L}$
SL	$I_0(1+g)^{t-L}\left[\frac{(2+g)[(1+g)^L-1]}{2Lg}\right]$
SD	$I_0(1+g)^{t-L}\left[\frac{(2+g)[1+(1+g)^L(Lg-1)]}{g^2L(L+1)}\right]$
COH	$I_0(1+g)^{t-L}[(1+\lambda)^{-L}]$
DM	$I_0(1+g)^t\left[\frac{LM(2+g)-m(2L-m)(1-m/L)^{t-1}(1+g)^{-(t-1)}}{2L(Lg+m)}\right]$
CDB	$I_0(1+g)^t\left[\frac{2L+L(\lambda+g+g\lambda)-(2L-1)(1-1/L)^{t-1}[(1+g)(1+\lambda)^{-(t-1)}]}{2L[1+L(g+\lambda+g\lambda)]}\right]$

appropriate modifications thereof in the case of the first four expressions in Table 4.2. If we also define

Y_t = gross domestic product at factor cost
Y_{Nt} = net domestic product at factor cost
$s = I_t/Y_t$
a = gross property income's share of gross product
r_g = gross rate of return on capital
k_g = gross capital-output ratio
a_N = net property income's share of net output
r_N = net rate of return on capital
k_N = net capital-output ratio

we can then examine the implications of choice of capital measurement assumptions on for the following popular measures: the capital-output ratio, the rate of return on capital, and property's share of income.

$$(4.5)\qquad \begin{aligned} Y_{Nt} &= Y_t - D_t \\ &= Y_0(1+g)^t - sY_0(1+g)^t(D) \\ &= Y_0(1+g)^t[1-s(D)] \end{aligned}$$

Capital-Output Ratios

$$(4.6)\qquad \begin{aligned} k_g = K_{t-1}/Y_t &= \frac{sY_0(1+g)^t(K)}{Y_0(1+g)^t} \\ &= s(K) \end{aligned}$$

$$(4.7)\qquad \begin{aligned} k_N = K_{t-1}/Y_{Nt} &= \frac{sY_0(1+g)^t(K)}{Y_0(1+g)^t[1-s(D)]} \\ &= \frac{s(K)}{1-s(D)} \end{aligned}$$

Since (K) decreases as λ or the speed of the write-off increases, it is obvious that the gross capital-output ratio will be the smaller the larger is the assumed rate of obsolescence. (D) rises as λ increases and the speed of the write-off increases $(g > 0)$. Thus k_n will fall with increasing λ, however, if the rate at which (D) increases is sufficiently less than the rate of decrease in (K), this turns out to be the case. Thus by the simple expedient of assuming a higher rate of obsolescence one can reduce the capital-output ratio and thus increase the measured average productivity of capital.

Factor Shares

Consider the functional distribution of gross income, with the reward of capital equal to aY_t and the reward of labor equal to $(1 - a)Y_t$. When we consider net output, it is non-labor income alone which is affected by the deduction of "depreciation" charges. We should therefore expect labor's share of net income to be larger than its share of gross income, but we may hope that this measure will not be affected by the choice of depreciation accounting technique. This hope is not realized.

(4.8)
$$a_N = \frac{aY_t - D_t}{Y_{Nt}} = \frac{aY_0(1+g)^t - sY_0(1+g)^t(D)}{Y_0(1+g)^t[1 - s(D)]} = \frac{a - s(D)}{1 - s(D)}$$

Thus as (D) increases as more allowance is made for obsolescence, property's share decreases, even though gross shares remain the same.

Rate of Return on Capital

The rate of return on capital, or property income as a per cent of the value of the capital employed in producing it, is also affected by the assumptions used to define and measure the stock.

(4.9)
$$r_G = \frac{aY_t}{K_{t-1}} = \frac{aY_0(1+g)^t}{sY_0(1+g)^t(K)} = a/s(K)$$

(4.10)
$$r_N = \frac{a_N Y_{Nt}}{K_{t-1}} = \frac{aY_0(1+g)^t - sY_0(1+g)^t(D)}{sY_0(1+g)^t(K)} = \frac{a - s(D)}{s(K)}$$

It is obvious that r_G increases as (K) falls with greater allowance for obsolescence. What may be less apparent is that r_n also increases, since (K) falls more rapidly than (D) rises. Thus the measured marginal as well as average productivity of capital can be increased, gross and net, by the assumption of higher rates of obsolescence.

