THE SOCIAL ACCOUNTS FROM A CONSUMER'S POINT OF VIEW

AN OUTLINE AND DISCUSSION OF THE REVISED UNITED NATIONS SYSTEM OF NATIONAL ACCOUNTS

BY RICHARD STONE

Department of Applied Economics,
University of Cambridge

After a short introduction, the first part of this paper (section 3 through 9) provides an outline of the revisions proposed to the System of National Accounts (SNA) of the United Nations which are now under discussion. These proposals were considered by an expert group at the end of 1964 and were accepted by the Statistical Commission of the United Nations in 1965 as the basis for further work on the extension and revision of the SNA. The aim of the revision is to provide a fully integrated system of accounts and balance sheets in which input-output, flows-of-funds and sector balance sheets are incorporated in a generalised accounting framework. Whereas the real side of the economy has been studied analytically in many countries (input-output analysis, demand analysis and so on) much less experience is available on modelling the financial side of the economy, apart from econometric work on saving behaviour, which is fairly widespread. Accordingly, the second part of the paper (sections 10 through 14) contains a discussion of financial model-building in which a number of possibilities are explored. The final topic discussed (section 15) is demographic accounting, by which is meant a framework for recording and analysing human, as opposed to economic, flows and stocks. The development of such a system arose out of the emphasis placed by the expert group on the integration of demographic and economic information.

1. INTRODUCTION

The purpose of this paper is to look at social accounting from the point of view of a user and, in particular, to examine the recent proposals for revising and extending the United Nations' *System of National Accounts* (SNA) in this light. The stage of development of the subject to be considered here can be summarised as follows. The last ten years have seen the application in many parts of the world of the SNA or systems very like it: the *Yearbook of National Accounts Statistics* is sufficient testimony to this. Efforts to improve statistics have led to a reconsideration of taxonomic problems in many fields: private consumption, capital formation, public sector statistics, foreign trade and the balance of payments are cases in point. At the same time, input-output studies, which are directed to the disaggregation and elaboration of the flows in the production account of an economy, have been widely applied and in some degree related to the social accounts. The same can be said, though the applications are much less widespread, of flow-of-funds studies, which concentrate on an important financial aspect of economic systems namely the transactions in financial claims which accompany the translation of sectoral saving into the finance of sectoral investment. In this period we can even find a few examples of statistical work on national and sectoral balance sheets, thus completing a study of flows in an economy with a corresponding study of stocks.

With the widespread attention to so many lines of research, it was natural that, in planning a revision of the SNA, the Statistical Office of the United Nations should look well beyond the original framework of 1953 and the minor amendments later introduced into it [17] and aim at a full and detailed treatment of flows and stocks in an economy. A standard system is expected to last for at least a decade and would clearly get off to a bad start if it did not take account of developments already successfully pioneered in many countries.

This brief statement expresses a producers' point of view. Economic statisticians all over the world have carried their work so far beyond the original SNA that a much wider framework is now needed. The same can be said without much fear of contradiction from the point of view of consumers. I realise, of course, that there are almost endless uses for social accounting data but I do not think there is much conflict in the case of particular pieces of information, between the narrower and the broader uses. It is true that one economist, being mainly interested in the real side of the economy, might prefer a concentration on, say, production, consumption and other uses of product, input-output and employment, whereas another economist, being mainly interested in the financial side of the economy, might prefer a concentration on, say, income, saving, flow-of-funds and balance sheets. But neither partial user would lose anything if the information in which he was particularly interested was presented as part of an integrated system. There are, moreover, many uses which require a coherent and detailed view of the whole of the economy. Whether we are interested in knowing what has taken place in an economy, in understanding why this should have been so, in forecasting what will take place or in exploring what might take place, we have to interest ourselves in all the pieces of the statistical jigsaw puzzle and try to put them together. In doing this we need the kind of framework envisaged in the proposals for revising the SNA.

Accordingly, I feel justified in taking as my archetypal consumer of social accounting data the modeller of national economic systems. He will have to look at all the pieces and so, in a general way, more specialised interests will be taken into account. Modellers of the world economy will simply find the same old problems in aggravated form.

2. Models of National Economic Systems

In 1964 I conducted an international survey of planning models [13] from which a number of relevant facts emerged. First, there is a considerable number of these models, built or under construction. I have received reports on over forty such models and, since I am in no position to carry out a truly systematic survey but must rely on personal contacts and general knowledge, there is little doubt that my sample falls far short of complete coverage. Second, almost all these models make use to a greater or less extent of a social accounting framework, and comments on the adequacy of local data vary considerably. Third, the models tend to belong to one of two main classes: short-term forecasting models and long-term planning models. In either case a social accounting framework provides the best means of ensuring that the various parts of the model give consistent results that satisfy arithmetical and accounting identities.

It is sometimes said that there is a third type of model that expresses the more modest aim of understanding the past rather than forecasting or planning the future. This distinction, it seems to me, turns more on the use to which a model is put than on the nature of the model itself. Forecasting and planning presuppose that we do understand the past and that, if we had enough data about the past, our model would give a reasonable approximation to what had happened.

The requirements of forecasting and planning models, though different, are not as different as might be supposed. A forecasting model is usually a dynamic short-term model in which we try to work out what would happen if, over the next few quarters, the economy were allowed to develop 'naturally'. I say quarters rather than years because the error terms in our equations are likely to yield forecasts that diminish in accuracy rapidly through time. A planning model, on the other hand, is usually a static long-term model in which the initial conditions which appear in the forecasting model are replaced by a set of aims and the problem is to decide what parameters to change in order to realise these aims.

A very simple example will illustrate this distinction. Let us consider a saving-investment model in which the possibilities open to an economy are determined by the following relationships.

$$\sigma = \beta\mu - \alpha\omega \tag{2.1}$$

$$\dot{\omega} = \kappa\dot{\mu} \tag{2.2}$$

$$\sigma = \dot{\omega} \tag{2.3}$$

where σ, μ and ω denote, respectively, saving, income (or net output) and wealth, a dot denotes the first derivative with respect to time of the variable it surmounts and α, β and κ denote positive constants. The first equation states that savers respond positively to income and negatively to wealth. The second equation states that the rate of change of wealth over time is a constant multiple of the rate of change of income over time. And the third equation states that saving is equal to the rate of change of wealth over time, that is to investment.

The solution of this system for any time θ can be written as

$$\mu = \frac{\alpha}{\rho\kappa}(\omega_0 - \kappa\mu_0) + \frac{1}{\rho\kappa}(\beta\mu_0 - \alpha\omega_0)\, e^{\rho\theta} \tag{2.4}$$

$$\omega = \frac{\beta}{\rho\kappa}(\omega_0 - \kappa\mu_0) + \frac{1}{\rho}(\beta\mu_0 - \alpha\omega_0)\, e^{\rho\theta} \tag{2.5}$$

$$\sigma = (\beta\mu_0 - \alpha\omega_0)\, e^{\rho\theta} \tag{2.6}$$

where ω_0 and μ_0 denote respectively the values of ω and μ at time $\theta = 0$ and $\rho \equiv (\beta/\kappa) - \alpha$. The term $(\omega_0 - \kappa\mu_0)$ denotes the excess of initial wealth over the product of income and the marginal wealth-income ratio; and the term $(\beta\mu_0 - \alpha\omega_0)$ denotes the initial level of saving.

In this simple situation, a forecaster would need to know ω_0, μ_0, α, β and κ and could then predict μ, ω and σ at time θ from (2.4), (2.5) and (2.6). A planner, interested in increasing the rate of growth of the economy, would see that his chances of success depended on his ability to increase β or to diminish κ or α. The relative effectiveness of these different possibilities could be measured by working out the elasticity of the growth rate, ρ, with respect to α, β and κ. Thus

$$\frac{\partial\rho}{\partial\alpha}\cdot\frac{\alpha}{\rho} = -\frac{\alpha\kappa}{\beta - \alpha\kappa} \tag{2.7}$$

$$\frac{\partial\rho}{\partial\beta}\cdot\frac{\beta}{\rho} = \frac{\beta}{\beta - \alpha\kappa} \tag{2.8}$$

$$\frac{\partial\rho}{\partial\kappa}\cdot\frac{\kappa}{\rho} = -\frac{\beta}{\beta - \alpha\kappa} \tag{2.9}$$

And so a given proportionate increase in β would have exactly the same effect on the growth rate as the same proportionate reduction in κ. The same proportionate reduction in α would necessarily have a smaller effect since, if the economy is growing initially, $\beta > \alpha\kappa$. The effort required to bring about these different effects would indicate a course of action for the planner.

The purpose of this theoretical digression is to illustrate the different tasks of forecasters and planners. The information they need about the way the economy works is very similar; the difference comes in the use to which this information is put. But plans, perhaps more so than forecasts, should exhibit many types of consistency [11] and certainly consistency in an arithmetical and accounting sense. The establishment of a satisfactory framework and the ability to use it in model-building is therefore highly relevant to a wide range of tasks which nowadays occupy economists and statisticians.

Once we are agreed that we need a framework, we must next decide on the form this framework is to take. Clearly it must satisfy a number of requirements. For example, it must: (i) provide a complete set of arithmetical and accounting checks; (ii) reflect important economic and accounting distinctions, such as intermediate and final or current and capital; (iii) indicate compromises between what is desirable analytically and what is practicable statistically; (iv) relate the varied classifications needed to connect the different parts of an economy; and (v) avoid the unnecessary piling of one set of categories on another. Let us now look at the proposals for revising the SNA in the light of these requirements.