Values of the gross and net capital-output ratio, of capital's share of output, and of the rate of return on capital are shown in Table 4.3 for an economy in which gross output is growing at a rate of 2% per annum, in which 10% of each year's output is invested in the form of gross additions to the stock of fixed capital; and in which 35% of gross income (= output) accrues in the form of non-labor income. Two asset lives are considered—fifty years and twenty-five years; two rates of embodied technical change are investigated—2% and 4% per annum; and the values of these statistics are computed for two years, when $t = L + 1$ and as $t \rightarrow \infty$.

Several remarks may be made about these figures. In the first place for the declining balance estimates both (K) and (D) increase as $t \rightarrow \infty$. This means that the capital-output ratio will increase and that capital's share of net output and both net and gross rates of return will decline over time. In the second place the gross capital-output ratio falls quite predictably as more allowance is made for improvement in capital goods. This is also true of the net capital-output ratio with the exception of CDB when $\lambda = 1/L$; in this case the extremely low depreciation allowance offsets the slightly higher value of (K) and thus k_N for CDB is lower than that for DM2.

Third, capital's share of net output falls as more rapid write-offs are used so long as we confine ourselves to traditional depreciation accounting methods. Using the embodied technical change assumptions, however, we find that net capital's share rises as λ increases since depreciation charges are so small. This deserves some comment. Looking at the formulae in Table 4.2 we observe that the depreciation allowances for COH and CDB are the lowest of all for their particular survival assumptions; they are, in fact, even less than those generated by the gross stock assumptions, since these assets which are physically retiring are valued in terms of their productive potential in today's technology. Thus for the embodied stocks, depreciation is a measure of the loss in productive capacity in terms of today's technology which results from the physical depreciation which accrues on the gross stocks. The conventionally defined net stocks are reduced by an amount covering both physical decay and the revaluation of existing assets.

Fourth, the rate of return on capital, both net and gross, increases the more one allows for obsolescence. In the case of COH a λ of $1/L$ leads to a rate of return somewhat less than the SL rate; CDB with $\lambda = 1/L$ has a yield in excess of that on a DM2 stock if returns are measured net, though not in the case of the gross return. Within each survival assumption, however, and within each type of accounting—conventional depreciation or embodiment—the greater the allowance for obsolescence, the higher the rate of return.

Finally, for any given weighting and mortality assumption, the calculated values of these ratios will also depend on L, the service life. Capital-output ratios, property's share ratios, and both rates of return fall as the service life is reduced, other things being equal.

This example is, of course, highly idealized. Investment and output are well-behaved, the λ's were selected with some care, and the two service life assumptions are rather far apart. The results do serve to illustrate a rather

TABLE 4.3

SELECTED AGGREGATE MEASURES FOR AN ECONOMY WITH OUTPUT GROWING AT 2% PER ANNUM, A SAVING RATIO OF 10%, AND PROPERTY'S SHARE 35% OF INCOME

	L = 50				L = 25			
	Gross		Net		Gross		Net	
Method	$t = L + 1$	$t \to \infty$	$t = L + 1$	$t \to \infty$	$t = L + 1$	$t \to \infty$	$t = L + 1$	$t \to \infty$
A. *Capital-Output Ratios*								
OH		3.15		3.27		1.95		2.08
SL		1.82		1.95		1.06		1.15
SD		1.34		1.45		0.73		0.80
COH (λ = 0.02)		2.18		2.21		1.59		1.65
COH (λ = 0.04)		1.62		1.63		1.32		1.35
DM1	2.15	2.48	2.25	2.61	1.28	1.63	1.35	1.75
DM5	1.82	1.97	1.93	2.10	1.06	1.21	1.19	1.31
DM2	1.56	1.63	1.67	1.75	0.90	0.96	0.97	1.09
DCB (λ = .02)	1.59	1.67	1.64	1.73	1.08	1.24	1.13	1.31
CDB (λ = .04)	1.25	1.27	1.28	1.30	0.93	1.01	0.97	1.05
B. *Capital's Share of Output*								
OH		0.35		0.325		0.35		0.308
SL		0.35		0.306		0.35		0.295
SD		0.35		0.298		0.35		0.290
COH (λ = 0.02)		0.35		0.341		0.35		0.325
COH (λ = 0.04)		0.35		0.347		0.35		0.334
DM1		0.35	0.320	0.316		0.35	0.314	0.313
DM5		0.35	0.312	0.308		0.35	0.304	0.296
DM2		0.35	0.329	0.327		0.35	0.321	0.316
COH (λ = 0.02)		0.35	0.329	0.327		0.35	0.321	0.316
COH (λ = 0.04)		0.35	0.333	0.333		0.35	0.324	0.322
C. *Rate of Return on Capital*								
OH		0.111		0.099		0.179		0.148
SL		0.192		0.157		0.330		0.256
SD		0.261		0.206		0.481		0.364
COH (λ = 0.02)		0.160		0.154		0.220		0.197
COH (λ = 0.04)		0.216		0.213		0.265		0.248
DM1	0.163	0.141	0.142	0.121	0.273	0.215	0.232	0.176
DM5	0.192	0.178	0.161	0.147	0.330	0.289	0.267	0.226
DM2	0.224	0.215	0.183	0.174	0.389	0.365	0.305	0.280
CDB (λ = 0.02)	0.220	0.209	0.200	0.189	0.324	0.287	0.283	0.242
CDB (λ = 0.04)	0.280	0.275	0.260	0.255	0.374	0.347	0.336	0.306

important point, however. The basic facts of this economy—its rate of growth, its gross investment ratio, and gross property income are given. We may even consider that it has in all instances the same collection of tangibles physically present. The differences, then, in the description of the economy provided by the statistics calculated above are simply a reflection of different bookkeeping assumptions. It thus becomes obvious that movements in these statistics for an economy or firm over time or differences between economies or firms operating at the same time may simply reflect different accounting procedures applied to basically similar collections of assets. Real differences may, therefore, be obscured by differences attributable to assumption rather than to economic reality. This is, however, a highly unrealistic example. What happens when we look at such statistics for the United States during the period under discussion?

The capital aggregate used in the following analysis is the replacement cost value of the stock of all assets other than consumer durables and assets owned by the government. Since residential structures with their extremely long service lives constitute an appreciable fraction of the stock, the embodiment estimates are extremely close to the SL and DM2 series.

The output concept is domestic rather than national product; estimates of gross product and income were reduced by the amount of net factor income originating in the rest of the world. The gross income and output series are official estimates of the U.S. Department of Commerce, Office of Business Economics, supplemented for years before 1929 by estimates adapted from Kendrick's Commerce Concept series.[22] Net income and output were derived by subtracting the depreciation estimates generated by the stock calculation from these gross income and product data.

"Property income" is taken to be equal to the sum of the components of domestic income whose value is influenced by depreciation allowances; it includes corporate profits before tax, rental income of persons, and entrepreneurial income; it does not include net interest. Thus the ratio of property income to total income should not, strictly speaking, be interpreted as a measure of capital's share unless one is willing to assume that such labor income as is included in entrepreneurial income is equal in amount to the net interest flow which is excluded.

Turning first to Tables 4.4 and 4.5, we find that the relationships among the various estimates of both net and gross capital output ratios are those we would expect in the light of the results shown in Table 4.3. Not only is there considerable fluctuation over time in the capital-output ratio based on a single method, but there is also considerable variation among the estimates for a particular year. The trends in the ratios also differ, the most extreme example being the decline in the sum-of-digits ratio between 1929 and 1939, a period over which all other methods produced increases. Had inventories been included in the numerator, the ratios using gross product would all have been increased by the same constant, and the discrepancies shown in Table 4.4 would have

22. Kendrick, *Productivity Trends in the United States*, *op. cit.*, pp. 296–297. For further details, see Tice, *op. cit.*, Appendix C.