3. The Revised SNA in Outline

The proposals in [15] emerged from the discussion of an expert group, convened by the Statistical Office of the United Nations, which met at the end of 1964. They were considered by a working group of the Conference of European Statisticians [18] in the spring of 1965 and, shortly afterwards, came before the Statistical Commission of the United Nations at its thirteenth session [16]. The Commission accepted, subject to certain modifications, the general approach and the general structure of accounts and tables proposed by the expert group as the basis for further work on the extension and revision of the SNA. It was contemplated that, before a final version could be published, further work would be needed in two main areas: (i) the ultimate details of the classifications and definitions proposed; (ii) a number of topics (such as the scope of national and sector balance sheets, the integration of data on the distribution of income and the development of demographic data capable of being precisely related to the social accounts) which had been put forward by the expert group but not spelled out in detail by them.

These brief remarks are intended to indicate the present position of the proposals now to be outlined.

Like its predecessor, the new system is based on the national accounts. These are, essentially, four in number: (i) a production account; (ii) an income and outlay account; (iii) a capital transactions account; and (iv) an external account (or account for the rest of the world). The first three of these accounts contain not only transactions with the rest of the world, which are mirrored in the external account, but also all transactions internal to the economy we are studying. The first two accounts relate to current transactions and the third relates to capital transactions.

The revised system extends this broad framework in two ways. First, the capital transactions account is divided into two: the first part, called a capital expenditure account, relates to real investment, that is investment in fixed assets and stocks; the second part, called a capital finance account, relates to the finance of this investment and the accompanying transactions in financial claims. Second, to the set of accounts there is added an opening and a closing national balance sheet and also a revaluation of existing assets and liabilities, real and financial, needed to place the entries in the closing balance on a current basis of valuation. The result of these extensions before any further detail is introduced is set out in table 1.

TABLE I
THE NATIONAL ACCOUNTS AND BALANCE SHEETS IN MATRIX FORM

			1	2	3	4	5	6	7	8
Opening balance sheet		1					A_0			
Current accounts	Production	2			C	V		E		
	Income and outlay	3		P			$-D$	Z		
Capital accounts	Capital expenditure	4					V			
	Capital finance	5	L_0		S			K	R	L_1
Rest of the world account		6		I	G		B			
Revaluations		7					R			
Closing balance sheet		8					A_1			

The meaning of the symbols in this table can conveniently be explained in terms of the accounting relationships to which it gives expression. Each row and column pair balances and so we obtain the following equations.

The first row and column lead to

$$A_0 = L_0 \tag{3.1}$$

that is, opening assets are equal to opening liabilities.

The second row and column lead to

$$C + V + E = P + I \tag{3.2}$$

that is, consumption plus gross domestic capital formation plus exports, which are the sources of revenue for domestic production, are equal to gross value added plus imports, which are the corresponding outgoings.

The third row and column lead to

$$P - D + Z = C + S + G \tag{3.3}$$

that is, net value added plus net income from abroad equals consumption plus saving plus net current transfers abroad.

The fourth row and column lead to the apparently empty statement

$$V = V \tag{3.4}$$

and, indeed, the distinction between the two capital accounts is not needed at the national level. However, in disaggregating this consolidated system, we shall take as the institutional unit in the real accounts (2 and 4) the technical unit or establishment, which can be combined to form industries, whereas we shall take as the institutional unit in the financial accounts (3 and 5) the financial unit, such as enterprises or households, which can be combined to form sectors. Equation (3.4), therefore, simply relates alternative classifications: gross investment in all industries equals gross investment in all sectors.

If we confine ourselves to the flows in the fifth row and column, we see that

$$S + K = V - D + B \tag{3.5}$$

that is, national saving plus net capital transfers from abroad (or foreign saving transferred to the nation) equals net investment at home and abroad. Ignoring revaluations for the moment, this total represents the net addition to the nation's wealth.

The sixth row and column lead to

$$I + G + B = E + Z + K \tag{3.6}$$

that is, imports plus net current transfers to abroad plus net investment abroad equals exports plus net income from abroad plus net capital transfers from abroad.

The seventh row and column lead to another apparently empty statement

$$R = R \tag{3.7}$$

This means that, in total, revaluations of assets are equal to revaluations of liabilities. Since the balance sheet entries which we can attempt to revalue are restricted to real assets and financial claims (which may be held as assets or as liabilities), (3.7) implies a particular revaluation of accumulated saving or net worth, that is the wealth of the nation.

Finally, the eighth row and column lead to

$$A_1 = L_1 \tag{3.8}$$

that is, closing assets equal closing liabilities.

If we review this system of accounts and balance sheets in the light of the five requirements set out at the end of the preceding section, the position can be summarised as follows.

(i) The set of accounting checks is complete since not only must each account balance but so must the opening and closing balance sheets. Further the opening and closing balance sheets are reconciled by the entries in the capital finance account and the revaluations.

(ii) Important distinctions appear although few can be made at such a high level of aggregation. In addition to the distinctions between home and abroad and between current and capital, which appeared in the original SNA, we also find a distinction between real and financial and an explicit treatment of stocks as well as flows.

(iii) Problems of reconciling analytical requirements with statistical possibilities can hardly show up at the highest level of aggregation. The revised SNA goes into considerable detail and is, in principle, capable of endless further development. But by consolidation the detail can be reduced to the outline; and by consolidating 4 and 5 in table 1 and by omitting 1, 7 and 8, we can return to the simplest system of four national accounts. The system is therefore very flexible; parts of it can be treated in detail, in line with analytical interests and the availability of statistics in different countries while other parts are treated in very summary manner. The respective parts can vary from country to country but there will always be some level of aggregation at which any pair of countries can be compared.

(iv) This requirement can only be met in a very general way in a consolidated system. The main distinctions, as in (ii) above, will make their appearance but little else. We shall return to this subject in the following sections.

(v) The real-financial distinction is introduced mainly to avoid an excessive piling up of categories in the different parts of the system. For example, in studying industrial structure, it is important to know how much coal goes into steel-making but not to know how much coal mined in private unincorporated mines goes into nationalised steel mills. Again, in studying the flow of funds, it is important to know how sectoral saving finds its way into the finance of sectoral investment but not to know how the saving of coal mines and their staffs finds its way into the finance of capital expenditure in steel mills. The two sides of the economy, the real and the financial must, of course, be firmly linked together; but, to do this, it is not essential that the primary classification of each part should be reproduced as a subsidiary classification in the other.

Let us now turn to the various parts of the system outlined in table 1 and see what further details are desirable.

4. The Production Accounts

When we subdivide the national production account, we have to face the fact that the ultimate production units from which we collect information (which I shall call establishments) typically produce more than one product or commodity. The usual way of dealing with this situation is to examine the composition of production in the different establishments and, from this experience, draw up a set of distinct principal product groups to which establishments can be assigned. The total output of the set of establishments assigned to a particular group defines the output of the corresponding industry. Typically these establishments will produce subsidiary products in addition to their principal products and, if we define commodities as principal product groups, we can arrange all our information about products in a square matrix with, say, industries in the rows and commodities in the columns. The row totals of such a matrix measure industry outputs and the column totals measure commodity outputs. The two sets of totals will, in general, be different unless each industry produces only its own principal products. In practice, with even a moderate degree of disaggregation, the two sets of totals are different because establishments, and therefore industries, in fact produce subsidiary products. By a suitable choice of principal product groups, subsidiary production can be diminished but it cannot be got rid of altogether.

If we now turn to industrial inputs of intermediate products we find that what we can observe is the input of commodities into establishments and, therefore, into industries. We do not know from exactly which industry each commodity came nor do we know exactly how it is used in the production of different commodities. This state of affairs holds more generally: buyers buy commodities, inputs are made into industries. This is just as true of final buyers as of intermediate buyers and of primary inputs as of intermediate inputs.

Thus, in arranging data in an orderly way, it is useful to make a distinction between commodities and industries. In the revised SNA there is a set of accounts for commodities which show the supply of each commodity from some branch of domestic production or from abroad and the absorption of each commodity into some form of intermediate or final demand. There is also a set of industry accounts whose incomings are obtained exclusively from the commodity accounts and whose outgoings are devoted to intermediate products and primary inputs, including provisions for the consumption of fixed capital and indirect taxation.

But this is not the end of the story. The principal final buyers, namely private and public consumers, group their expenditures on goods and services into categories which do not exactly correspond to industrial commodities. In all our statistical work we necessarily employ a considerable degree of grouping; we employ tens or at most hundreds of commodity categories and never the tens of thousands that actually exist. Accordingly, we find that a consumers' category, like clothing and footwear, puts demands on several commodity groups, such as textiles, clothing and rubber, and also on transport and distributive services since, in input-output analysis, these are necessarily treated as margin activities.

In order to make demand analyses and projections we need a third classification whose categories relate to consumers' goods and services. But we can only study the industrial implications of these demands if consumers' goods and services are related to industrial commodities. This can be achieved if we introduce a set of accounts for consumers' goods and services whose incomings come from the income and outlay accounts (and also the account for the rest of the world in respect of expenditure by foreign tourists) and whose outgoings show the cost composition of these outlays in terms of commodities, indirect taxes, direct income payments, depreciation and expenditure abroad.

An exactly similar problem arises with public consumers but this time the classification required relates to purposes rather than to goods and services. In order to study public consumption we need a fourth set of accounts, for government purposes, whose incomings come from the income and outlay accounts and whose outgoings show the cost composition of one of the purposes.