TABLE 4.4

THE GROSS CAPITAL-OUTPUT RATIO

Depreciable Assets/GDP

Year	OH	SL	SD	COH	DM1	DM5	DM2	CDB
1900	2.92	2.03	1.59	2.20	2.22	1.96	1.73	1.74
1912	2.85	1.88	1.43	2.09	2.15	1.85	1.61	1.63
1922	3.37	2.06	1.50	2.37	2.47	2.06	1.75	1.79
1929	3.25	2.00	1.49	2.31	2.43	2.03	1.74	1.78
1933	5.06	2.94	2.11	3.49	3.71	3.03	2.54	2.61
1939	3.79	2.08	1.44	2.50	2.80	2.22	1.81	1.86
1945	2.26	1.21	0.82	1.45	1.67	1.31	1.06	1.07
1951	2.84	1.58	1.12	1.89	2.15	1.70	1.40	1.43
1958	3.27	1.86	1.36	2.24	2.50	2.00	1.66	1.72
1962	3.05	1.76	1.29	2.11	2.36	1.89	1.58	1.63

TABLE 4.5

THE NET CAPITAL-OUTPUT RATIO

Depreciable Assets/NDP

Year	OH	SL	SD	COH	DM1	DM5	DM2	CDB
1900	3.00	2.20	1.76	2.23	2.35	2.11	1.90	1.82
1912	2.93	2.04	1.58	2.12	2.28	1.99	1.76	1.71
1922	3.52	2.26	1.67	2.41	2.64	2.24	1.93	1.88
1929	3.42	2.19	1.66	2.36	2.61	2.22	1.92	1.87
1933	5.43	3.40	2.47	3.60	4.13	3.44	2.92	2.81
1939	4.20	2.30	1.61	2.62	3.03	2.43	2.00	1.96
1945	2.36	1.28	0.88	1.48	1.75	1.38	1.12	1.11
1951	2.97	1.72	1.23	1.92	2.30	1.85	1.53	1.50
1958	3.55	2.06	1.52	2.32	2.70	2.20	1.85	1.78
1962	3.23	1.93	1.43	2.17	2.54	2.06	1.73	1.72

been the same. The ratios using net product, however, would have been increased by amounts whose size is inversely related to the value of net output. Since the rank ordering of net output is not the same as the rank ordering of the capital-output ratios, the inclusion of inventories in the capital concept would in many instances lead to even greater disparities among the ratios derived under the various assumptions than those shown in Table 4.5.

We next examine the estimates of "property's share" of net domestic income presented in Table 4.6. There has been a decline over the period covered by the analysis in the share of property income in gross income; this decline is of course reflected in all net estimates as well. As one would expect, there are differences in this ratio among the various estimates pertaining to any given year. These discrepancies, moreover, are of such a size that were they to occur over time for a particular method, they might well be taken as evidence of slight secular movement in property's share. It is obvious that an income series based, as are the OBE's estimates, on enterprise records may show secular movements which are more the reflection of changing accounting practice than of real shifts in factor shares, since over time service lives and depreciation accounting techniques are changing.

TABLE 4.6

SHARE OF INCOME ATTRIBUTABLE TO THE OWNERSHIP OF TANGIBLES IN NET DOMESTIC INCOME

Year	OH	SL	SD	COH	DM1	DM5	DM2	CDB
1900	0.435	0.402	0.386	0.444	0.415	0.403	0.394	0.423
1912	.440	.408	.395	.448	.420	.410	.402	.429
1922	.381	.348	.336	.398	.364	.353	.345	.378
1929	.367	.337	.325	.387	.353	.342	.332	.367
1933	.202	.125	.114	.239	.166	.144	.132	.200
1939	.270	.271	.267	.317	.289	.279	.274	.312
1945	.310	.302	.299	.331	.311	.306	.303	.324
1951	.343	.313	.303	.362	.325	.315	.308	.340
1958	.262	.247	.236	.299	.264	.252	.243	.298
1962	0.266	0.236	0.228	0.291	0.253	0.242	0.234	0.272

The last ratio to be considered is the rate of return on capital, or income attributable to the ownership of depreciable assets as a percent of the value of these assets. Results of this calculation for the United States are shown in Tables 4.7 and 4.8. The relationships which we would expect from the results shown in Table 4.3 are again confirmed here, with a few exceptions in the case of CDB estimates. The net rate of return on the CDB stock exceeds that on the SD stock in 1929, 1933, and 1958. Again the differences in rates of return shown for any single year are in many cases as large as differences exhibited for one method over time; and it is again likely that some of the differences in measured rates of return among firms or among economies may reflect differences in accounting conventions rather than differences in the productivity of capital.

V. CONCLUSIONS

It is clear that the assumptions which define a country's stock of tangible capital are of considerably greater importance than has often been supposed. Stock levels are extremely sensitive to this choice of assumption; the trends in the stock series, though not remarkably different when gross investment is

TABLE 4.7

GROSS PROPERTY INCOME AS A PER CENT OF DEPRECIABLE ASSETS

Year	OH	SL	SD	COH	DM1	DM5	DM2	CDB
1900	14.56	20.92	26.75	19.32	19.14	21.76	24.49	24.40
1912	15.15	22.92	30.19	20.63	20.06	23.38	26.82	26.43
1922	11.22	18.35	25.18	15.96	15.29	18.34	21.55	21.12
1929	11.24	18.30	24.53	15.85	15.04	18.00	21.02	20.55
1933	4.41	7.60	10.56	6.40	6.01	7.37	8.79	8.55
1939	8.19	14.92	21.52	12.40	11.10	14.00	17.11	16.72
1945	13.78	25.68	37.79	21.40	18.58	23.78	29.41	28.93
1951	11.88	21.31	30.17	17.90	15.71	19.82	24.10	23.62
1958	9.05	15.90	21.80	13.20	11.84	14.79	17.79	17.21
1962	9.18	15.96	21.77	13.28	11.86	14.81	17.79	17.19

TABLE 4.8

NET PROPERTY INCOME AS A PER CENT OF DEPRECIABLE ASSETS

Year	OH	SL	SD	COH	DM1	DM5	DM2	CDB
1900	13.70	17.20	20.52	18.80	16.57	17.94	19.43	21.84
1912	14.25	18.93	23.59	20.08	17.38	19.42	21.60	23.79
1922	9.94	14.08	18.31	15.14	12.60	14.36	16.28	18.44
1929	9.71	13.84	17.55	14.87	12.22	13.86	15.58	17.76
1933	3.06	2.99	3.75	5.52	3.28	3.42	3.68	5.88
1939	5.60	10.25	14.48	10.66	8.36	10.01	11.90	14.00
1945	11.73	21.00	30.49	20.02	15.83	19.76	24.11	26.16
1951	10.41	16.26	21.99	17.10	12.61	15.28	18.00	20.46
1958	6.65	10.80	13.92	11.65	8.80	10.30	11.83	15.13
1962	7.36	10.93	14.19	12.06	8.88	10.43	12.02	14.18

relatively free of fluctuations, become much more sensitive to choice of method in the case of a volatile investment series.

Because of the link with the income and product accounts provided by the depreciation estimates, the way in which one defines the stock of capital also influences the definition of many other descriptive statistical measures. Thus what we may observe as a trend in an indicator over time may be a statistical artifact reflecting nothing more than changes in the conventional treatment of physical decay and obsolescence.

Les estimations de l'aggrégat du stock de capital des Etats-Unis dépendent de la méthode de dépréciation choisie par le statisticien. Le montant traditionnel de la dépréciation peut être considéré comme englobant les dotations de déterioration physique et d'obsolescence. Si le stock brut est défini comme le stock des avoirs subsistants, les divers stocks nets définis en comptabilisant la dépréciation peuvent être définis comme une réévaluation de ces avoirs à l'aide d'un indice du changement technologique incorporé. Les estimations du stock de capital des Etats-Unis ont été faites selon huit groupes d'hypothéses. Ces estimations sont comparées quant à leur niveau, leur tendance, et leurs implications pour d'autres indicateurs statistiques aggrégés. L'auteur en conclut que les hypothèses qui définissent le stock de capital d'un pays sont d'une importance beaucoup plus grande qu'on ne le suppose souvent.