When we come to capital expenditures and exports, we can usually work directly in terms of commodities and so no new categories are needed. Of course, we might not wish to do this: for example, we might prefer to keep a separate export account for each region of the rest of the world and we should then have to convert regional demands into demands for commodities.

So far, we have accumulated four classes of production accounts relating, respectively, to commodities, industries, consumers' goods and services and government purposes. In the revised SNA a fifth class is introduced relating to taxes on commodities. The main object of this class is to assist input-output analysis by increasing homogeneity in the commodity accounts. If input-output analysis is based on money values, as it almost always is, then it is important that valuations should be reasonably homogeneous along the commodity rows so that £1's worth of demand for a particular commodity will have an effect on the productive system which is independent of the buyer. This will only be the case if commodities are valued exclusive of at least the more important commodity taxes. Thus if eighty per cent of the cost of cigarettes is duty and if this duty is remitted in the case of export sales, then the stimulus given directly to the production of cigarettes, and so indirectly to the productive system in general, by £1's worth of export sales, will be matched by £5's worth of domestic sales. Only by valuing both these sales in the same way can we avoid serious distortions in input-output analysis due to the unequal incidence of taxation. Moreover distortion may creep in in other ways even when the tax levied on individual commodities is independent of the buyer. We have seen that our definitions of commodities are likely to be extremely composite. One of these commodities might be petroleum products which would include both fuel oil and petrol. These products are used by different classes of buyers. If one is virtually untaxed and the other is highly taxed, misleading conclusions will result unless the tax element is removed from the composite commodity.

For reasons such as these a separate set of accounts for commodity taxes are introduced into the revised SNA and the commodity flows are valued excluding these taxes. At the same time these taxes are debited to uses, intermediate or final. If to any entry in the commodity accounts the corresponding entry in the commodity tax accounts is added, there results an estimate of the commodity flow in question at producers' values. Although commodity taxes are indirect taxes, the introduction of commodity tax accounts (which need only relate to the more important of these taxes) does not affect the treatment of indirect taxes as a part of value added. As is usual, the revised SNA divides value added between compensation of employees, income from property and entrepreneurship, provisions for the consumption of fixed capital and net indirect taxes.

Thus, the production accounts in the revised SNA are devised to make possible a full analysis of the flow of products in the economy. The accounts for commodities (industrial products) show where the various commodities come from and the uses to which they are put. The distinction between competitive and complementary products, which differs in content from one economy to another, is not made explicitly but emerges through the categories of the commodity classification. Thus the difference in importance of the entries for imports and domestic production in the columns of the commodity accounts brings out the extent to which imports contribute to total supply and makes it possible to relate imports to total supply. Whenever, as in the cases of private and public consumption, the analysis of final demand requires a classification different from that of industrial products, this classification is provided for and related to the classification of commodities. Industries are distinguished from commodities and so a place is found for the components of industrial costs which, from an observational point of view, must, in general, be debited in the first instance to productive units and, therefore, to industries rather than to commodities. The final step of transforming information on commodities used by industries into estimates of commodities used in making commodities is discussed and the methods that might be adopted are described.

The accounting structure designed to meet these analytical needs is such that, when all the production accounts are consolidated, the familiar national account for production is obtained.

5. The Income and Outlay Accounts

Let us turn now from production to income and outlay. When we were considering production our main interest lay in groups of establishments, called industries, their output of commodities, the uses to which these commodities are put and the inputs, primary and intermediate, needed for their production. Now our categories are different. We are mainly interested in groups of financing units, called sectors, the income originating in them, the transfers of income between them and the division of ultimate, or disposable, income between the satisfaction of current needs (consumption) and provisions for the future (saving).

The problem of grouping economic units into sectors is an old one; indeed, the division of these units into companies, households and government was the only institutional division in the accounting system of the original SNA. In the revision of that system a number of changes are proposed, and yet others are under consideration. First, in view of the desire to increase the value of the system for financial analysis by introducing flow-of-funds information and sector balance sheets into the accounting structure, it seemed desirable to separate financial institutions from non-financial enterprises and to show these institutions in some detail. Second, some consideration was given to the further division of non-financial enterprises into broad categories along industrial lines. I put the matter in this way because the units to be grouped in this case are enterprises rather than establishments so that the categories would need to be different and, in general, broader than the industrial categories used for the production accounts. Third, a number of changes were proposed to the boundaries between the sectors; in particular, that government enterprises should no longer be separated from public corporations and that both categories should appear among enterprises rather than general government; and that while small unincorporated enterprises, such as farms and shops, should remain in the household sector, large ones, which permit an adequate accounting separation of the concern from its proprietors, such as family banking and shipping businesses, should be transferred to the company sector. Fourth, consideration was given to the need for a thorough reappraisal of private non-profit institutions and their eventual separation from households in view of the very great increase in the importance of this form of organisation. Finally, consideration was given to the desirability of introducing information on the distribution of income into the accounting structure by subdividing the household sector by income size or by socio-economic class. The introduction of information on the distribution of household income and outlay into the system is highly important from the users' point of view because, without this information, economic models can have nothing to say on the indirect effects of changes in the distribution of income. But the subject is not an easy one, since so much statistical work on the distribution of income makes use of units, the individual or the tax group, which do not fit readily into a classification of households; and, further, tax sources, which are so often used, do not as a rule give a complete coverage either of income or of income recipients. Any recommendations made in the present context would have to be suitable for general application and much remains to be done before an international standard can be established.

Given the sector classification, the next question is the treatment of income flows. The proposals are that all compensation of employees should be paid directly into the household sector (or abroad in the case of border workers), all indirect taxes (less subsidies) should be paid into general government and all income from property and entrepreneurship should be paid into the sector in which it arises. This means that we need to know only the two marginal totals in a matrix connecting the flow of non-labour income in industries and sectors. All further redistributions of this kind of income are treated as transfers. A separate set of accounts are set up for the various classes of transfer and sectors are shown either paying into these accounts or withdrawing from them.

Sectoral saving is paid into the sectoral capital finance accounts. It is not essential that the boundaries between sectors be drawn at the same places in the current and capital accounts. Thus it might be important to subdivide the current account for households, as suggested above, but unnecessary to subdivide the corresponding capital account. Contrariwise, it would certainly be important to subdivide the capital account for financial institutions but hardly necessary to subdivide the corresponding current account. In either case the link would be saving which would have to be estimated for the full range of sectors distinguished in either set of accounts. In the scheme just envisaged, household saving would flow from a number of current accounts into a single capital account whereas the saving of financial institutions would flow from a single current account into a number of capital accounts.

We have already seen that private and public current expenditures on goods and services are divided respectively into consumer goods and services and into government purposes. It is usual to keep these two classifications separate and in practice there is no great difficulty in doing this. For many purposes, however, it would be useful to have a combined classification so that the allocation of resources to consumption purposes could be seen as a whole independently of whether certain components were provided collectively or by individual initiative. A specific recommendation was not made in the revised SNA because the subject has not been studied sufficiently. It would seem that the main problems are likely to arise in connection with expenditures for health, education and culture generally.

Again, as with production, when all the income and outlay accounts are consolidated, the corresponding national account is obtained.

6. The Capital Expenditure Accounts

These accounts are concerned with gross investment. This investment is undertaken by industries or for various private and public purposes. The accounts, therefore, relate to industries or to those consumer goods and services or government purposes which can give rise to capital expenditure. In the case of industries, investment in stocks of goods and work in progress is separated from investment in fixed assets and shown in a separate set of accounts. The outgoings of these accounts are incomings for the commodity and commodity tax accounts, thus providing a link connecting the investment by industries and for non-industrial purposes with the products (machines, vehicles, construction) which this investment calls for.

By combining the industrial capital expenditure accounts with the industrial production accounts we obtain much of the information needed for the construction of production functions. Indeed, if we work with the type of vintage production functional introduced recently by Pyatt [5] we have all the regular accounting information we need provided it is understood that we measure not only values but also the associated quantities and prices. If we accept such ideas, we can forget about the problem of measuring capital employed when analysing the real side of the economy. If, however, we feel the need for a measure of capital employed, it is to these accounts that we should attach it although, as we saw in section 3 above, this is not done in the revised SNA. Such a measure would involve accumulating gross investment in fixed assets, industry by industry, and subtracting accumulated scrapping or retirements. This would give us some idea of capital capacity, the one-dimensional concept of capital. It would be important not to confuse scrapping with provisions for the consumption of fixed capital since, if the stock of capital is growing, the former will be smaller than the latter and, often, very considerably smaller.

While the outgoings of the capital expenditure accounts connect with the commodity and the commodity tax accounts, the incomings connect with the capital finance accounts. Thus, these accounts link the commodities required directly for gross investment in different industries and for different non-industrial purposes with the finance to be raised on account of these investments by the various sectors. The form in which this finance is raised, provisions for the consumption of fixed capital, saving or the issue or sale of financial claims is a matter for sectors and is set out in detail in the capital finance accounts.

If this set of accounts is consolidated we obtain two totals of gross investment: the first represents the total value of commodities added to the stock and the commodity taxes paid in respect of these additions; and the second represents the total value of finance obtained from the sectors to meet these outgoings.

7. The Capital Finance Accounts

As with the income and outlay accounts, the institutional units to which these accounts relate are sectors. These accounts show the sources and uses of capital funds and relate these flows to the opening and closing balance sheets of the sectors. In the case of the income and outlay accounts we saw that it was convenient to introduce a set of accounts for current transfers and some other accounts which would provide a link between value added in industries (together with income payments and indirect taxes debited directly to consumer goods and services and government purposes) and income originating in sectors. In the present case it is convenient, for similar reasons, to introduce a set of accounts for financial claims, which may be held as assets or as liabilities, and some other accounts to handle capital transfers and land and to enable industrial capital formation to be treated as a pair of marginal vectors rather than as a complete industry $\times$ sector matrix.

The incomings into the capital finance accounts for sectors are of three types: (i) sectoral saving received from the income and outlay accounts; (ii) new issues less redemptions of financial claims of all kinds; and (iii) net capital transfers received from all other sectors and the rest of the world. The corresponding outgoings are four in number: (i) outlays to finance gross investment; (ii) a negative item corresponding to provisions for the consumption of fixed capital; (iii) outlays for the net acquisition of land; and (iv) net acquisitions of financial claims as assets.

The incomings into the accounts for financial claims represent the net acquisition of these claims as assets by each of the sectors and the rest of the world; the outgoings represent the corresponding net issues by each of the sectors and the rest of the world.

If we consolidate all the capital finance accounts we obtain, apart from revaluations, all the links between the opening and closing balance sheets for the nation. On the incoming side we have: (i) total saving; and (ii) net acquisitions of financial claims and land by the rest of the world. On the outgoing side we have: (i) net domestic capital formation; (ii) net capital transfers received by the rest of the world; and (iii) net issues of financial claims by the rest of the world. The particular arrangement of these flows enables the capital finance account for each sector to provide the link (apart from revaluations) between the sector's opening and closing balance sheets, but it can easily be rearranged to give the identity: saving plus net capital transfers received equals net domestic capital formation plus net lending to the rest of the world.

If we also include the capital expenditure accounts in the consolidation, nothing is changed since, as we have seen, they connect alternative classifications of gross domestic capital formation.

When discussing the earlier parts of the system, I did my best to indicate their uses for model-building purposes but I did not attempt to explain these uses in detail. There is a large literature on such topics as consumption functions, production functions and input-output analysis, and on the ways in which they can be combined in a general model. But when we come to the financial end of things and, particularly, to the capital finance accounts and balance sheets there seems to be much less experience to go on and almost nothing on the use of such material in highly disaggregated models. Accordingly in sections 10-14, when I have completed my description of the system and its uses, I shall indicate a few avenues that might be worth exploring in attempting a financial analysis comparable in scope to the real analysis with which we are familiar.

8. The Rest of the World Account

In the revised SNA, the rest of the world is treated in the usual summary manner: it is provided with a single account which brings together the external transactions of the economy we are studying. There is, however, a discussion of regional accounting in which a consolidation of the complete system, in fact the internal national accounts and balance sheet, are set out for a number of regions. As explained more fully in [7] such a system can be simplified in various ways depending on the amount of information assumed to be available about interregional flows.

The obvious application of such an extension of a national system is to regional planning; without it, a national model can say nothing about locative problems. Looking a little to the future, similar ideas could be used to provide an integrated picture of a region defined as a set of countries. The uniform input-output tables recently published by the European Economic Community [2, 6] should perhaps be regarded as a step in this direction.

9. Revaluations and Balance Sheets

In venturing into the area of revaluations and balance sheets, the revised SNA has done something to correct a serious imbalance in the development of social accounting: the concentration on flows to the exclusion of stocks. Two generations ago there was not much difference in the attention paid to the national capital and to the national income. In the tremendous upsurge of interest that has followed the introduction of national accounting, work on the national capital has been left far behind. The reason for this change of emphasis can hardly be attributed to great disparities in available data or to the much greater conceptual difficulties in accounting for stocks compared with accounting for flows, though these factors may have played a part. The main reason is probably a greater concern, in the immediate postwar years, with problems of production and employment than with problems of finance, coupled with the belief that at the national level financial problems solve themselves. However this may be, the neglect of wealth has had some unfortunate results. At the level of statistical compilation, a full set of accounting checks has not been available with the result that many flows and, in particular, saving have not been measured as accurately as they might have been. At the level of econometrics, great efforts have been made to devise consumption functions which do not depend explicitly on wealth as well as income, as if economic systems were always in a steady state with wealth in a fixed ratio to income. In general, the description of the economic system has been left incomplete so that analyses of it are necessarily incomplete too.

In the revised SNA, balance sheets relate to sectors. Assets are divided into four classes: (i) reproducible tangible assets, which have entered the balance sheet through capital formation in the past and which appear in it at written down replacement cost; (ii) non-reproducible tangible assets, the most important items being land and mineral wealth, which are valued, as far as possible, at market value; (iii) intangible assets matched by liabilities, that is financial claims of all kinds, which are valued, as far as possible, at market values whether they are held as an asset or as a liability; and (iv) other intangible assets, such as goodwill, which represent the excess of the value of an economic unit as a going concern over the sum of the preceding items. Since items (i), (ii) and (iii) are all valued at current replacement cost or market value, it is proposed that (iv) should be omitted, at least in the first instance.

The liabilities that match these assets take two main forms: (i) financial claims held as liabilities (corresponding to (iii) above); and (ii) accumulated saving, including net capital transfers received, which can be regarded as liabilities to self. This last item is the only entry in the balance sheet which cannot, even in principle, be revalued directly; its revaluation must be deduced, therefore, from the revaluations of tangible assets and of claims.

The introduction of balance sheets offers great possibilities to model builders. I have mentioned the importance of a term in wealth in the consumption, or saving, function [3, 9] and this can be illustrated further with the help of the little model in section 2 above.

If we assume income to grow exponentially, so that $\dot{\mu}/\mu = \rho$ say, then, apart from a transient term which will appear if initial income and wealth are not in balance, the saving-income ratio given by (2.1) is

$$\sigma/\mu = \beta\rho/(\alpha + \rho) \tag{9.1}$$

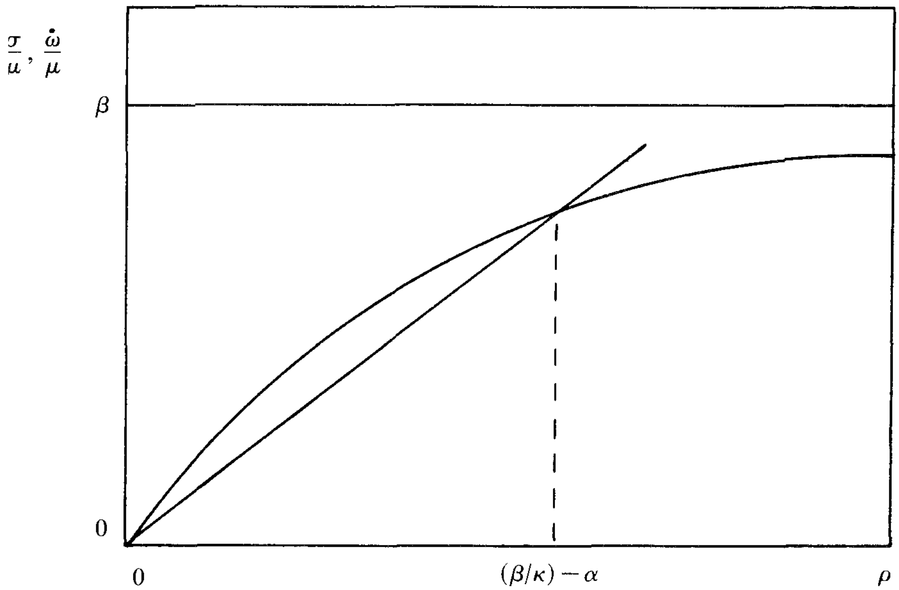
By dividing (2.2) by μ we can see that the corresponding investment-income ratio is

$$\dot{\omega}/\mu = \kappa\rho \tag{9.2}$$

By equating (9.1) and (9.2) we can see that either $\rho = 0$ or $\rho = (\beta/\kappa) - \alpha$ as before. Thus we can connect the growth rate with the saving supply and investment demand as follows.

DIAGRAM

SAVING AND INVESTMENT RATIOS AND THE RATE OF GROWTH OF INCOME



On the simple assumptions of the model, the saving ratio is related to the growth rate by the curve and the investment ratio is related to it by the straight line. An economy rigidly governed by these relationships will either be stationary or will grow at the rate $(\beta/\kappa)-\alpha$. If, for example, $\beta=0.25$, $\kappa=2.5$ and $\alpha=0.07$ the growth rate will be 0.03.

But, apart from such simple uses of information from balance sheets, there are the more complicated problems of the structure of the capital market and the forces leading to transactions in financial claims. The problem of portfolio selection has been analysed by Markowitz [4] and notion of preferred portfolio patterns has become familiar through the work of Tobin and Watts [14, 19]. In the following section I shall explore some of the more obvious possibilities for using flow-of-funds and balance sheet data for the purposes of model building. It is not too soon to consider how this kind of information could be used and, in particular, to see if it is at all amenable to the kind of manageable simplifications that have proved so useful in studying the flow of products. Even if the results presented here are not very impressive, we shall at least have developed some techniques and recognised a few blind alleys.

10. MODELLING THE FINANCIAL SYSTEM

The models I shall describe in the sections that follow relate to many sectors and many claims. They can conveniently be set out in a matrix notation of the kind that has become familiar to economists through input-output analysis. Before proceeding, it may be convenient if at this point I summarise the main conventions on which my notation is based.

(a) In almost every case a capital letter denotes a matrix. The exceptions, mainly Greek capitals, are the operators: Σ for summation; Π for forming a product; E for shifting a variable (thus $E^{\theta}A(\tau) \equiv A(\tau + \theta)$; and Δ for forming first differences (thus $\Delta \equiv E - 1$).

(b) Small roman letters denote vectors. These are written as column vectors: a row vector is written with a prime superscript, as is the transpose of a matrix. The letter i is used to denote the unit vector: that is $i = \{1, 1, \ldots, 1\}$ where $\{\ \}$ denotes that the elements of a column vector are written out in a row. With one exception, diagonal matrices are denoted by a symbol for a vector surmounted by a circumflex accent. The exception is the familiar I, used in place of $\hat{i}$ to denote the unit matrix.

(c) Small Greek letters denote scalars.

(d) In some cases, for example in section 12, it is easy to become confused about the dimensionality of the different symbols. In order to avoid this as far as possible, I have added subscripts: j to denote the typical sector and k to denote the typical claim. Thus A_{jk}, I_{jj} and i'_k denote respectively: a matrix whose rows relate to sectors and whose columns relate to claims; the unit matrix of order equal to the number of sectors; and the unit row vector with elements equal in number to the number of claims.

With these conventions the nature of the different symbols soon becomes apparent and scalar and matrix algebra can be mixed without confusion. With this digression let us return to the question of financial models.

If we are looking for fairly constant ratios in this general area we should expect to find them in the composition of assets and liabilities in the balance sheets of the various sectors rather than in the corresponding year-to-year transactions. For example, if we take the invested assets measured at book value of British insurance companies, we find that the broad composition remained fairly constant over the years 1955 to 1963 except that there was a tendency for mortgages to rise in importance and there was a considerable shift from British government securities to ordinary shares. These movements had largely come to an end by 1960 and the portfolio patterns for 1961, 1962 and 1963 were all very alike. Over the same period the composition of balance sheet changes were altogether more variable. This seems to be a fairly general phenomenon and suggests that the estimation of changes in preferred portfolio patterns may be no more difficult than the estimation of changes in input-output coefficients. Equally, though there may be considerable variation, whole sectors seem to have fairly stable patterns of liabilities; in other words, sectors have typical ways of raising outside finance just as they have typical ways of holding that part of their capital that they devote to financial assets.

Accordingly, let us now set up the balance sheets of a closed economy in a standard matrix form, distinguishing on the assets side between financial assets and real assets and on the liabilities side between liabilities to third parties and liabilities to self, that is accumulated saving. For a system of n sectors and m claims, this can be done as follows.

TABLE 2
A SYSTEM OF SECTOR BALANCE SHEETS IN MATRIX FORM

	n sector statements	m financial claim statements	Real asset/ accumulated saving statement	Row totals
n sector statements		A_{jk}	e_j	w_j
m financial claim statements	L'_{jk}			l_k
Real asset/ accumulated saving statement	z'_j			ζ
Column totals	x'_j	a'_k	ϵ	

In this table the first n row-and-column pairs relate to sectors; each row contains a sector's assets and the corresponding column contains its liabilities. The following m row-and-column pairs relate to financial claims; each row contains the holdings of a particular claim as a liability and the corresponding column contains the holdings of the same claim as an asset. The penultimate row-and-column pair relate to the real assets and accumulated saving in the various sectors and the final row-and-column pair relate simply to totals.

The degrees of freedom of this system are easily calculated. By simply counting the symbols we can see that there are $2nm + 4n + 2m + 2$ variables. These are connected by $2n + 2m + 2$ independent arithmetic identities corresponding to the fact that the entries in each row and column add up to the relevant total, and by $n + m$ independent accounting identities since, in a closed system of $n + m + 1$ balancing statements, $n+m$ of these statements are independent. The number of degrees of freedom is, therefore, $2nm+n-m$.

If we apply the first-difference operator, Δ, to the entries in the table, we shall obtain the flows in the system for the accounting period following the date for which the balance sheets were drawn up. ΔA_{jk} and $\Delta L'_{jk}$ contain the flow-of-funds information for this period, the elements of Δe_j are the investments of the sectors in real assets and the elements of $\Delta z'_j$ are the saving by the sectors in the period. The application of Δ does not change either the number of variables or the number of independent identities and so in the flow matrix the number of degrees of freedom remain as stated above.

In what follows I shall assume that we know Δe_j, the sectors' investment programmes, and $\Delta z'_j$, the sectors' intended saving, and I shall examine possible methods of calculating ΔA_{jk} and $\Delta L'_{jk}$. On this basis we now have $2n$ additional variables assumed to be known, and so that the degrees of freedom are reduced to $2nm - n - m$, that is to $n(m-1) + m(n-1)$.

11. Exogenous and Endogenous Variables

I have just said that I propose to take the sectors' investment programmes and their saving intentions as actual or potential exogenous variables, leaving the acquisition and issue of different claims by the various sectors as endogenous variables. The reason for this is that I can imagine the investment programmes to come from another part of the model dealing with continued production and I can imagine saving intentions coming from another part of the model dealing with spending and saving behaviour. In a balanced model, total investment would equal total saving and so we should be left, at this stage, with the creation of new assets and liabilities which would enable the financial capital accounts to balance by sector as well as in total. I realise, of course, that all these aspects of the general economic process are interdependent and should, ideally, be solved simultaneously. I see no harm, however, in treating the different aspects separately in the first instance but it must be recognised that by dividing the problem in this way I leave the repercussions of financing considerations on the saving and investment of individual sectors for separate treatment later on.

The scope of the exogenous variables I have chosen is, I think, clear but may well not be the most convenient; it might be desirable to include some of the transactions in financial claims among the exogenous variables. Thus, if we are estimating saving, we might at the same time estimate certain types of contractual saving, such as saving through life insurance. If we did this it would seem logical to move the financial claim 'liability to policy holders' into the exogenous part of the acquisition of assets by individuals and into the exogenous part of the issue of additional liabilities by insurance companies.

12. Financial Input-Output

The arrangement of the balance sheet entries in the table of section 10, which would be more in conformity with the arrangement in the revised SNA if it were transposed, bears a superficial formal resemblance to the account entries in an input-output system where a distinction is made between commodities and industries. Thus, with a change of headings, we could identify e_j with final demands, z'_j with primary inputs, L'_{jk} with intermediate product flows and A_{jk} with the mix of commodity outputs by the different industries. The discrepant element in the analogy is that in the input-output case we should find not e_j but e_k: final demands would be expressed as demands for commodities not as demands on industries.

In the case of input-output we know that the distinction between industries and commodities leads to the possibility of alternative formulations, as described in [1]. The same is true in the present case. Thus suppose we were to define two coefficient matrices, A^{**}_{jk} and L'^{*}_{jk} as follows:

$$A^{**}_{jk} = A_{jk}\,\hat{a}_k^{-1} \qquad (12.1)$$

and

$$L'^{*}_{jk} = L'_{jk}\,\hat{x}_j^{-1} \tag{12.2}$$

By premultiplying (12.1) by i'_j we can see that the column sums of A^{**}_{jk} are all equal to 1; and so (12.1) adds $m(n-1)$ new independent relationships. Similarly, by premultiplying (12.2) by i'_k we can see that the column sums of L'^{*}_{jk} are all equal to 1 minus the proportion of the capital of one of the sectors that is represented by that sectors' accumulated saving; and so (12.2) adds $n(m-1)$ new independent relationships. Thus we have just enough new relationships to enable us to construct a fully determined model. This model takes the form:

$$w_j = A^{**}_{jk}\, a_k + e_j \tag{12.3}$$

$$l_k = L'^{*}_{jk}\, x_j \tag{12.4}$$

$$w_j \equiv x_j \tag{12.5}$$

$$a_k \equiv l_k \tag{12.6}$$

from which it follows that

$$\begin{aligned} w_j &= A^{**}_{jk}\, L'^{*}_{jk}\, w_j + e_j \\ &= (I_{jj} - A^{**}_{jk}\, L'^{*}_{jk})^{-1}\, e_j \end{aligned} \tag{12.7}$$

and

$$\begin{aligned} l_k &= L'^{*}_{jk}\,(I_{jj} - A^{**}_{jk}\, L'^{*}_{jk})^{-1}\, e_j \\ &= (I_{kk} - L'^{*}_{jk}\, A^{**}_{jk})^{-1}\, L'^{*}_{jk}\, e_j \end{aligned} \tag{12.8}$$

If e_j is taken from the table from which the coefficient matrices were calculated, (12.7) and (12.8) will exactly reproduce w_j and l_k in that table and, by the use of the other relationships specified, the whole of the table can be filled in. Similarly, if e_j is replaced by Δe_j a completely balanced flow table will be obtained.

This flow table may not be very acceptable because of the highly restrictive assumptions of the model. On the liabilities side, the model reproduces in detail for each sector the pattern of financing, including self-financing from accumulated saving, as it appeared in the original table. The total of each kind of financial claim is obtained by summing liabilities to third parties, the elements of L'_{jk}, over sectors. These claims are then assumed to be held by the various sectors in fixed proportions, the proportion relevant to a particular claim being given by the elements in the relevant column of A^{**}_{jk}.

It is not very difficult to criticise this model. Bearing in mind that it is intended to apply to sectors and not to the individual transactors of which sectors are composed, it is probably a point in its favour that it recognises normal patterns in the issue by the various sectors of liabilities to third parties, the relative size of the elements in each of the columns of L'^{*}_{jk}. It seems much less reasonable to assume: (i) that self-financing through accumulated saving will have the same relative importance, sector by sector, in this year's investment programme as it has had, on the average, in all past investment programmes; and (ii) that sectors will be deflected away from the normal composition of their holdings of financial assets merely by the consideration that some one has to hold the claims that they and others choose to issue.

We shall now see that if we try to meet these criticisms by an alternative formulation of the model, we can only do so by destroying what I have described as its more reasonable feature and by replacing one set of dubious assumptions by another set.

The alternative model is as follows. Let us define two coefficient matrices, A'^{*}_{jk} and L^{**}_{jk} as follows:

$$A'^{*}_{jk} = A'_{jk}\,\hat{w}_j^{-1} \tag{12.9}$$

and

$$L^{**}_{jk} = L_{jk}\,\hat{l}_k^{-1} \tag{12.10}$$

Like (12.1) and (12.2) above, (12.9) and (12.10) add $n(m-1)+m(n-1)$ new independent relationships. The resulting model is therefore fully determined and takes the form

$$x_j = L^{**}_{jk}\,l_k + z_j \tag{12.11}$$

$$a_k = A'^{*}_{jk}\,w_j \tag{12.12}$$

$$x_j \equiv w_j \tag{12.13}$$

$$a_k \equiv l_k \tag{12.14}$$

from which it follows that

$$\begin{aligned} x_j &= L^{**}_{jk}\,A'^{*}_{jk}\,x_j + z_j \\ &= (I_{jj} - L^{**}_{jk}\,A'^{*}_{jk})^{-1}\,z_j \end{aligned} \tag{12.15}$$

and

$$\begin{aligned} a_k &= A'^{*}_{jk}\,(I_{jj} - L^{**}_{jk}\,A'^{*}_{jk})^{-1}\,z_j \\ &= (I_{kk} - A'^{*}_{jk}\,L^{**}_{jk})^{-1}\,A'^{*}_{jk}\,z_j \end{aligned} \tag{12.16}$$

The story now repeats itself with z_j in place of e_j. If z_j is taken from the table from which the coefficient matrices were calculated, (12.15) and (12.16) will exactly reproduce x_j and a_k in that table and the whole of the table can be filled in; and if z_j is replaced by Δz_j a completely balanced flow table will be obtained.

The two reproductions of the original table will, of course, be the same but, in general, the two flow tables will be different. This will happen unless the exogenous elements of the flow tables are equal to the corresponding elements in the original table multiplied by a constant. Since we can always write

$$\Delta e_j = \hat{b}_j\,e_j \tag{12.17}$$

and

$$\Delta z_j = \hat{c}_j z_j \tag{12.18}$$

where b_j and c_j are vectors of constants, the equality of the two flow tables is equivalent to the condition that

$$b_j = c_j = \alpha i_j \tag{12.19}$$

where α is a constant.

The assumptions of the second model are different from those of the first but equally restrictive. On the assets side, the model reproduces in detail for each sector the pattern of asset holding, whether these assets be real or financial, as it appeared in the original table. The total of financial claims is obtained by summing financial assets, the elements of A_{jk}, over sectors. These claims are then assumed to be issued by the various sectors in fixed proportions, the proportions relevant to a particular claim being given by the elements in the relevant column of L^{**}_{jk}.

With a suitable change of words, the criticism of the second model follows the same pattern as the criticism of the first model.

The first model produces a balanced flow table which accords with our initial assumptions about the investment programmes of the different sectors; and the second model does the same in respect of our initial assumptions about the sectors' saving intentions. If we denote these two tables by T^* and T^{**}, we could always obtain a third balanced flow table, T say, defined as

$$T = \lambda T^* + (1 - \lambda) T^{**} \tag{12.20}$$

where λ is a constant. As we increase λ from zero to one we shall give less and less weight to our initial assumptions about saving intentions and more and more weight to our initial assumptions about investment programmes.

In a preliminary and formal treatment like the present one it does not seem very useful to go into practical details such as how to treat an open economy, what to do with capital transfers or where to put many forms of land and buildings, in particular, dwellings, shops and offices, which, though real assets, have many of the properties of financial assets. But it does seem worthwhile to make a few points about the idea of normal coefficients in the financial sphere. First, these must clearly be derived from holdings, that is from balance sheets, and not from transactions, that is from accounts. Second, there is no reason why coefficients should be derived unchanged from actual past balance sheets. Just as we know that input-output coefficients change, so we know that portfolio patterns change too; in the preceding section we saw an example of this in the gradual switch from British government securities to industrial ordinary shares. Third, there is the whole question of current versus constant prices. As in input-output, though for different reasons, it would seem best to regard normal coefficients as leading to estimates at constant prices which have then to be transformed to current prices.

Although the point is perhaps an obvious one, what has emerged from this section is that the structure of assets and liabilities is so interconnected that the use of enough fixed coefficients to yield a fully determined model leads to excessive rigidity. Let us now, therefore, explore the possibilities of systematically changing the coefficients in the interests of achieving a balance while at the same time ensuring, as far as possible, that these coefficients relate to the preferences of sectors rather than to the proportions in which different financial assets are held and different financial liabilities are issued.

13. Preferred Portfolio Patterns

Let us begin with the sectors' holdings of assets, real and financial, as set out in the first n rows of the table in section 10 above, and let us premultiply these rows by the diagonal matrix $\hat{b}_j$ defined in (12.17) of the preceding section. The result of the operation can be written in the form

$$\begin{aligned} \Delta w_j &= \hat{b}_j w_j \\ &= \hat{b}_j A_{jk} i_k + \hat{b}_j e_j \\ &= \Delta A_{jk} i_k + \Delta e_j \end{aligned} \tag{13.1}$$

Using the symbol $\mathrm{E} \equiv 1 + \Delta$ we can write out the equation for the stocks of assets held by the various sectors at the end of the period. Thus

$$\begin{aligned} \mathrm{E} w_j &= (I_{jj} + \hat{b}_j) w_j \\ &= (I_{jj} + \hat{b}_j) A_{jk} i_k + (I_{jj} + \hat{b}_j) e_j \\ &= \mathrm{E} A_{jk} i_k + \mathrm{E} e_j \end{aligned} \tag{13.2}$$

If we could now accept $A'_{jk}(i_j + b_j)$ as an estimate of $\mathrm{E} a_k$, then by the RAS technique [1, 8], we could try to construct an estimate of $\mathrm{E} L'_{jk}$ of the form

$$\mathrm{E} L'_{jk} = \hat{r}_k L'_{jk} \hat{s}_j \tag{13.3}$$

where r_k and s_j are vectors of constants, such that

$$\begin{aligned} \mathrm{E} L'_{jk} i_j &= \mathrm{E} l_k \\ &= \mathrm{E} a_k \end{aligned} \tag{13.4}$$

and

$$\begin{aligned} \mathrm{E} L_{jk} i_k &= \mathrm{E} x_j - \mathrm{E} z_j \\ &= \mathrm{E} w_j - \mathrm{E} z_j \end{aligned} \tag{13.5}$$

where $\mathrm{E} w_j$ is given by (13.2) and $\mathrm{E} z_j = z_j + \Delta z_j$ embodies our initial estimates of the intended saving of the various sectors. If every sector issued a wide variety of claims we should undoubtedly succeed since we know that, subject to very general conditions, the RAS technique converges to a unique solution when applied to non-negative matrices. The small number of types of claim issued by some sectors may, however, lead to a singular situation to which the technique cannot be applied. The problem is apparent if we consider a sector that issues only a single type of claim. In this case, a particular amount will be needed to balance the sectors' balance sheet but this may not equal the amount of this type of claim that other sectors, following their preferred portfolio patterns, would wish to hold at the end of the accounting period. Since the amount of the holding is fixed there may simply be no way to balance the sector's balance sheet.

If we are successful, we should obtain a completely balanced set of closing balance sheets and holdings of claims as assets and as liabilities. By subtracting the elements of our original table from the corresponding elements of this table, we should obtain a completely balanced system of flows. This system of flows would respect the sectors' investment programmes and also their saving intentions. Each sector would add to its portfolio of assets according to its preferred pattern and what have been made to give in order to balance the system are the quantities of different financial claims issued by the various sectors.

In this case, just as with the input-output models of the preceding section, there is an alternative way of setting up the model. In this alternative, we start with the sectors' liabilities at the end of the period, rather than with their assets, and form the equation

$$\begin{aligned} \mathrm{E}x_j &= (I_{jj} + \hat{c}_j)\, x_j \\ &= (I_{jj} + \hat{c}_j)\, L_{jk}\, i_k + (I_{jj} + \hat{c}_j)\, z_j \\ &= \mathrm{E}L_{jk}\, i_k + \mathrm{E}z_j \end{aligned} \tag{13.6}$$

where the diagonal matrix $\hat{c}_j$ is defined in (12.18).

If we could accept $L'_{jk}\,(i_j + c_j)$ as an estimate of $\mathrm{E}l_k$ then by the RAS technique we could try to construct an estimate of $\mathrm{E}A_{jk}$ of the form

$$\mathrm{E}A_{jk} = \hat{p}_j A_{jk}\, \hat{q}_k \tag{13.7}$$

where p_j and q_k are vectors of constants, such that

$$\begin{aligned} \mathrm{E}A'_{jk}\, i_j &= \mathrm{E}a_k \\ &= \mathrm{E}l_k \end{aligned} \tag{13.8}$$

and

$$\begin{aligned} \mathrm{E}A_{jk}\, i_k &= \mathrm{E}w_j - \mathrm{E}e_j \\ &= \mathrm{E}x_j - \mathrm{E}e_j \end{aligned} \tag{13.9}$$

where $\mathrm{E}x_j$ is given by (13.5) and $\mathrm{E}e_j = e_j + \Delta e_j$ embodies our initial estimates of the sectors' investment programmes.

In this case we are more likely to be successful because sectors hold varied portfolios of financial assets and it is the composition of these portfolios that is made to give in order to achieve a balance.

If we were successful we should again obtain a completely balanced set of closing balance sheets and holdings of claims as assets and as liabilities.

By subtraction we could again reach a completely balanced system of flows.

If we denote the two tables of flows by T^* and T^{**}, we can see that any linear combination of these tables, as in (12.20), would also be a balanced table and, moreover, a table that respected the sectors' investment programmes and their saving intentions. As we increased λ from zero to one, we should be insisting less and less on sectors' preferred patterns in the holding of assets and more and more on their preferred patterns in the holding of liabilities. At the same time changes would be taking place in the extent to which the wealth (or capital) of the different sectors increased and in the total amount of new financial claims.

In applying the RAS technique we are at liberty to fix some of the entries at the outset, adjust the remainder and add back the entries that have been fixed so as to obtain a complete table. This procedure would be likely to alter some of the account totals and so lead to adjustments in both $\mathrm{E}L'_{jk}$ and $\mathrm{E}A_{jk}$. It would be useful in such a case to have a convergent, iterative procedure, a kind of *tâtonnement* that would simulate the adjusting mechanism of the market itself. At the moment I have no practical suggestions as to how this might be obtained.

Some readers may have been bothered by a purely technical point in the description of the models. Why, it may be asked, do we estimate the end-period balance sheets and then obtain the flow tables by difference instead of estimating the flows directly; in other words why do we begin by multiplying the initial balance sheets by E rather than by Δ. The reason is that the flow tables may contain negative elements and the RAS technique is applicable to matrices with non-negative elements. This means that we should begin by applying E rather than Δ to the initial tables. Though at first sight it might appear otherwise, it is not a matter of indifference which we do since, if we accept (13.3), then

$$\Delta L'_{jk} = \hat{r}_k L'_{jk} \hat{s}_j - L'_{jk} \tag{13.10}$$

that is $\Delta L'_{jk}$ is not related to L'_{jk} by an RAS transformation.

14. Programming and the Capital Market

The portfolio models of the last section, though less rigid than those of the section preceding it, are still, perhaps, too rigid for practical purposes. A step in the right direction might be, therefore, to allow portfolio patterns to change in response to market forces rather than to change in a mechanical and one-sided way to meet arithmetic and accounting identities. I cannot claim to have made much progress with this problem but the formulation that follows may show the kind of consideration that we have to take into account.

Consider a matrix, C say, with elements c_{ghk}, where g and h relate to typical sectors and k denotes a typical kind of claim. This matrix shows transactions in claims in a wholly disaggregated form: c_{ghk} denotes the money received by sector g from sector h in respect of parting, either by issue or sale, with a claim of type k. We can conveniently think of the main subsets of row-and-column pairs in this matrix as relating to sectors and the elements of each of these subsets as relating to a particular claim. Let us now see if we can devise a linear programme that will determine, or at least circumscribe, the elements in this matrix.

Linear programming consists of maximising, or minimising, a linear function of certain variables (the objective function) subject to linear inequalities (the constraints). Let us consider: first, what constraints could reasonably be put on the c_{ghk}; and, second, what function of these elements could reasonably be maximised or minimised.

There are certainly five types of constraint to be considered.

(a) Since there is no netting, all transactions are non-negative. That is

$$c_{ghk} \geqslant 0 \tag{14.1}$$

for all g, h and k.

(b) Since no-one can part with something he does not possess, there is an upper bound on the sales of each type of asset by each sector. That is

$$\sum_j c_{gjk} \leqslant \bar{c}_{g \cdot k} \tag{14.2}$$

where j denotes a typical sector, for all g and at least for all k which are not issued as a liability by sector g.

(c) Since no-one can sell without someone else buying, the total amount of each claim sold (or issued) by all sectors must equal the total amount bought (or acquired) by all sectors. That is

$$\sum_g \sum_j c_{gjk} = \sum_j \sum_h c_{jhk} \tag{14.3}$$

for all k.

(d) Since the account for each sector balances, the financial assets acquired plus the financial liabilities redeemed plus the real investment of a sector are equal to the financial assets sold plus financial liabilities issued plus the saving of the same sector. That is

$$\sum_j \sum_k c_{gjk} + \Delta e_g = \sum_j \sum_k c_{jgk} + \Delta z_g \tag{14.4}$$

for all g.

(e) Since sectors have to consider not only the present but also the uncertainties of the future, there are upper and lower bounds to the changes in the composition of their holdings of financial assets and liabilities that they can allow to result from the transactions of a single period. That is

$$a_{g \cdot m} \sum_j \sum_m c_{gjm} \leqslant \sum_j c_{gjm} - \sum_j c_{jgm} \leqslant b_{g \cdot m} \sum_j \sum_m c_{gjm} \tag{14.5}$$

where $a_{g \cdot m}$ and $b_{g \cdot m}$ are lower and upper bounding proportions for claim m, held as a liability by sector g, and

$$a_{\cdot gn} \sum_j \sum_n c_{jgn} \leqslant \sum_j c_{jgn} - \sum_j c_{gjn} \leqslant b_{\cdot gn} \sum_j \sum_n c_{jgn} \tag{14.6}$$

where $a_{\cdot gn}$ and $b_{\cdot gn}$ are lower and upper bounding proportions for claim n, held as an asset by sector g. For the degree of aggregation of claims that we are likely to use in this kind of analysis, it is probable that (14.6) will dominate (14.2): normal patterns will be violated before the complete holding of a particular claim is disposed of.

We come now to possible objective functions. From the point of view of borrowers we should like to minimise the cost of the elements in the matrix since this would minimise the total cost of all borrowing. If we assume, for simplicity, that a certain return, r_k say, is received per unit of claim k independently of the borrowing and lending sectors, then we could denote by s_{ghk} the return received by sector h from sector g in respect of the sale (or issue) of a unit of claim k by sector g to sector h. Then

$$s_{ghk} = r_k \, c_{ghk} \tag{14.7}$$

and from the borrowers' point of view we should want to have

$$\sum_g \sum_h \sum_k s_{ghk} = \min \tag{14.8}$$

Correspondingly, from the lenders' point of view, we should want to have

$$\sum_g \sum_h \sum_k s_{ghk} = \max \tag{14.9}$$

For a fixed vector $\{r_k\}$, the solution of (14.8) and (14.9) subject to (14.1) through (14.6) would represent the position if (i) borrowers and (ii) lenders were completely dominant in the market. A linear combination of these solutions with positive weights would represent an intermediate position in which neither borrowers as a whole nor lenders as a whole had their way completely.

Finally, we might seek a vector, $\{r_k^*\}$ say, such that, if it were in force, the solutions of (14.8) and (14.9), subject to the constraints, would coincide. This would imply a set of rates of return on different claims at which the interests of borrowers and lenders would be the same.

For the present, I shall not try to go further with this type of model; perhaps I have said enough to show the kind of thing that is involved if we want to get away from the rigidities of the models described in the preceding sections.

15. Demographic Accounting

A matter of great importance to users, which is being considered in connection with the revision of the SNA but on which recommendations have not yet been formulated, is the provision of demographic information which can be properly compared with the information in the social accounts. Not only the population itself and the labour force but the distribution of these totals over households, occupations and industries have often to be related to accounting data and it is very difficult to do this satisfactorily if the two types of information come from more or less unrelated publications.

As I have shown in greater detail in [12], the organisation of demographic information, which usually involves combining data from a wide range of sources, can be greatly helped by the application of accounting ideas. In accounting for people, as in accounting for money, it is useful to have a framework that provides a place for all the information needed and imposes as many arithmetical checks as possible. One way to do this is to trace the population in different age groups and activities from one year to the next. Thus if we start with those born in a particular year, some will die in the year and the remainder will go on to be one year olds in the following year. If we consider the one year olds, some, again, will die and the remainder will go on to be two year olds in the following year. Whereas the one year olds were all at home, some of the two year olds go to nursery school. We now have to follow the future course of two streams: the children based on the home and the children based on the nursery school. At age three, those in school will tend to stay there and their numbers will be increased by children going to school for the first time at age three. When the age of compulsory attendance at school is reached we shall find virtually the whole population of that age at one or other type of school and we can trace their flow through the school system. As they advance in their educational career, we can further subdivide them into arts and science streams, trace them through various stages of further education and, from the age at which education ceases to be compulsory, trace them into various types of employment. Eventually we shall trace them back into the home on retirement and at all ages they may go temporarily or permanently, into some form of institution: asylums, prisons, homes for old people and the like. At any age and in any activity the numbers may be increased by immigration or diminished by emigration.

In this outline, I may have given the impression that we are tracing a particular vintage of births through its life cycle. We could, of course, do this but for many purposes it is more useful to concentrate on a particular year and measure the flows into the various categories at the beginning of the year and the corresponding outward flows at the end of the year. If we consider three adjacent years we can represent the accounting structure as follows.

TABLE 3
DEMOGRAPHIC ACCOUNTS IN MATRIX FORM

	b'_0	b'_1	b'_2	β
d_0		P_{01}		n_0
d_1			P_{12}	n_1
d_2				n_2
δ	n'_0	n'_1	n'_2	

In this table the suffixes 0, 1, 2 relate to three successive years. The first row and column pair with the row vectors b'_θ in the row and the column vectors d_θ in the column represents the world outside the society we are considering and is the source of births and immigrants and the destination of deaths and emigrants. It is not strictly an account; in general, it does not balance for limited periods but only asymptotically as the time period is increased indefinitely.

The inflow into a year, say year 1, is made up to two parts: births and immigration, which appear in b'_1 and the carry in from the preceding year which appears in the submatrix P_{01}. Similarly the outflow is made up of two parts: deaths and emigration which appear in d_1 and the carry out into the following year which appears in the submatrix P_{12}. Since inflow into each category is equal to outflow in each category, we have

$$\begin{aligned} n_1 &= b_1 + P'_{01} i \\ &= d_1 + P_{12} i \end{aligned} \tag{15.1}$$

In this method of accounting, we consider only flows over year ends. Since we are interested in other categories as well as age, we might wish to represent a change of category within the year. This can be done by introducing a submatrix, P_{11} say, at the intersection of the rows and columns for period I. With this addition (15.1) becomes

$$\begin{aligned} n_1 &= b_1 + P'_{01} i + P'_{11} i \\ &= d_1 + P_{11} i + P_{12} i \end{aligned} \tag{15.2}$$

Comparing (15.1) and (15.2), we must recognise that while $i'n_1$ and $i'd_1$ are the same in the two representations, their elements are, in general, different.

This method of setting out demographic data makes it possible to connect future states of the system with its present state. The entries in row j of any P-matrix say $P_{\theta(\theta+1)}$ show the distribution by category at the outset of year $\theta+1$ of individuals who were in category j at the end of year θ. As a first approximation, it would seem reasonable to assume that these distributions remain constant over time. Accordingly, let us put

$$\hat{n}_\theta^{-2} P_{\theta(\theta+1)} = C' \tag{15.3}$$

for all values of θ. The elements of C' are non-negative constants and the row sums of C' are less than one, since in each year there are some deaths in each category.

From (15.3) and the first row of (15.1) we can write

$$\begin{aligned} n_1 &= b_1 + P'_{01} i \\ &= b_1 + C n_0 \end{aligned} \tag{15.4}$$

or, in general, using an operator rather than a suffix notation,

$$\mathrm{E}n = \mathrm{E}b + C n \tag{15.5}$$

where E^{θ} applied to a variable transforms that variable into its value θ years later. If we multiply (15.5) by E, substitute for En in the new equation from (15.5) and repeat this operation $\tau-1$ times, we obtain

$$E^{\tau}n = \sum_{\theta=0}^{\tau-1} C^{\theta} E^{\tau-\theta} b + C^{\tau} n \tag{15.6}$$

which expresses the numbers in the various categories τ periods hence in terms of the present number and future births and immigration up to and including those that take place in period τ. In view of the properties of C (which are those normally associated with an input-output coefficient matrix) it follows that C^{τ} approaches zero as τ increases. Eventually, the initial conditions, n, become irrelevant and everything depends on births and immigration, $E^{\theta}b$ for $\theta=\tau, \tau-1, \ldots$.

In practice, we are likely to find that the elements of C change through time; indeed, if they do not change naturally, we may be interested in discovering how to change them so as to bring about, let us say, a more highly educated population [10]. In any case, if we can form an opinion of how the elements of C are likely to change, we can substitute $E^{\theta}C$ for C at time θ and go through the steps described above to connect (15.5) and (15.6). In this case (15.6) is replaced by

$$E^{\tau}n = E^{\tau}b + \sum_{\theta=1}^{\tau-1} \left(\prod_{\lambda=\tau-\theta}^{\tau-1} E^{\lambda} C \right) E^{\tau-\theta} b + \left(\prod_{\theta=0}^{\tau-1} E^{\theta} C \right) n \tag{15.7}$$

where Π denotes the operation of forming a product over the range indicated.

If (15.1) is replaced by (15.2) then (15.5) is replaced by

$$\begin{aligned} En &= Eb + Cn + DEn \\ &= A(Eb + Cn) \end{aligned} \tag{15.8}$$

where

$$\hat{n}_{\theta}^{-1} P_{\theta\theta} = D' \tag{15.9}$$

and

$$A = (I-D)^{-1} \tag{15.10}$$

With this change, (15.6) is replaced by

$$E^{\tau}n = \sum_{\theta=0}^{\tau-1} (AC)^{\theta} A E^{\tau-\theta} b + (AC)^{\tau} n \tag{15.11}$$

and the expression corresponding to (15.7) can easily be derived.

Thus we see that the use of an accounting structure not only helps us in the compilation of coherent demographic information but also provides a framework within which we can model many aspects of demographic change.

16. Conclusion

This completes my account of the revised SNA and the main ideas that it is designed to bring together and integrate into a coherent system. It can be seen that the revised version goes far beyond the original. It aims at completeness but at the same time is flexible and firmly linked to the familiar aggregates of national accounting. My treatment of the different topics is based on the conceptual framework of the revised system; I have not attempted to describe the tables to be extracted from this framework for purposes of international reporting, the precise classifications to be used or the frequency with which reports on different details of the system would be desirable.

I have tried to look at the revised system from the point of view of a particularly demanding user whose aim is nothing less than a detailed model of the economic system in all its aspects. I have not discussed the relationships that might be used in the more familiar parts of the model: consumptions functions, input-output relationships and so on. But I have examined some of the possibilities in attempting to model the financial aspect of the economic process and I have tried to reflect the growing view, frequently expressed by the architects of the revised SNA, that economic analysis requires in many cases demographic and social analysis as well. It can fairly be claimed, I think, that in the revision of the SNA the views of consumers as well as of producers are adequately represented.

17. A List of Works Cited

1. Cambridge, Department of Applied Economics. *Input-Output Relationships, 1954-1966*. No. 3 in *A Programme for Growth*. Chapman and Hall, London, 1963.
2. European Economic Community: Statistical Office. *Input-Output—Tabellen für die Länder der Europäischen Wirtschaftgemeinschaft*. E.E.C., Brussels, 1964.
3. Lydall, H. F. Saving and wealth. *Australian Economic Papers,* vol. 2, no. 2, Dec. 1963, pp. 228-50.
4. Markowitz, Harry M. *Portfolio Selection*. Cowles Foundation, monograph 16, John Wiley, New York, 1959.
5. Pyatt, G. A production functional model. In *Econometric Analysis for National Economic Planning,* Butterworths, London, 1964.
6. Schumacher, H. Das input-output-system des Statistischen Amtes der Europäischen Gemeinschaften. *Statistische Informationen,* no. 2, 1964, pp. 13-36.
7. Stone, Richard. Social accounts at the regional level: a survey. In *Regional Economic Planning: Techniques of Analysis,* O.E.E.C., Paris, 1961.
8. Stone, Richard. Multiple classifications in social accounting. *Bulletin de l'Institut International de Statistique,* vol. xxxix, no. 3, 1962, pp. 215-33.
9. Stone, Richard. Private saving in Britain, past, present and future. *The Manchester School of Economic and Social Studies,* vol. xxxii, no. 2, 1964, pp. 79-112.

10. Stone, Richard. A model of the educational system. *Minerva,* vol. III, no. 2, 1965, pp. 172-86.

11. Stone, Richard. Consistent projections in multi-sector models. To be published by the International Economic Association.

12. Stone, Richard. Input-output and demographic accounting: a tool for educational planning. *Minerva,* vol. IV, no. 3, 1966, pp. 365-80.

13. Stone, Richard, and Colin Leicester. The methodology of planning models: a preliminary survey. To be published.

14. Tobin, James. The theory of portfolio selection. In *The Theory of Interest Rates,* Macmillan, London, 1965.

15. United Nations, Economic and Social Council. *A System of National Accounts (Proposals for Revision of SNA, 1952).* E/CN.3/320, mimeographed, 1965.

16. United Nations, Economic and Social Council. *Statistical Commission: Report of the Thirteenth Session.* United Nations, New York, 1965.

17. United Nations, Statistical Office. *A System of National Accounts and Supporting Tables.* U.N., New York, 1953; *Revision 1,* 1960.

18. United Nations, Statistical Commission and Economic Commission for Europe Conference of European Statisticians. *Report of the Working Group on National Accounts and Balances.* Conf. Eur. Stats./WG.22/7, mimeographed, 1965.

19. Watts, Harold W., and James Tobin. Consumer expenditures and the capital account. In *Consumption and Saving* (ed. Friend and Jones), vol. II, University of Pennsylvania, 1960.

Après une brève introduction, la première partie de cette étude (sections 3-9) résume les revisions proposées au Système de Comptes Nationaux (SCN) des Nations Unies qui sont maintenant en discussion. Ces propositions furent étudiées par un groupe d'experts à la fin de 1964 et acceptées en 1965 par la Commission Statistique des Nations Unies comme base pour les futurs travaux concernant l'extension et la revision du SCN. Le bût de cette revision est de fournir un système complètement intégré de comptes et bilans dans lesquels les flux réels et monétaires, ainsi que les bilans sectoriels sont incorporés dans un cadre comptable généralisé. Tandis que les aspects réels de l'économie ont été etudiés analytiquement dans de nombreux pays (flux réels, analyse de la demande, etc.) l'expérience disponible est beaucoup moindre concernant les aspects financiers de l'économie excepté pour les travaux économétriques sur les épargnes qui ont reçu un développement substantiel. Dès lors, la second partie du papier (sections 10-14) discute la construction de modèles financiers pour lesquels un nombre des possibilités sont explorées. La dernier sujet discuté (section 15) est la comptabilité démographique, c'est-à-dire, un cadre pour l'enregistrement et l'analyse des flux et stocks humains plutôt qu'économiques. Le développement d'un tel système trouve son origine dans l'accent placé par le groupe d'experts sur l'intégration de l'information démographique et économique.